MERGER ANTITRUST LAW

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Georgetown University Law Center
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Tuesdays and Thursdays, 3:30-5:30 pm
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CLASS 11 WRITTEN ASSIGNMENT—INSTRUCTOR'S ANSWER

Instructions

Submit by email by 3:30 pm on Tuesday, October 8
Send to dale.collins@shearman.com
Subject line: Merger Antitrust Law: Assignment for Class 11

Assignment

Calls for short answers. Since the assignment calls for some equations, feel free to write your answers using a pencil and paper (rather than a computer) if that is easier. Attach either a scan or a photograph to your email.

Assumptions: Consider a single firm.

1. The market price is p (whatever that may be)
2. The firm's residual demand function is $q = 100 - 8p$
3. The firm has fixed costs $F = 25$ and constant marginal costs $c = 5$

Questions:

1. Explain the concept of a demand curve. Why is it downward sloping?
2. What is the equation for the firm's inverse demand curve? If $q = 20$, what is the market clearing price?
3. What is the equation for the firm's revenues? What are the firm's revenues when $q = 20$?
4. Explain the concept of marginal revenue and how it relates to gross revenue gains and losses associated with incremental sales. What is the equation for the firm's marginal revenue at a production level q? What is the firm's marginal revenue when $q = 20$?
5. Explain the concepts of total cost, fixed cost, total variable cost, average variable cost, marginal cost, and constant marginal cost. What are the equations for these variables? What are these various costs when q = 20?
6. Explain why the firm maximizes profit when marginal revenue equals marginal cost.
7. What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?

8. Say the firm's fixed costs F decrease to 20. What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?
9. Say the firm's fixed costs F remain at 25 but its (constant) marginal cost c decreases to 4. What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?
10. Explain what the firm should do and why if it finds that marginal revenue is greater than its marginal cost at current production (say because of a shift in demand).

If you have any questions, send me an e-mail. See you in class.

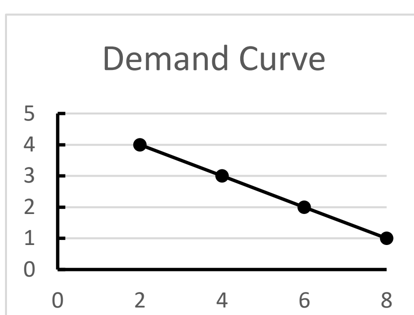
Questions:

1. Explain the concept of a demand curve. Why is it downward sloping?

Every customer has a maximum willingness to pay for the firm's product. Some customers will not value the product very much and so will have a low maximum willingness to pay, while other customers will place greater value on the product and have a higher willingness to pay. Customers will purchase a product only if the product's price at or below the customer's maximum willingness to pay. Accordingly, at low prices, more customers will purchase the product than at higher prices. For example, say there are four customers in the market with the following willingness to pay:

Customer	MWP
A	8
B	6
C	4
D	2

Then one customer would purchase the product at a price of 8, two at 6, 3 at 4, and 4 at two. This traces out a downward-sloping demand curve.



2. **What is the equation for the firm's inverse demand curve? If *q* = 20, what is the market clearing price?**

From the assumptions, the firm's residual demand function is $q = 100 - 8p$. The firm's inverse demand function is the price at which clear the market (for the firm's product) at a production level q. To obtain the firm's inverse demand function, rearrange the demand function to express price p as a function of quantity q:

Demand function:	$q = 100 - 8p$
Add 8p to both sides:	$q + 8p = 100 - 8p + 8p = 100$
Subtract q from both sides	$q + 8p - q = 100 - q$
So	$8p = 100 - q$
Divide both sides by 8:	$p = \frac{100 - q}{8}$
Simplify:	$p = 12.5 - \frac{1}{8}q$

The last expression is the firm's inverse demand function when the demand function is q=100–8p.

If q = 20, then the market-clearing price p is 12.5 – 1/8 * 20 =12.5 – 2.5 = 10.

3. **What is the equation for the firm's revenues? What are the firm's revenues when *q* = 20?**

Revenues r are equal to the price p times the quantity sold q. So $r = pq$.

When q =20, p equals 10 from the inverse demand function. So r = 10*20 = 200.

4. **Explain the concept of marginal revenue and how it relates to gross revenue gains and losses associated with incremental sales. What is the equation for the firm's marginal revenue at a production level *q*? What is the firm's marginal revenue when *q* = 20?**

Technically, marginal revenue is the slope of the revenue curve. It is a function of the quantity q. Heuristically, marginal revenue is the addition revenue the firm earns with the sale of one additional unit of product.

Marginal revenue is closely related to incremental unit revenue. Incremental unit revenue is the revenue the firm earns for selling some additional units of output, whether the number of additional units are large or small. Incremental unit revenue increasingly approximates marginal revenue as the additional number of units becomes smaller compared to the level of total production.[1]

Incremental revenue (and therefore marginal revenue) consists of two parts:

a. The gross gain in revenue due to the sale of the additional product, and
b. The gross loss in revenue due to the fact that the firm, which faces a downward-sloping demand curve, has to lower its price on *all* products by some amount in order to sell the additional unit.

Let q be the original level of sales and $q+1$ be the original sales plus one unit. Let p be the original price and Δp be the decrease in price required to sell the additional unit. Then the gross gain in revenues on the additional sale is equal to one times $p-\Delta p$ (the new price charged for the additional unit sale) and the gross loss on the lower price that the firm now has to charge is q times the decrease in price Δp. In other words:

Incremental unit revenue = [1] the gain in revenue due to the sale of one additional at the lower market clearing price, or $1 \times (p - \Delta p)$, where Δp is the market price decrease necessary to clear the market with the sale of an additional unit

[2] minus the loss of margin on prior units sold due to the decrease in the market-clearing price, or $[q\Delta p]$

So $IUR = [1 \times (p - \Delta p)] - [q\Delta p]$ (when $\Delta q = 1$)

$= p - (q+1)\Delta p$

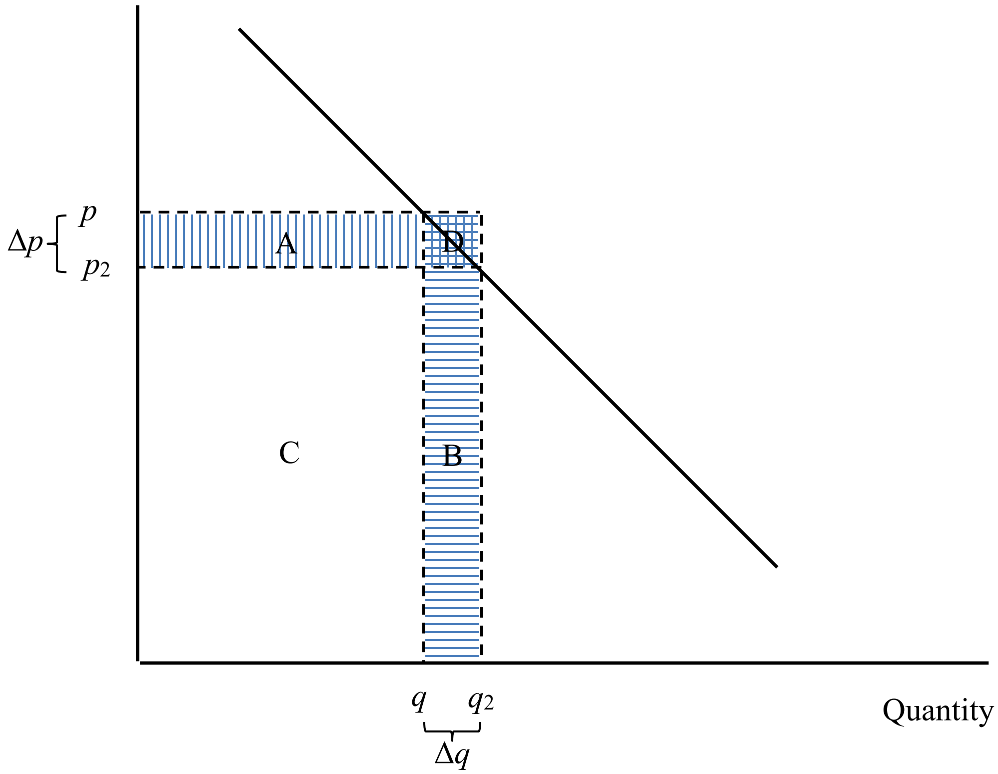
The diagram below illustrates this.

[1] Mathematical aside (optional): Marginal revenue is the limit of incremental unit revenue as $\Delta q/q$ approaches zero. In calculus terms:

$$\text{Revenue (r)} = pq$$

$$\text{Marginal revenue (mr)} = \frac{dr}{dq} = p + q\frac{dp}{dq}$$

Note that dp/dq is the derivative of a downward-sloping (inverse) demand curve and so is a negative number.



The gain in revenue due to the sale of Δq additional units at the lower market-clearing price is $\Delta q \times (p - \Delta p)$, which is represented by area B. When $\Delta q = 1$, this becomes $1 \times (p - \Delta p)$.

The loss of margin on the prior units sold due to the decrease in the market-clearing price is $q\Delta p$, which is represented by area A.

So the incremental gain in revenue to the firm of increasing output from q_1 to q_2 is area B – area A.

When $\Delta q = 1$, this becomes $1 \times (p - \Delta p) - q\Delta p$, or $p - (q+1)\Delta p$ (as shown on the previous page). This is equivalent to:

Area B+D (= p, the additional revenue earned by selling one unit of output at the original price)

minus Area A+D (= $(q+1)\Delta p$, the price adjustment required on all q+1 units of output).

When q is 20, p is 10. When production is increased by one unit, q become 21 an p becomes 9.875. So $\Delta p = 9.875 - 10 = -0.125$. Then IUR = $p - (q+1)\Delta p$ = 10 – 21*0.125 = 7.375.

5. **Explain the concepts of total cost, fixed cost, total variable cost, average variable cost, marginal cost, and constant marginal cost. What are the equations for these variables? What are these various costs when q = 20?**

Total cost *TC* is the sum of all costs of manufacturing a production level *q*. Conventionally, costs are broken down into two types: *fixed costs* (*F*), which do not vary with the production level (such as the CEO's salary or the maintenance of the headquarters building), and (total) *variable costs* (V), which change with the level of production.

$$TC(q) = F + V(q)$$

Fixed cost (*F*) are costs that the firm must incur into order to enter into production (with a given maximum capacity) but which do not vary with the level of production output *q*, as long as *q* is less than the maximum capacity. *F* is a fixed number and is not a function of *q*.

Total variable costs V are costs of production that vary with the production level and that are incurred producing a level *q*. Total variable cost is a function of *q*.

Average variable cost (*AVC*) are the total variable costs for producing output q divided by *q*:

$$AVC(q) = \frac{VC(q)}{q}$$

Marginal cost (*c*) is the cost to produce one additional unit. Since marginal cost may depend on the current production level *q*, it is a function of *q*.

$$\begin{aligned} MC(q) &= TC(q+1) - TC(q) \\ &= F + V(q+1) - F + V(q) \\ &= V(q+1) - V(q) \end{aligned}$$

In general, marginal costs are a function of *q*.

In calculus terms (optional):

$$TC(q) = F + V(q)$$
$$c(q) = \frac{dTC}{dq} = \frac{dV}{dq}.$$

A *constant marginal cost* is a marginal cost that does not change with the production level. If *c* is a constant marginal cost, then $V(q) = cq$ and $TC(q) = F + cq$.

6. **Explain why the firm maximizes profit when marginal revenue equals marginal cost.**

If marginal revenue is greater than marginal cost, then by producing one additional unit the firm earns more revenues than it expends in producing the additional unit. A profit-maximizing firm would therefore increase its production by one unit, and then ask again whether marginal revenue would be greater than marginal cost for the production of yet another addition unit. If so, the firm should produce the additional unit. This iterative process should continue until marginal revenue is less than marginal cost, at which point the firm would lose profits by producing an additional unit. This means that the firm should produce a level of output so that marginal revenue equals marginal cost.

$$\text{Profits } (\pi) = \text{Revenues - total costs}$$
$$= pq - \left[F + V(q)\right]$$

In calculus terms (optional): The first-order condition for a profit maximum set the derivative of profits with respect to quantity equal to zero, so marginal revenue is equal to marginal total costs (which is equal to marginal cost):

$$\pi(q) = r(q) - TC(q)$$
$$\frac{d\pi}{dq} = \frac{dr}{dq} - \frac{dTC}{dq} = 0$$
$$= MR - MC = 0$$

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7. **What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?**

At the firm's profit-maximizing level of output q^*, the firm's marginal revenue is equal to its marginal cost.

When the firm's residual demand curve is $q = 100 - 8p$, the firm's inverse demand curve is $p = 12.5 – 1/8\ q$ (from Question 2). From the formula in the slides for marginal revenue in the case of a linear demand curve, marginal revenue $mr = 12.5 – 1/4q$. The assumptions state that marginal cost are constant at c = 5.

Setting marginal revenue equal to marginal cost at the profit-maximizing level of output q^*:

$$12.5 - \frac{1}{4}q = 5$$

Solving for q^* yields $q^* = 30$. Then:

$p^* = 12.5 - 1/8*30 = 8.75$ from the inverse demand curve

$r^* = p^*q^* = 30*8.75 = 262.5$ from the definition of revenues

$TC^* = F + V(q^*)$

$= F + cq^*$ since marginal costs are constant

$= 25 + 5*30 = 175$ since $F = 25$ and $c = 5$

$\pi = r(q^*) - TC(q^*)$

$= 262.5 - 175 = 87.5$

8. Say the firm's fixed costs F decrease to 20. What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?

None of the equations for the firm's profit-maximizing price, output, revenues, or total variable costs depend on *F*. So those are the same as in Problem 7:

$q^* = 30.$

$p^* = 8.75$

$r^* = 262.5$

$V^* = 150$

Fixed costs, however, decrease to 20. So total cost becomes:

$TC^* = 20 + 150 = 170$

So profits increase by 5:

$\pi = r(q^*) - TC(q^*)$

$= 262.5 - 170 = 92.5$

9. Say the firm's fixed costs *F* remain at 25 but its (constant) marginal cost *c* decreases to 4. What is the firm's price, output, revenues, total costs, and profits at the profit-maximizing level of output?

At the profit-maximizing of output, marginal revenue is equal to marginal cost. Marginal cost has changed, so the profit-maximizing level of output will change as well.

Setting marginal revenue equal to the new marginal cost $c = 4$ at the profit-maximizing level of output q^*:

$$12.5 - \frac{1}{4}q = 4$$

Solving for q^* yields $q^* = 34$. Then:

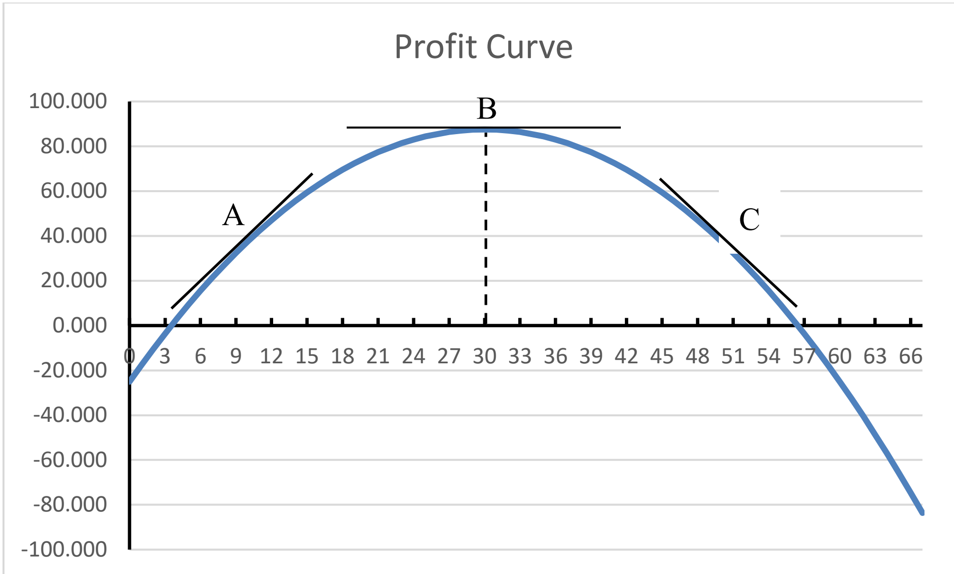
$p^* = 12.5 - 1/8*34 = 8.25$	from the inverse demand curve
$r^* = p^*q^* = 8.75*34 = 280.5$	from the definition of revenues
$TC^* = F + V(q^*)$	
$= F + cq^*$	since marginal costs are constant
$= 25 + 4*34 = 161$	since $F = 25$ and $c = 4$
$\pi = r(q^*) - TC(q^*)$	
$= 280.5 - 161 = 119.5$	

So a change in fixed costs will not affect the profit-maximizing level of output or price, but a change in marginal cost will, In particular, a decrease in marginal cost will increase output and lower the market-clearing price, while a increase in marginal cost will decrease output and increase the market-clearing price.

10. Explain what the firm should do and why if it finds that marginal revenue is greater than its marginal cost at current production (say because of a shift in demand).

For the reason explained in the answer to Question 6, whenever the firm finds that its marginal revenue is greater than its marginal cost, it should increase its output until marginal revenue is again equal to marginal cost.

More generally, the profit curve is a parabola and the slope of the curve at any point q is the marginal profit.



At point A on the profit curve, the slope is positive, meaning that marginal revenues are greater than marginal costs and so the firm would increase profits by increasing output. At point C on the profit curve, the slope is negative, meaning that marginal revenues are less than marginal costs and so the firm would increase profits by decreasing output. At point B on the profit curve, the slope is zero, meaning that marginal revenues are equal to marginal costs. At this point, the firm would loss money if it either increased to decreased its level of production. According, point B (where q = 30) is the profit-maximizing point.