What You Really Need to Know

Unit 8. Competition Economics

Part 1. Demand, Costs, and Profits

Merger Antitrust Law

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Motivation

- The purpose of merger antitrust law
  - Section 7 of the Clayton Act prohibits mergers and acquisitions that "may be substantially to lessen competition, or to tend to create a monopoly"[1]
  - In modern terms, a transaction may substantially lessen competition when it threatens, with a reasonable probability, to create or facilitate the exercise of market power to the harm of consumers.
  - Operationally, a transaction harms consumer when it result in—
    - Higher prices
    - Reduced market output
    - Reduced product or service quality in the market as a whole
    - Reduced rate of technological innovation or product improvement n the market

    Merger antitrust analysis typically focuses on price effects (see Unit 2)

  compared to what would have been the case in the absence of the transaction (the "but for" world) and without any offsetting consumer benefits

Consequently, a central focus in merger antitrust law is the effect a merger is likely to have on the profit-maximizing incentives and ability of the merged firm to raise price in the wake of the transaction. In the first instance, this requires us to know how a profit-maximizing firm operates. The basic tools to enable us to do this analysis is the subject of this unit. These same tools are also fundamental to an understanding of merger antitrust law defenses.

[1] 15 U.S.C. § 18.

What you should be able to do after Part 1

For a firm—
- Facing a downward sloping residual (inverse) demand curve $p = a + bq$
- With fixed costs F and constant marginal costs c

1. Determine and graph the profit-maximizing levels of—
   - Output q^*
   - Price p^*
   - Profits π^*
2. Determine and graph the net incremental revenue for a firm increasing output by Δq, including—
   - The gross gain in revenues from the increase in output, and
   - The gross loss in revenues from the reduction of price for sales at the original price
3. Derive and graph an inverse demand curve given a demand curve

1. Profit Maximization

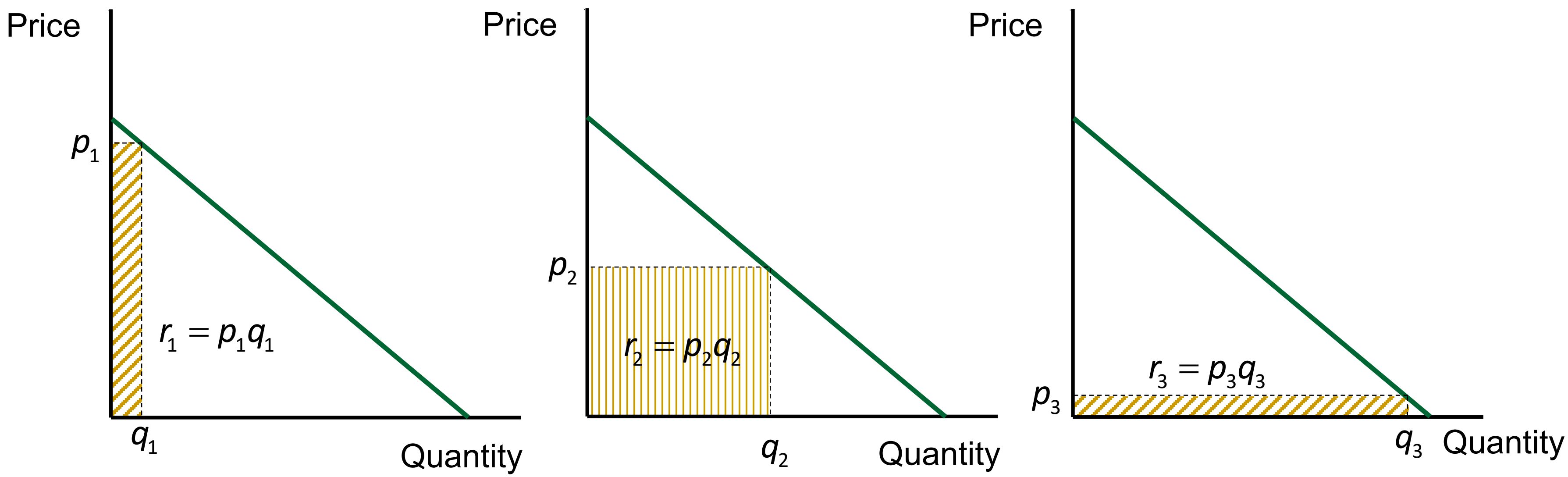
Profits

1. When the firm produces output q, its profits $\pi(q)$ are equal to its revenues $R(q)$ minus its total costs $TC(q)$:

$$\pi(q) = R(q) - TC(q)$$

2. Revenues $R(q)$ are equal to price p times output q:

$$R(q) = pq$$

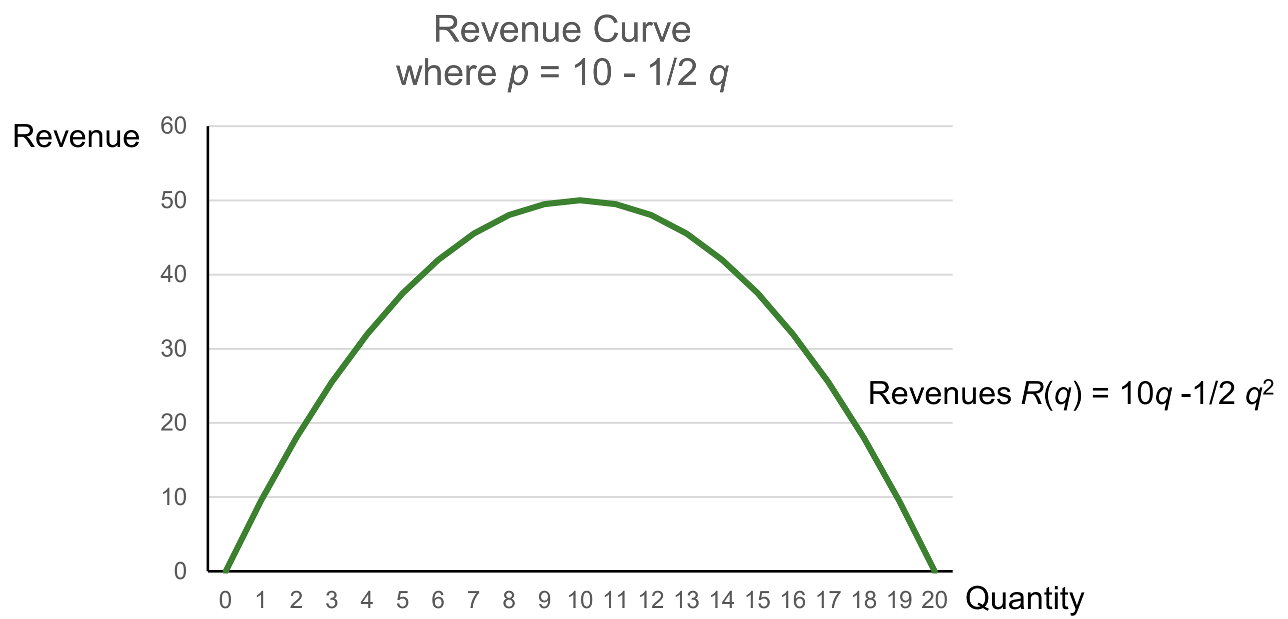


Profits

3. When the firm faces a downward-sloping residual (inverse) demand curve $p = a + bq$:

$$\begin{aligned} R(q) &= pq \\ &= (a + bq)q \\ &= aq + bq^2 \end{aligned}$$

- The graph of the firm's revenues as a function of q is a parabola:



Profits

4. At output q, total costs $TC(q)$ are equal to fixed costs F plus variable costs $V(q)$:

$$TC(q) = F + V(q)$$

- With constant marginal costs c, variable costs $V(q)$ are equal to marginal cost c times output q:

$$V(q) = cq$$

- Then total costs $TC(q)$ may be expressed as:

$$\begin{aligned} TC(q) &= F + V(q) \\ &= F + cq \end{aligned}$$

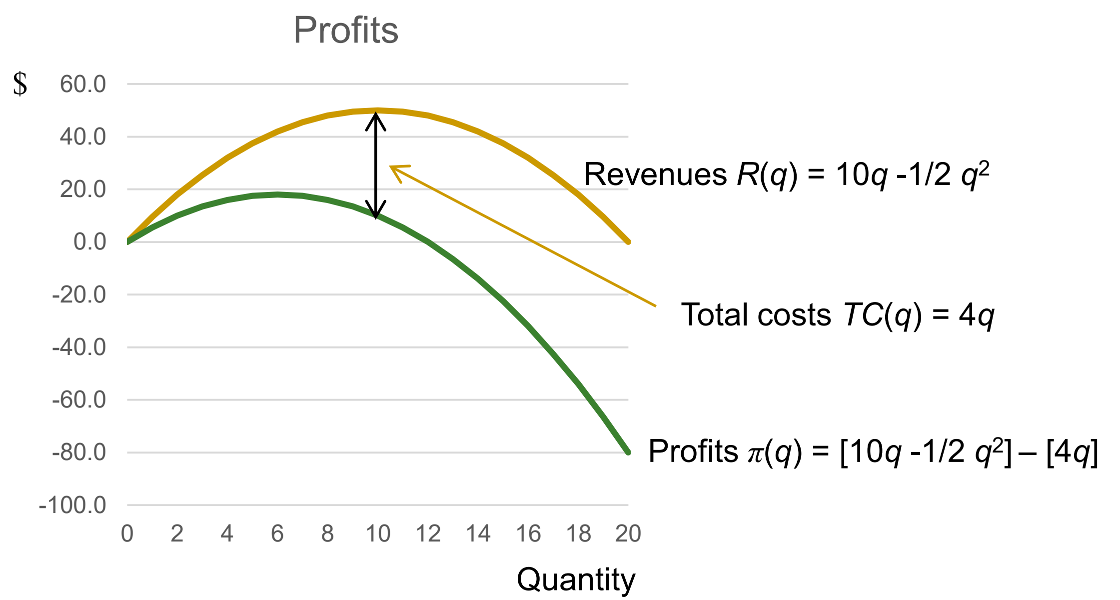
Profits

5. Now we can express total profits $\pi(q)$ as:

$$\begin{aligned}\pi(p) &= R(q) - TC(q) \\ &= (a + bq)q - cq \\ &= \left[aq + bq^2\right] - cq\end{aligned}$$

- Graphically

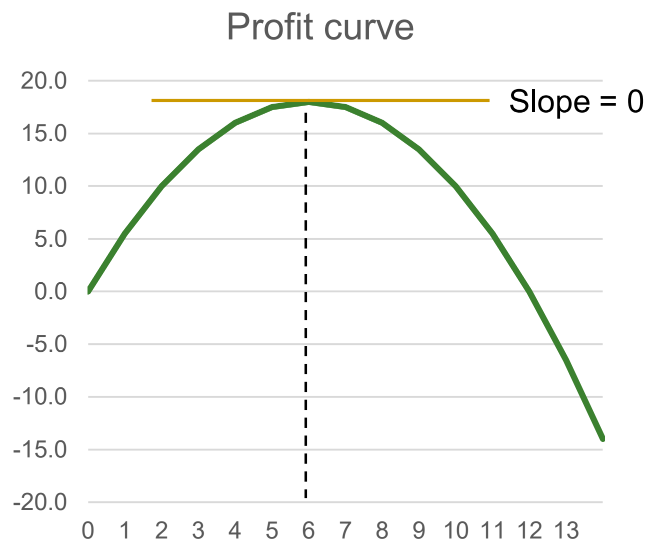
where
$p = 10 - ½\ q$
$F = 0$
$c = 4$



Profit maximization

6. The slope at the top of the profit "hill" is zero (a horizontal line):

where
  $p = 10 - \frac{1}{2} q$
  $F = 0$
  $c = 4$



- From the chart we see that the profit-maximizing output q^* is 6.
- From the inverse demand curve, we can calculate $p^*(6) = 10 - (1/2)(6) = 7$
- $R^* = R(6) = p^*q^* = (7)(6) = 42$
- $F = 0$ (from the hypothetical)
- $V^* = V(6) = cq^* = (4)(6) = 24$
- $TC^* = TC(q^*) = F + V(q^*) = 0 + 24 = 24$
- $\pi^* = \pi(q^*) = r^* - TC^* = 42 - 24 = 18$

Profit maximization

7. Marginal analysis—Some definitions
   - The slope of the revenue curve at an output q is called the *marginal revenue* $mr(q)$
     - Think of marginal revenue as the revenue the firm would earn if it produced one additional unit
     - If $R(q) = aq + bq^2$ (the revenue function for a linear inverse demand curve), then:

$$mr(q) = a + 2bq$$

   - The slope of the total cost curve at an output q is called the *marginal cost* $mc(q)$
     - Think of marginal cost as the cost the firm would earn if it produced one additional unit
     - If $TC(q) = F + cq$ (total costs with constant marginal costs), then:

$$mc(q) = c$$

   - The slope of the profit curve at an output q is called the *marginal profit* $m\pi(q)$
     - Think of marginal profit as the profit the firm would earn if it produced one additional unit
     - Marginal profit is marginal revenue minus marginal cost:

$$m\pi(q) = mr(q) - mc(q)$$

Optional: The marginal function is the derivative of the primary function. So, for example, the marginal revenue function is the derivative of the revenue function.

Profit maximization

8. First order condition (FOC)
   - From Slide 9, we know that profits are maximized at the top of the profit "hill," which is where the slope of the profit curve is zero
   - From Slide 10, we know that the slope of the profit curve at an output q is the marginal profit $m\pi(q)$ evaluated at output q.
   - From Slide 10, we also know that the marginal profit $m\pi(q)$ is equal to the marginal revenue $mr(q)$ minus the marginal cost $mc(q)$, all evaluated at output q, that is:

$$m\pi(q) = mr(q) - mc(q)$$

   - The *first order condition* for a profit-maximizing level of output q^* is that the marginal profit at q^* equals zero, that is:

$$m\pi(q^*) = mr(q^*) - mc(q^*) = 0$$

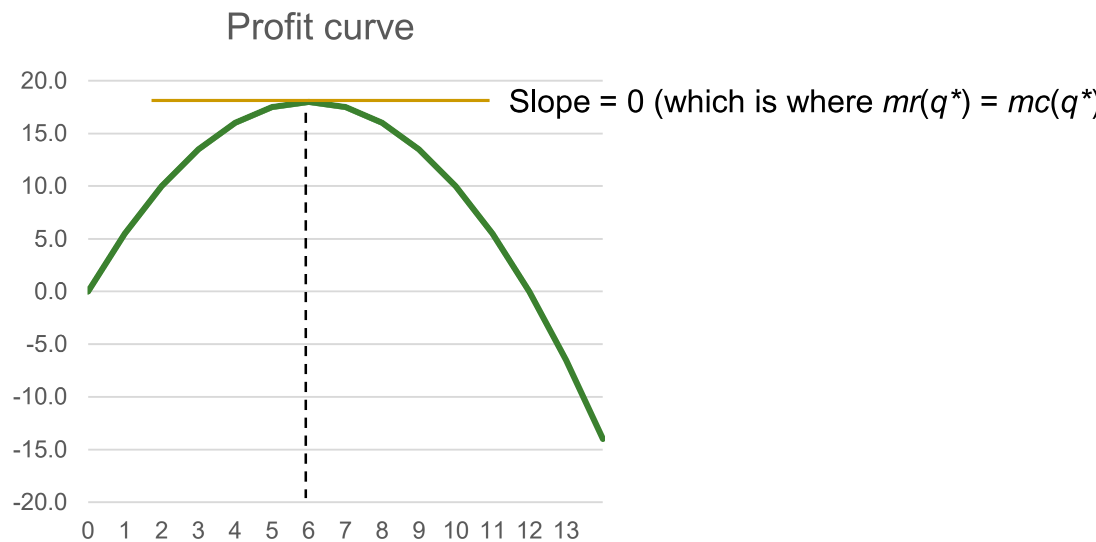
or equivalently:

$$mr(q^*) = mc(q^*)$$

Profit maximization

10. First order condition—Example

where
$p = 10 - ½ q$
$F = 0$
$c = 4$

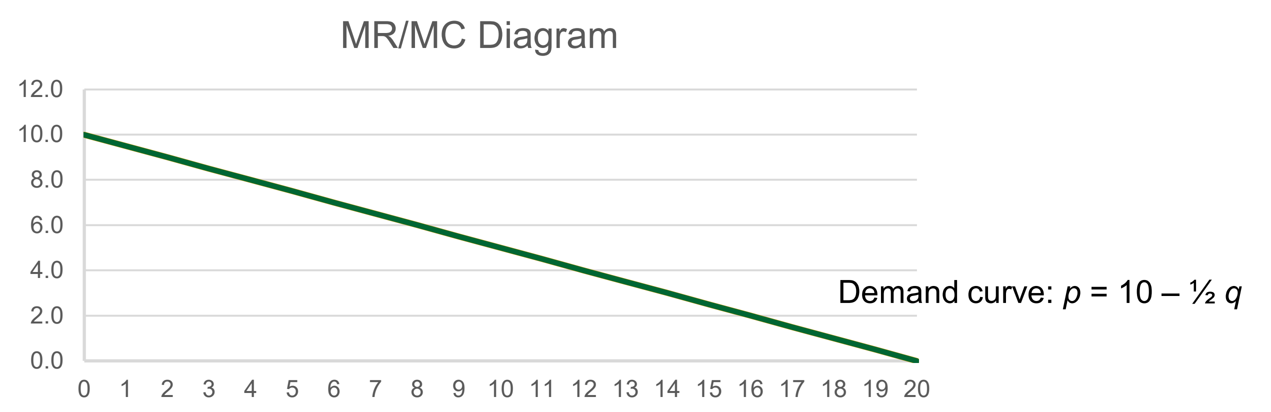


- $mr(q) = 10 - q$ (from the formula on Slide 10)
- $mc(q) = 4$ (from the hypothetical)
- FOC: $mr(q^*) = mc(q^*)$
  So $10 - q^* = 4$ or $q^* = 6$ (as shown in the diagram)
- $p^* = p(q^*) = 10 - ½ q^*$
  $= 10 - (½)(6) = 7$ (from the inverse demand curve)

Profit maximization

11. Marginal revenue/marginal cost diagrams
    - Will build this step-by-step

    a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$



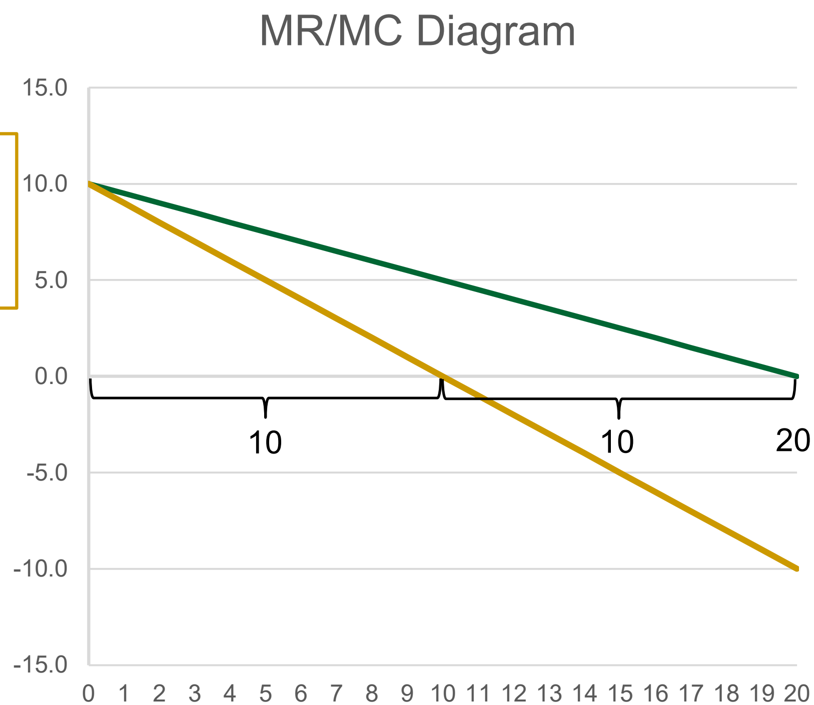
Profit maximization

11. Marginal revenue/marginal cost diagrams
    - Will build this step-by-step

a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$

b. Add the marginal revenue curve: $p = 10 - q$

Note: With linear demand, the marginal revenue curve falls twice as fast as the inverse demand curve



Demand curve: $p = 10 – \frac{1}{2} q$

Marginal revenue curve: $p = 10 - q$

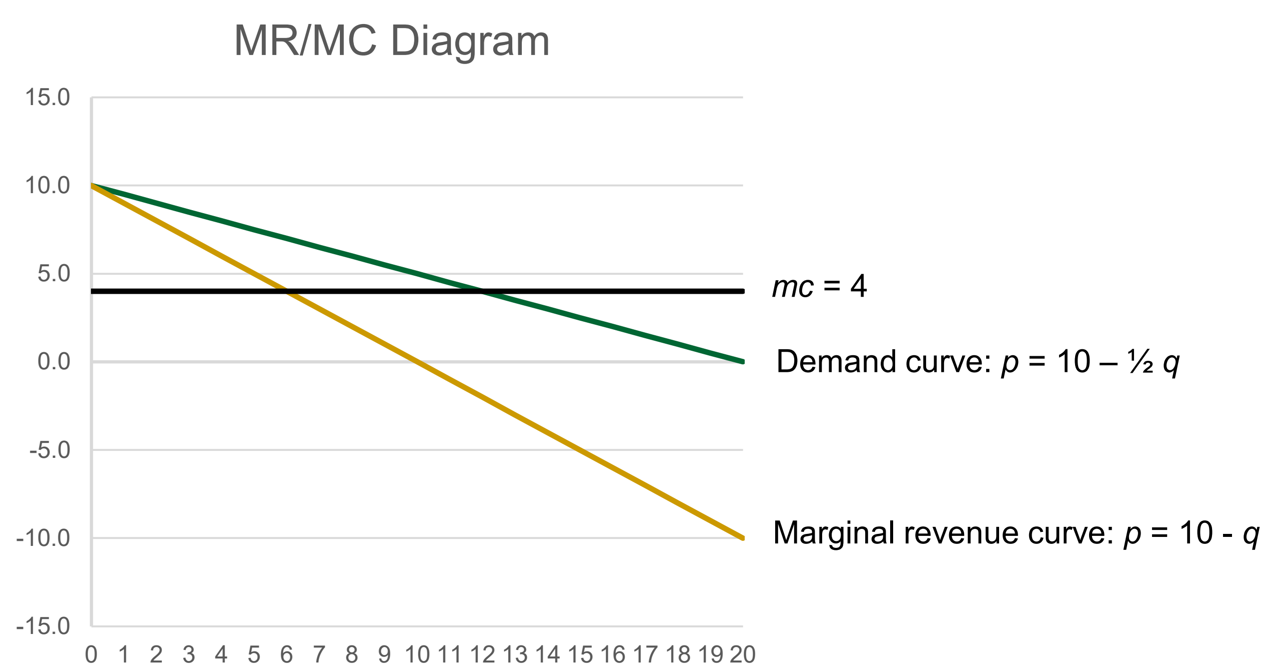
Profit maximization

11. Marginal revenue/marginal cost diagrams
    - Will build this step-by-step

    a. Consider an (inverse) demand curve: $p = 10 - ½ q$

    b. Add the marginal revenue curve: $p = 10 – q$

    c. Add the marginal cost curve: $c = 4$ (constant marginal cost)



Profit maximization

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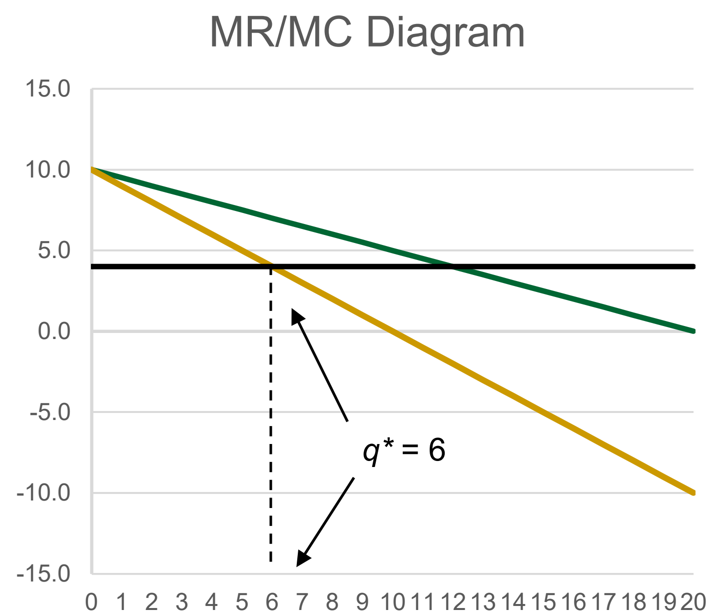
    a. Consider an (inverse) demand curve: $p = 10 - ½ q$
    b. Add the marginal revenue curve: $p = 10 – q$
    c. Add the marginal cost curve: $c = 4$ (constant marginal cost)
    d. Find intersection of *mr* and *mc* curves to determine profit-maximizing q^* (= 6)



Profit maximization

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    a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$

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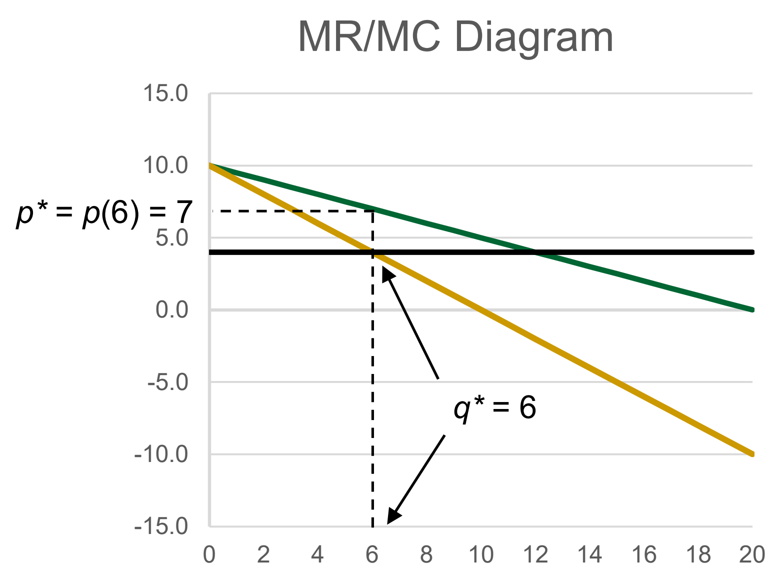
    c. Add the marginal cost curve: $c = 4$ (constant marginal cost)

    d. Find intersection of *mr* and *mc* curves to determine profit-maximizing q^* (= 6)

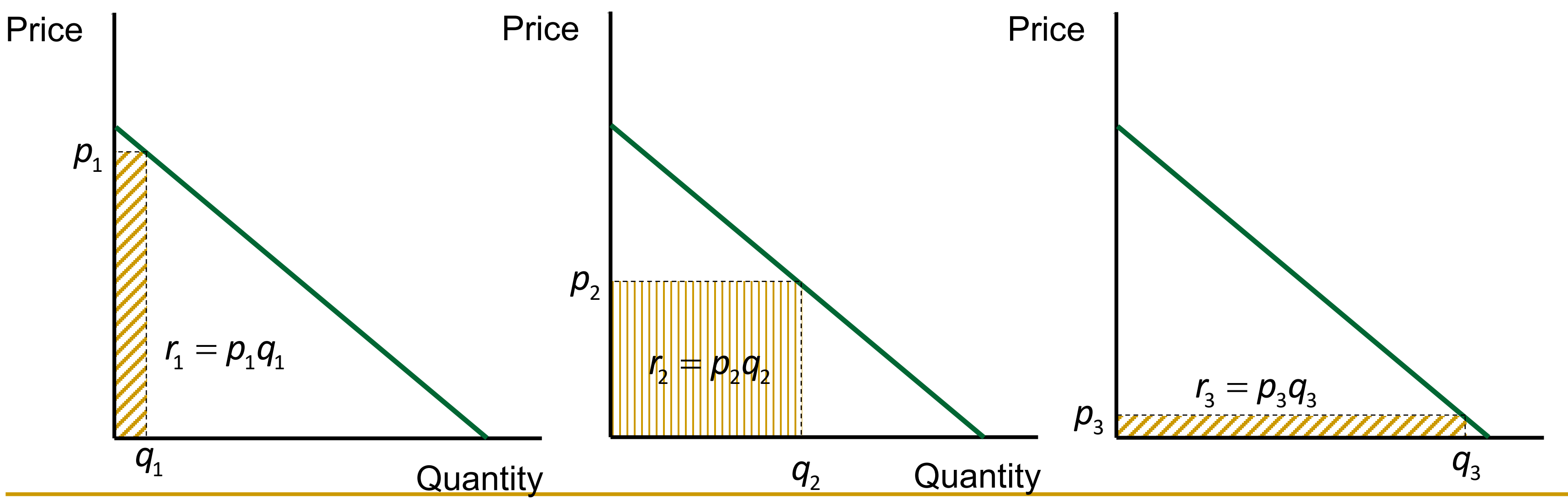
    e. Find $p^* = p(q^*)$ from the inverse demand curve ($p^* = 7$)



2. Incremental Revenue

Incremental revenue

- Introduction
  - *Incremental revenue* is the net gain in revenue that a firm could earn if it were to increase its product by some amount Δq
  - Incremental revenue is important when determining whether a firm should change its output level to increase its profits
  - Incremental revenue can be positive or negative
    - Moving from q_1 to q_2 increases revenue (incremental revenue is positive)
    - Moving form q_2 to q_3 decreases revenue (incremental revenue is negative)



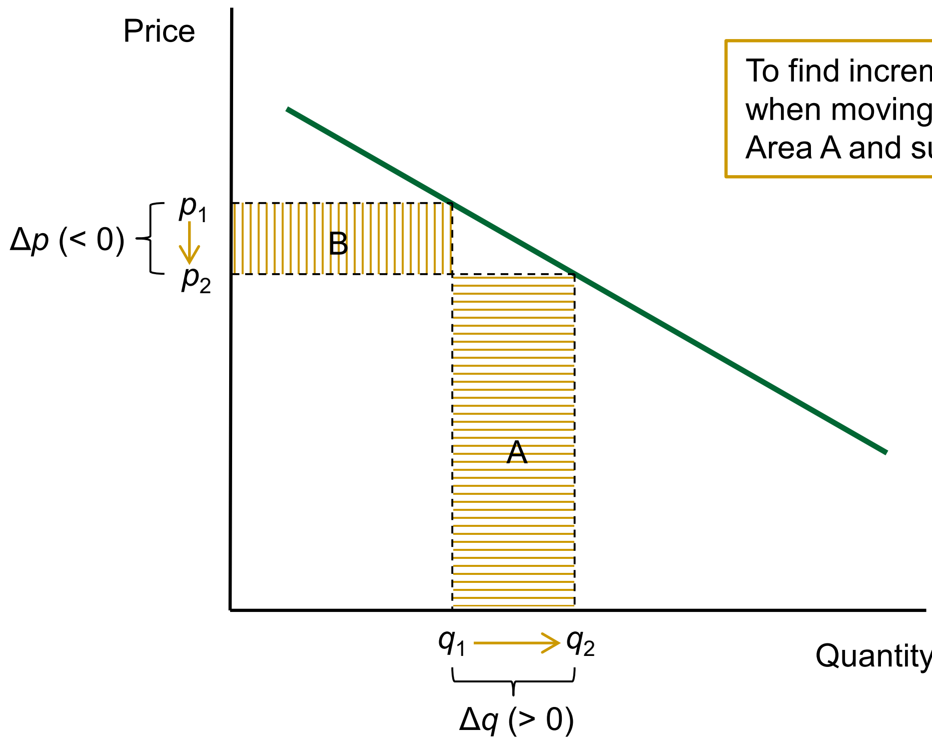
Incremental revenue

- Think about incremental revenue in two parts:
  1. The *gain* in revenue due to the sale of the additional units at the lower market-clearing price
     - Since there are more units to sell and demand is downward-sloping, the price will drop to clear the market
     - The gain in revenue is equal to $\Delta q \times (p - \Delta p)$, where
       - Δq is the additional quantity to be sold
       - Δp is the market price decrease necessary to clear the market with the sale of an additional unit
  2. Minus the *loss* of revenue on prior units sold due to the decrease in the market-clearing price
     - This loss of margin is the prior quantity *p* times the required price decrease, or $[q\Delta p]$
- So

$$IR = \Delta q\left(p - \Delta p\right) - q\Delta p$$

Incremental revenue

- Graphically



To find incremental revenue *IR* when moving from q_1 to q_2, add Area A and subtract Area B

Area A = $\Delta q(p_1 - \Delta p)$ is the gain in revenue from the additional sales Δq at the lower price $p_2 = p_1 - \Delta p$
Area B = $q_1 \Delta p_1$ is the loss in revenue due to the sales of q_1 at the lower price p_2

So

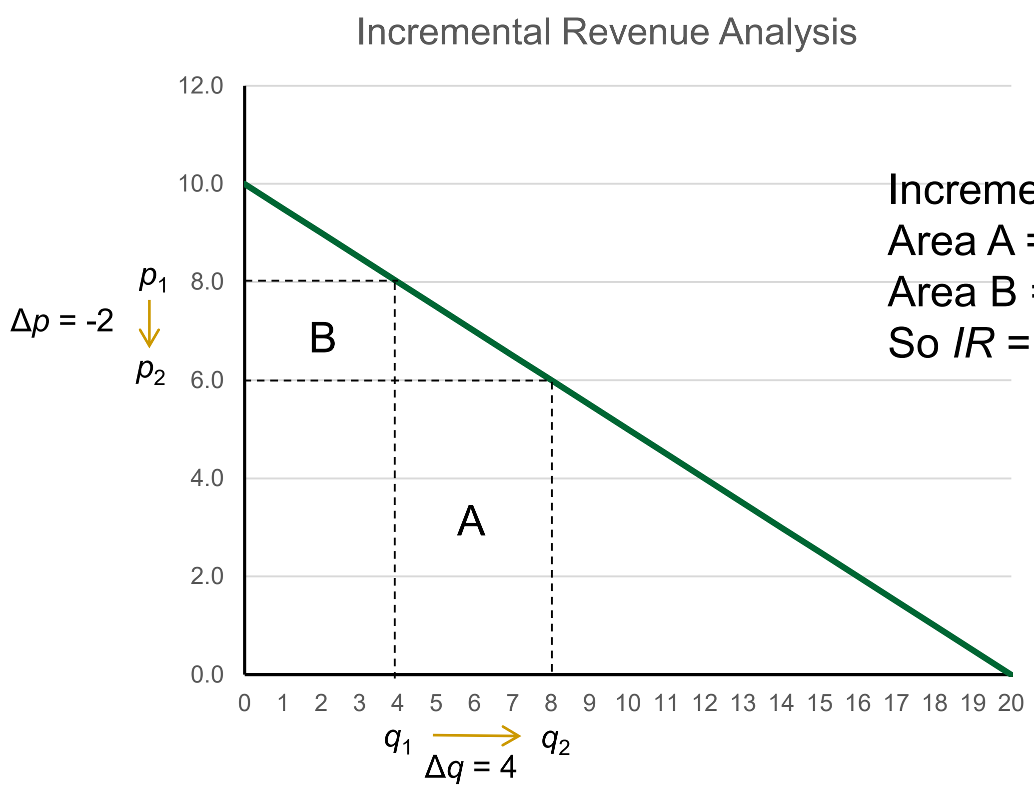
$$IR = \overbrace{\Delta q\left(p - \Delta p\right)}^{\text{Area A}} - \overbrace{q\Delta p}^{\text{Area B}}$$

Incremental revenue

- Example
  - (Inverse) demand: p =10 – ½ q
  - Starting point: $q_1 = 4$
  - End point: $q_2 = 8$

You need to calculate this:

So $p_1 = 8$	$\Delta q = q_2 - q_1 = 8 - 4 = 4$
So $p_2 = 6$	$\Delta p = p_2 - p_1 = 6 - 8 = -2$



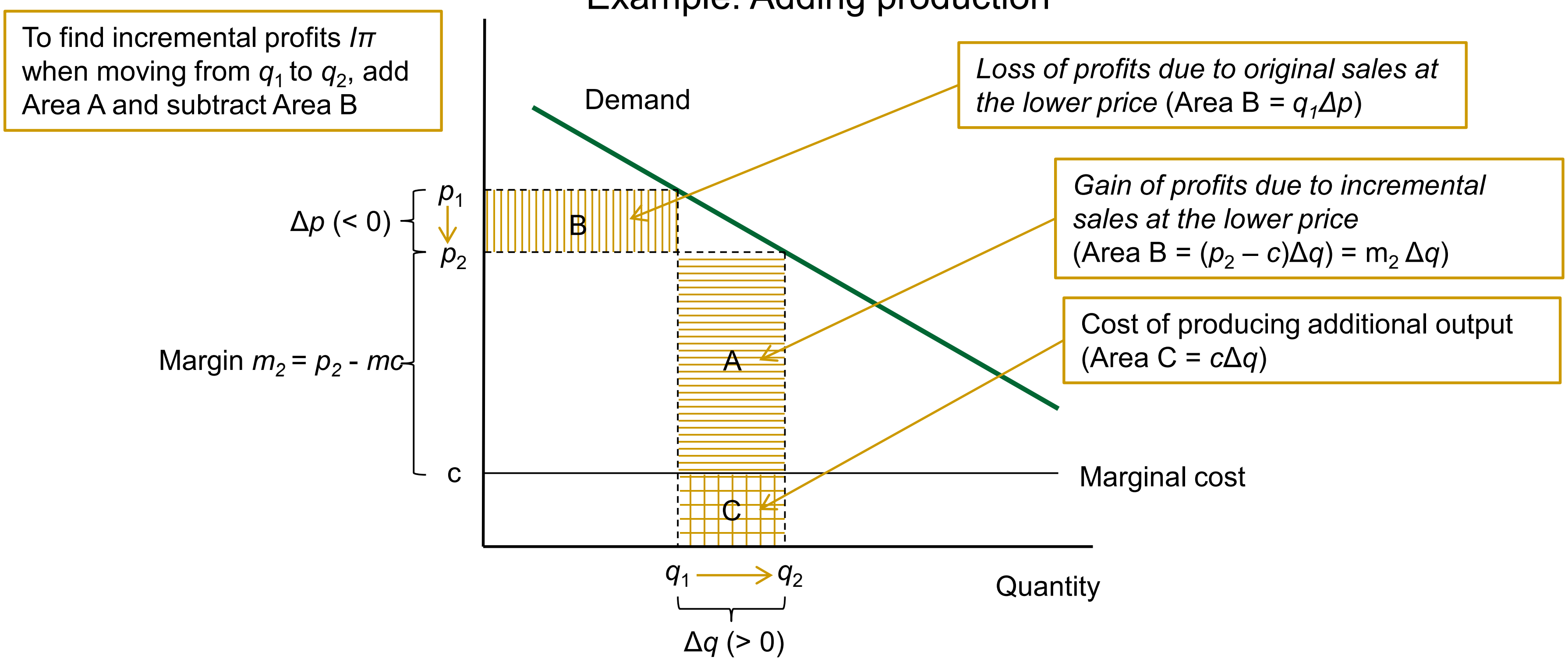
Incremental revenue = Area A – Area B
Area A = $p_2\Delta q$ = (6)(4) = 24
Area B = $q_1\Delta p$ = (4)(-2) = -8
So IR = 32 – 8 = 16

That is, the firm makes $16 more in revenues by moving from q_1 to q_2

Incremental profits

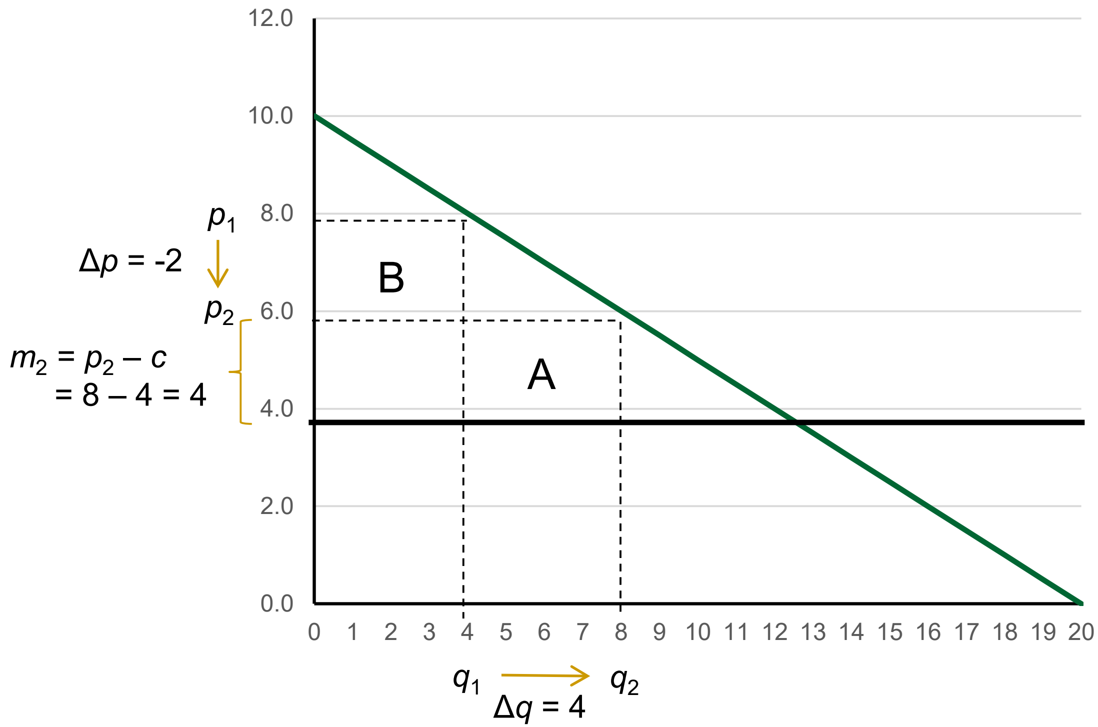
- We can easily extend the analysis of incremental revenues to incremental profits—We just have to:
  - Add the costs of additional production if we are adding to output (Δq > 0)
  - Subtract the costs of a reduction in output (Δq < 0)

Example: Adding production



Incremental profits

- Example: Output increase
  - (Inverse) demand: $p = 10 - \frac{1}{2} q$
  - Starting point: $q_1 = 4$ So $p_1 = 8$ $\Delta q = q_2 - q_1 = 8 - 4 = 4$
  - End point: $q_2 = 8$ So $p_2 = 6$ $\Delta p = p_2 - p_1 = 6 - 8 = -2$
  - Constant marginal cost $c = 4$

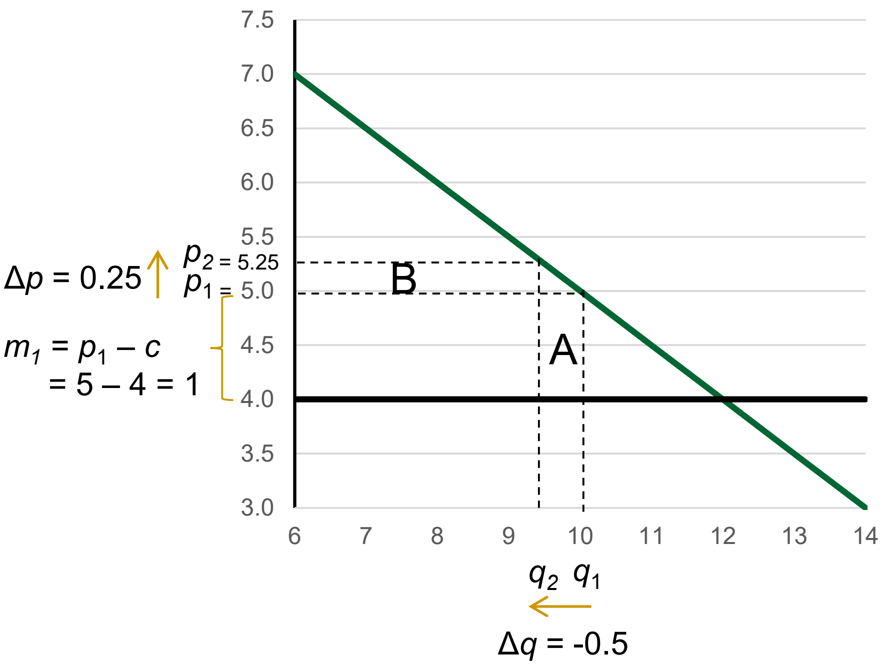


Incremental profits = Area A – Area B
Area A = $m_2 \Delta q$ = (4)(4) = 16
Area B = $q_1 \Delta p$ = (4)(-2) = -8
So $I\pi$ = 16 – 8 = 8

That is, the firm makes $8 more in profits by moving from q_1 to q_2

Incremental profits

- Example: Price increase
  - (Inverse) demand: $p = 10 - ½ q$ So $q = 20 - 2p$
  - Starting point: $p_1 = 5$ So $q_1 = 10$ $\Delta q = q_2 - q_1 = 9.5 - 10 = -0.5$
    End point: $p_2 = 5.25$ So $q_2 = 9.5$ $\Delta p = p_2 - p_1 = 5.25 - 5 = 0.25$
  - Constant marginal cost $c = 4$



With an increase price and a concomitant reduction in output, the roles of Areas A and B are reversed:

Area A now represents the *loss* of profits from lost sales that would have been made at original price p_1 (= $m_1 \Delta q$)

Area B represents the *gain* of profits from the increased price charged on the sales that continue to be made (= $q_2 \Delta p$)

Incremental profits = Area B – Area A

Area B = $q_2 \Delta p$ = (9.5)(0.25) = 2.375
Area A = $m_1 \Delta q$ = (1)(-0.5) = -0.5

So *incremental profits* = 2.375 – 0.5 = 1.875

Incremental profit

- Observations
  - The prior example shows that under the conditions of the hypothetical, a 5 percent price increase would be profitable to the firm

> This is mathematically identical to the exercise required by the *hypothetical monopolist test*, which is the primary analytical tool used by the agencies and the courts to define relevant markets. The hypothetical monopolist test asks whether a hypothetical monopolist of the candidate market could profitably sustain a "small but significant and nontransitory increase in price" (SSNIP), usually taken to be 5 percent. If so, the candidate market is a relevant market. In the prior example, if we assume that the demand curve is for the candidate market as a whole, this will be the residual demand curve for the hypothetical monopolist. If the original market price was $5 (as in the hypothetical), the hypothetical monopolist would find it profitable to reduce output in order to raise price by a 5 percent SSNIP.
>
> We will confront the hypothetical monopolist test in almost every case study going forward, starting with the Sanford/Mid Dakota Clinic case study next week. You will have plenty of opportunities to become familiar with the mechanics of the hypothetical monopolist test.

3. Inverting Demand and Inverse Demand Functions

Inverting demand and inverse demand functions

- Motivation
  - You will be given either the demand function or the inverse demand function in a problem. But you may need to derive the other function in order to solve the problem.
  - Example
    - In the price increase problem on Slide 25, you were given the inverse demand function:

$$p = 10 - \frac{1}{2}q$$

    - But the problem gave you p_1 and p_2 and required you to calculate q_1 and q_2. To do this, you need to convert the inverse demand function into the demand function, so that you could use the prices to calculate the associated quantities.
    - To create the demand function, you need to manipulate the inverse demand equation to isolate q on the left-hand side, so that quantities (which you need) are expressed in terms of prices (which the problem gives you)

Inverting demand and inverse demand functions

- Mechanics
  - An equality is maintained if you perform the same operation to both sides of the equation
  - Here are the steps to convert the above inverse demand function to a demand function:

Add ½ q to both sides:

$$p + \frac{1}{2}q = 10 - \frac{1}{2}q + \frac{1}{2}q$$
$$= 10$$

Subtract p from both sides:

$$p + \frac{1}{2}q - p = 10 - p$$

Simply:

$$\frac{1}{2}q = 10 - p$$

Multiply both sides by 2:

$$(2)\left(\frac{1}{2}q\right) = (2)(10 - p)$$

Simply:

$$q = 20 - 2p$$

This is the demand curve that you would need for the price increase incremental revenue problem

  - The same technique can be used to convert a demand curve into an inverse demand curve