What You Really Need to Know

Unit 8. Competition Economics

Part 2. Markets and Market Equilibria

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Substitutes, Complements, and Elasticities

Substitutes/Complements

- Substitutes
  - *Definition*: Two products or services are *substitutes* if, when consumer demand increases for one product, it will decrease for the other product
    - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} < 0$$

  - Examples
    - Coke and Pepsi
    - iPhone and Galaxy S series mobile phones
    - Nike and Adidas shoes
    - Hertz and Avis rental cars
    - Cars and oil
  - *Horizontal mergers* involve combinations of firms that offer substitute products

Substitutes/Complements

- Substitutes
  - Substitutes and prices
    - If products 1 and 2 are substitutes, then as the price of 1 increases the demand for 2 increases
    - Proof:

$$\underset{(-)}{\frac{\Delta q_2}{\Delta q_1}}\;\underset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \underset{(+)}{\frac{\Delta q_2}{\Delta p_1}} > 0$$

  - $\frac{\Delta q_2}{\Delta q_1}$ is a negative number (by definition of a substitute)
  - $\frac{\Delta q_1}{\Delta p_1}$ is a negative number (it is the slope of the demand curve for product 1)
  - A negative number times a negative number is positive, so $\frac{\Delta q_2}{\Delta p_1}$ is positive
  - If Δp_1 is positive (i.e., the price of product 1 goes up), then Δq_2 must be positive (i.e., demand for product 2 goes up)

Substitutes/Complements

- Complements
  - *Definition*: Two products are *complements* if, when a consumer demand increases for one product, consumer demand also will increase for the other product
  - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} > 0$$

  - Examples
    - *Vertical mergers* involve complements
      - Television LCD screens and TV sets
      - Car engines and cars
      - Cable TV programming and cable TV distribution (AT&T/Time Warner)
      - Drug manufacture and drug distribution
    - But some conglomerate mergers can also involve complements
      - Printers and ink cartridges
      - Razors and razor blades
      - Computers and computer software

Substitutes/Complements

- Complements
  - Complements and prices
    - If products 1 and 2 are complements, then as the price of 1 increases the demand for 2 decreases
    - Proof:

$$\overset{(+)}{\frac{\Delta q_2}{\Delta q_1}} \overset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \overset{(-)}{\frac{\Delta q_2}{\Delta p_1}} < 0$$

  - $\frac{\Delta q_2}{\Delta q_1}$ is a positive number (by definition of a complement)
  - $\frac{\Delta q_1}{\Delta p_1}$ is a negative number (it is the slope of the demand curve for product 1)
  - A negative number times a positive number is negative, so $\frac{\Delta q_2}{\Delta p_1}$ is negative
  - If Δp_1 is positive (i.e., the price of product 1 goes up), then Δq_2 must be negative (i.e., demand for product 2 goes down)

Elasticities

- Own-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product.

The Greek letter epsilon (ε) is the usual symbol in economics for elasticity

$$\varepsilon = \frac{\%\Delta q_i}{\%\Delta p_i}$$

Percentage change q_i in the quantity of product *i* demanded

Percentage change p_i in the price of product *i*

  - Examples:
    - If price increases by 5% and demand decreases by 10%, then the own-elasticity is -2 (= -10%/5%)
    - If price increases by 3% and demand deceases by 1%, then the own-elasticity is -1/3 (= -1%/3%)
  - Conventions
    - Own-elasticities are often simply called *elasticities*
    - Technically, own-elasticities are always negative numbers (given downward-sloping demand)
      - But economists often drop the negative sign and use the absolute value
      - The idea is that everyone knows that own-elasticities are negative, so why bother saying it?

Elasticities

- Own-elasticity of demand
  - The relation to the slope of the demand curve:

$$\varepsilon = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$

that is, the own-elasticity at a point on the demand curve is equal to the slope of the demand curve at that point times the ratio of price to quantity

- Proof

$$\varepsilon = \frac{\%\Delta q_i}{\%\Delta p_i} = \left(\frac{\frac{\Delta q_i}{q_i}}{\frac{\Delta p_i}{p_i}}\right)\left(\frac{\frac{q_i}{1}}{\frac{q_i}{1}}\right) = \left(\frac{\Delta q_i}{\Delta p_i}\right)\left(\frac{1}{\frac{q_i}{p_i}}\right) = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$

This fraction is equal to 1

Slope of the (residual) demand curve: Always negative

- Mathematical note (optional)
  - In calculus terms: $\varepsilon = \dfrac{dq_i}{dp_i}\dfrac{p_i}{q_i}$

Elasticities

For intuition only
(NOT technically correct)

- Some important definitions
  - *Inelastic demand*: Not very price sensitive

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} < 1$$

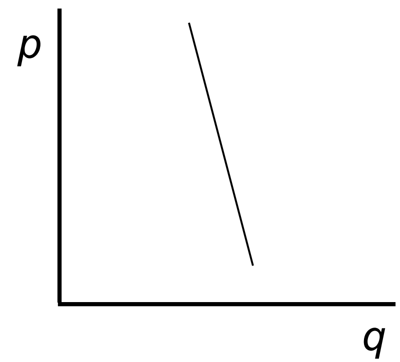
  - *Unit elasticity*:

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} = 1$$

  - *Elastic demand*: Price sensitive

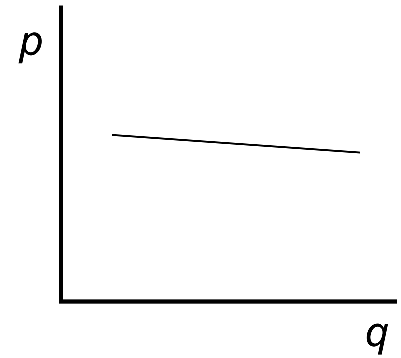
$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} > 1$$

Inelastic demand



Little sensitivity to price

Elastic demand



Demand is very sensitive to price

Note: |*x*| is the *absolute value* of *x*, which is the magnitude of *x* without the sign. So |3| = |-3| = 3.

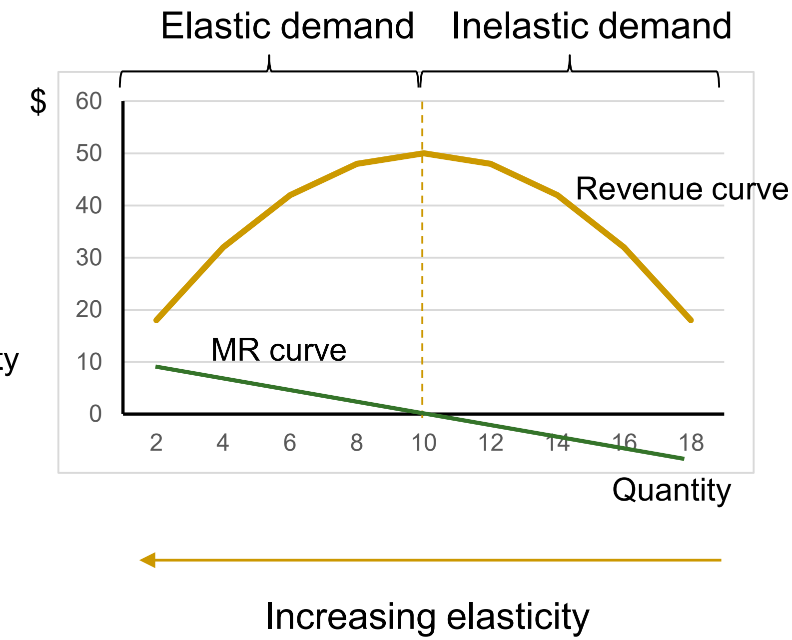
Elasticities

- Elasticity of demand and the slope of the demand curve
  - Even when the demand curve is linear (so that the slope is constant), elasticity varies along the demand curve

Demand curve:
$p = 20 - 2q$

p	q	Slope	p/q	ε	Total revenue
1	18	-2	0.0556	-0.1111	18
2	16	-2	0.1250	-0.2500	32
3	14	-2	0.2143	-0.4286	42
4	12	-2	0.3333	-0.6667	48
5	10	-2	0.5000	-1.0000	50
6	8	-2	0.7500	-1.5000	48
7	6	-2	1.1667	-2.3333	42
8	4	-2	2.0000	-4.0000	32
9	2	-2	4.5000	-9.0000	18

Inelastic demand (rows 1–4)
Unit elasticity (row 5)
Elastic demand (rows 6–9)



General rules:
Elasticity decreases as quantity increase and prices decrease
Elasticity increases as quantity decrease and prices increase

$$\varepsilon = \frac{\Delta q_i}{\Delta p_i} \frac{p_i}{q_i}$$

Elasticities

- Proposition
  - When a firm maximizes its revenues are maximized, the elasticity of its residual demand function is -1 (ε = -1)
    - We see this on the graph on the previous slide
    - Proof (optional)

$$r(q) = p(q)q$$ Definition of revenues

$$\frac{dr}{dq} = p + q\frac{dp}{dq} = 0$$ FOC for a revenue maximum

$$p = -q\frac{dp}{dq}$$ Rearranging FOC

$$\varepsilon = \frac{dq}{dp}\frac{p}{q}$$ Definition of elasticity

$$= \frac{dq}{dp}\frac{-q\frac{dp}{dq}}{q} = -1$$ Substituting for *p* and simplifying

Q.E.D.

Note: $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ That is, the derivative of a function *y*=*f*(*x*) is equal to the reciprocal of the derivative of the inverse function *x*=*g*(*y*)

Cross-elasticities

- Cross-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} = \frac{\%\Delta q_i}{\%\Delta p_j}$$

Percentage change q_j in the quantity of product *j* demanded

Percentage change p_i in the price of product *i*

- With a little algebra (as before):

$$\varepsilon_{ij} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$$

Positive for substitutes
Negative for complements

- Cross-elasticities are positive for substitutes and negative for complements

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- Mathematical note (optional)
  - In calculus terms:

$$\varepsilon_{ij} = \frac{dq_i}{dp_j}\frac{p_j}{q_i}$$

Cross-elasticities

- Elasticity of demand—More definitions
  - Cross-elasticities
    - *High cross-elasticity of demand*:
      - A small change in the price of product *i* will cause a large shift of demand to product *j*
      - As a result, product *j* brings a lot of competitive pressure on product *i*
    - *Low cross-elasticity of demand*:
      - A large change in the price of product *i* will cause only a small shift of demand to product *j*
      - As a result, product *j* brings little competitive pressure on product *i*

A important relationship

- Relationship of own-elasticities to cross-elasticities
  - Intuitively, the higher the cross-elasticities with the other products, the more elastic is the own-elasticity
  - Consequently, if a merger has the effect of decreasing the cross-elasticities of one or more substitute products, then the own-elasticity also decreases
    - *Key intuition*: This shifts the intersection of the marginal revenue curve and the marginal cost curve to the left, leading the firm to decrease output and increase prices
  - Formula:

$$\varepsilon_{11} = 1 + \frac{1}{s_1} \sum_{i=2}^{n} \varepsilon_{i1} s_i$$

Where ε_{11} is the own-elasticity of product 1 and ε_{i1} is the cross-elasticity of product *i* with respect to product 1

A important relationship

- Proof (optional):

  Let Δq_1 be the change in the quantity demanded of product 1 for a change in price Δp_1

  Let Δq_i be the associated change in the quantity demanded of product *i* for a change in price Δp_1 for substitutes 2, . . . *N*

  As consumers shift their purchases from product 1 to the substitute products, the amount they save on product 1 will be equal to the amount they spend on the substitutes:

Budget constraint:

$$p_1\Delta q_1 - \Delta p_1\left(q_1 - \Delta q_1\right) = p_2\Delta q_2 + \cdots + p_N\Delta q_N$$

Total amount now spent on other products

Amount saved by not by Δq_1 at price p_1

Increased spending because of the increased price on the remaining quantity of product 1 purchased

Rearrange terms:

$$p_1\Delta q_1 - \Delta p_1 q_1 + \Delta p_1\Delta q_1 = p_2\Delta q_2 + \cdots + p_N\Delta q_N$$

If Δq_i is very small, then Δq_i will also be very small, and $\Delta p_i \Delta q_i$ will be so small that we can ignore it.

A important relationship

- Proof (optional):

Expand and divide by $\Delta p_1 q_1$: $$\frac{\Delta q_1}{\Delta p_1}\frac{p_1}{q_1} - \frac{\Delta p_1}{\Delta p_1}\frac{q_1}{q_1} = \frac{\Delta q_2}{\Delta p_1}\frac{p_2}{q_1} + \cdots + \frac{\Delta q_N}{\Delta p_1}\frac{p_N}{q_1}$$

Multiply each term on the lhs by $\frac{p_1 q_i}{p_1 q_i} = 1$: $$= \frac{\Delta q_2}{\Delta p_1}\frac{p_2}{q_1}\frac{p_1 q_2}{p_1 q_2} + \cdots + \frac{\Delta q_N}{\Delta p_1}\frac{p_N}{q_1}\frac{p_1 q_N}{p_1 q_N}$$

Rearrange: $$= \frac{\Delta q_2}{\Delta p_1}\frac{p_1}{q_2}\frac{p_2 q_2}{p_1 q_1} + \cdots + \frac{\Delta q_N}{\Delta p_1}\frac{p_1}{q_N}\frac{p_N q_N}{p_1 q_1}$$

Recall $\varepsilon_{ij} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$ and let R_i be the total revenue spent on product *i*. Then:

$$\varepsilon_{11} - 1 = \sum_{i=2}^{N} \varepsilon_{i1} \frac{r_i}{r_1}$$

If R is the total revenue spent on all products and $s_i = r_i/R$ is the revenue share spent on product I, then:

$$\varepsilon_{11} = 1 + \sum_{i=2}^{N} \varepsilon_{i1} \frac{s_i R}{s_1 R}$$

$$= 1 + \frac{1}{s_1} \sum_{i=2}^{N} \varepsilon_{i1} s_i$$

Q.E.D.

A important relationship

- Proof (optional):

Let Δq_1 be the change in the quantity demanded of product 1 for a change in price Δp_1

Let Δq_i be the associated change in the quantity demanded of product *i* for a change in price Δp_1 for substitutes 2, . . . *N*

As consumers shift their purchases from product 1 to the substitute products, the amount they save on product 1 will be equal to the amount they spend on the substitutes:

Budget constraint: $p_1 \Delta q_1 - \Delta p_1 (q_1 - \Delta q_1) = p_2 \Delta q_2 + \cdots + p_N \Delta q_N$

$_i/q_i$:

Expanding and dividing by Δp_1: $$\frac{\Delta q_1}{\Delta p_1}\frac{p_1}{} - \frac{\Delta p_1}{\Delta p_1}\frac{q_1}{} = \frac{\Delta q_2}{\Delta p_1}\frac{p_2}{} + \cdots + \frac{\Delta q_N}{\Delta p_1}\frac{p_N}{}$$

Let *Q* be the aggregate quantity of all products 1 through *N* and let $s_i = \frac{q_i}{Q}$ be the market share of product *i.*

Divide by q_1: $$\frac{\Delta q_1}{\Delta p_1}\frac{p_1}{}\frac{s_1}{q_1} - q_1 \frac{s_1}{q_1} = \frac{\Delta q_2}{\Delta p_1}\frac{p_2}{}\frac{s_2}{q_1} + \cdots + \frac{\Delta q_N}{\Delta p_1}\frac{p_N}{}\frac{s_N}{q_1}$$

Recall $\varepsilon_{ij} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$:

$$\varepsilon_{11} s_i = \varepsilon_{21} s_i + \cdots + \varepsilon_{N1} s_i$$

$$\varepsilon_{11} = \sum_{i=2}^{N} \varepsilon_{i1} s_i$$

Q.E.D.

Markets and Market Equilibria

Price formation models

- Standard assumptions in the neo-classical model
  - Consumers
    - Individually maximize preferences (utility) subject to their individual budget constraints
    - Yields a consumer demand function, which gives the quantity demanded $q_i^{demanded}$ by consumer *i* for a given market price *p*
  - Firms
    - Individually maximize profits subject to their available production technology (production possibility sets)
    - Yields a production function that gives the quantity produced $q_j^{produced}$ by firm *j* for a given market price *p*
  - Equilibrium condition
    - No price discrimination (all purchases are made at the single market price)
    - Market clears at the market price (i.e., demand equals supply):

$$\sum_i q_i^{demanded} = \sum_j q_j^{produced}$$

$\sum$ simply means to add up the *q*'s. So if $q_1 = 10$. $q_2 = 7$, and $q_3 = 5$, then $\sum q_i = 10 + 7 + 5 = 22$.

Perfectly Competitive Markets

Perfectly competitive markets

- *Definition*: A market in which no single firm can effect price, meaning:
  - The firm's residual demand curve is horizontal
  - The firm can sell any amount of product without affecting the market price
  - $\frac{dp}{dq} = 0$
  - $p = \frac{dc}{dq}$ (i.e., price = marginal cost)

These four bullets are just different ways of saying exactly the same thing.

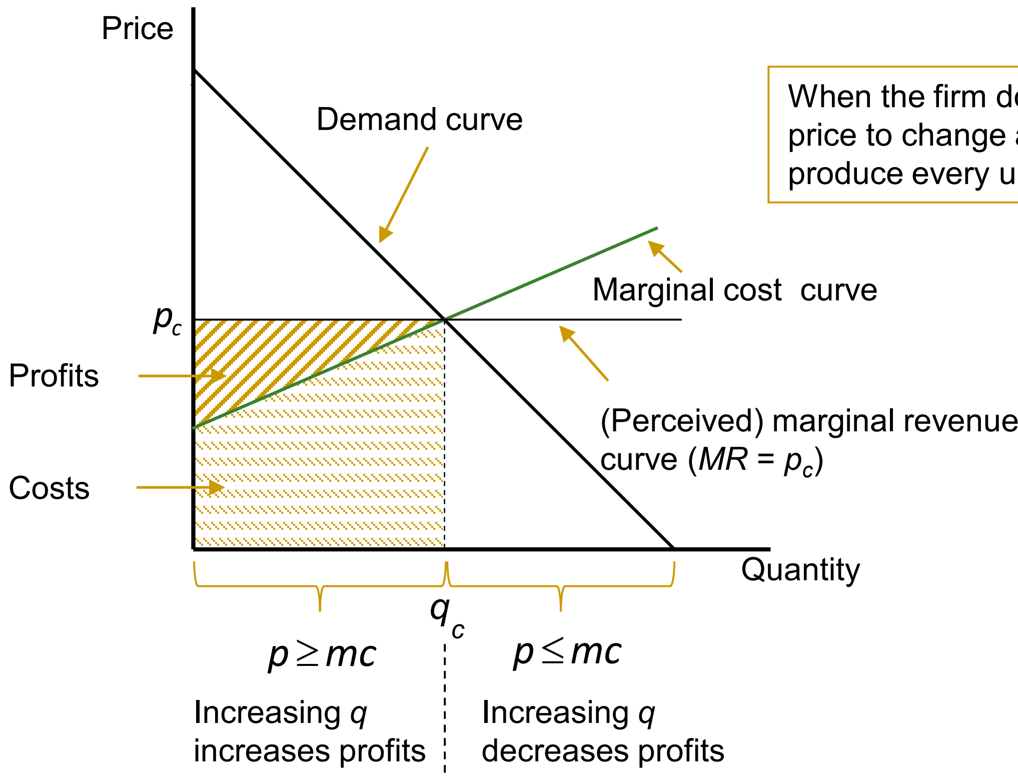
- Some more definitions
  - *"Price taking":* Competitive firms are called *price-takers*, that is, they take price as given and not something that they can affect
  - *Perfectly competitive equilibrium*: A market equilibrium where:
    - Aggregate supply equals aggregate demand, and
    - Each firm chooses its level of production so that the market-clearing price is equal to the firm's marginal cost of production

Perfectly competitive markets

- What could cause a market to be perfectly competitive?
  - *Traditional theory*: Each individual firm's production is very small compared to aggregate demand at any price, so that individual production changes cannot move significantly along the aggregate demand curve
    - This implies that there are a very large number of firms in the market
  - *Modern theory*: Competitors in the market place react strategically but non-collusively to price or quantity changes by a firm in ways that maintain the perfectly competitive equilibrium

Competitive firms

- Competitive firms take prices as given
  - Each individual firm perceives that its output decision does not affect the market-clearing price
  - This means that the firm acts *as if* $mr = p_c$



When the firm does not expect the market-clearing price to change as the firm expands output, the firm will produce every unit for which $p \geq mc$

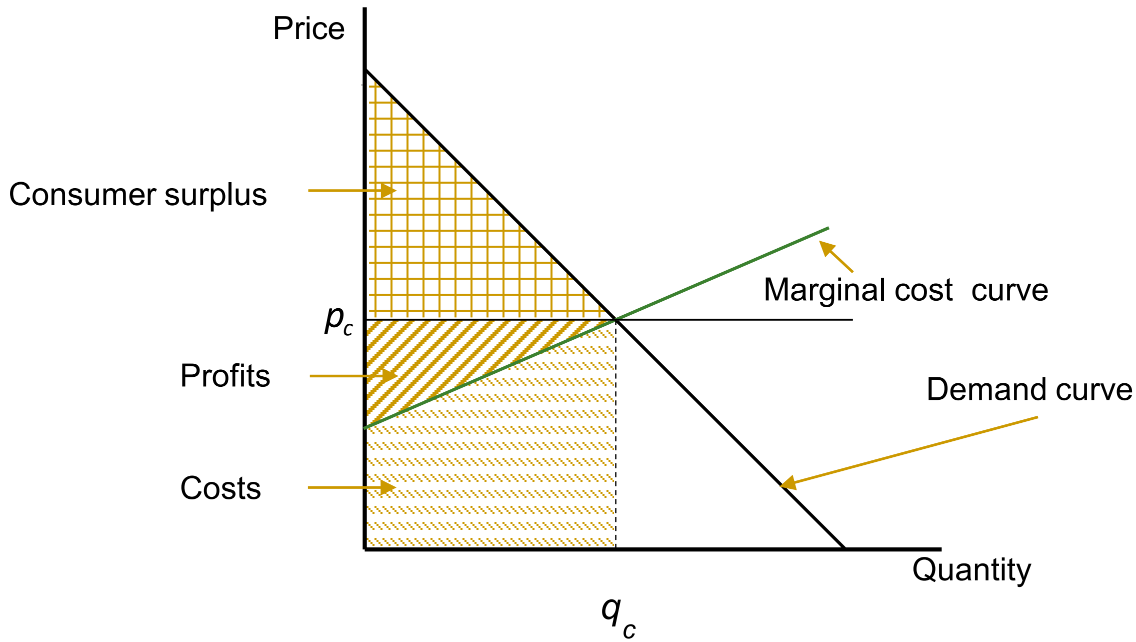
Rule: As always, the FOC is $mr = mc$.
If the firm is competitive, then
$mr = p_c$ and so FOC is $p_c = mc$.

Competitive firms

- Three take-aways
  1. Competitive firms do not perceive that their output decisions affects the market-clearing price
     - That is, they perceived that they face a horizontal demand curve
     - In fact, their output decisions do affect the market-clearing price but they do not perceive it
       - We know this since in the aggregate the output of all competitive firms does affect the market-clearing price
  2. Competitive firms chose their output so that $p = mc$
     - Competitive firm, like all other firms, choose output so that marginal revenue is equal to marginal cost ($mr = mc$)
     - Since a competitive firm does not perceive that its output decisions affect the market-clearing price, the firm does not perceive that there is any downward adjustment in market price when it expands its output.
     - Therefore, the firm perceives—and makes it output decision—on the premise that its marginal revenue is equal to the market price.
     - Hence, the firm select an output level so that $p = mc$.

Competitive firms

- Three take-aways
  3. A competitive market maximizes consumer surplus[1]
     - A competitive market exhausts all gains from trade



[1] We are assuming a simple market where there is only one product that sells at a single uniform price (i.e., there is no price discrimination).

Perfectly Monopolized Markets

Perfect monopoly

- Basic concepts
  - In a perfect monopoly market, there is only one firm that supplies the product
    - This is an economic concept
    - In law, a monopolist need not control 100% of the market
  - The aggregate demand curve defines the residual demand curve facing the firm

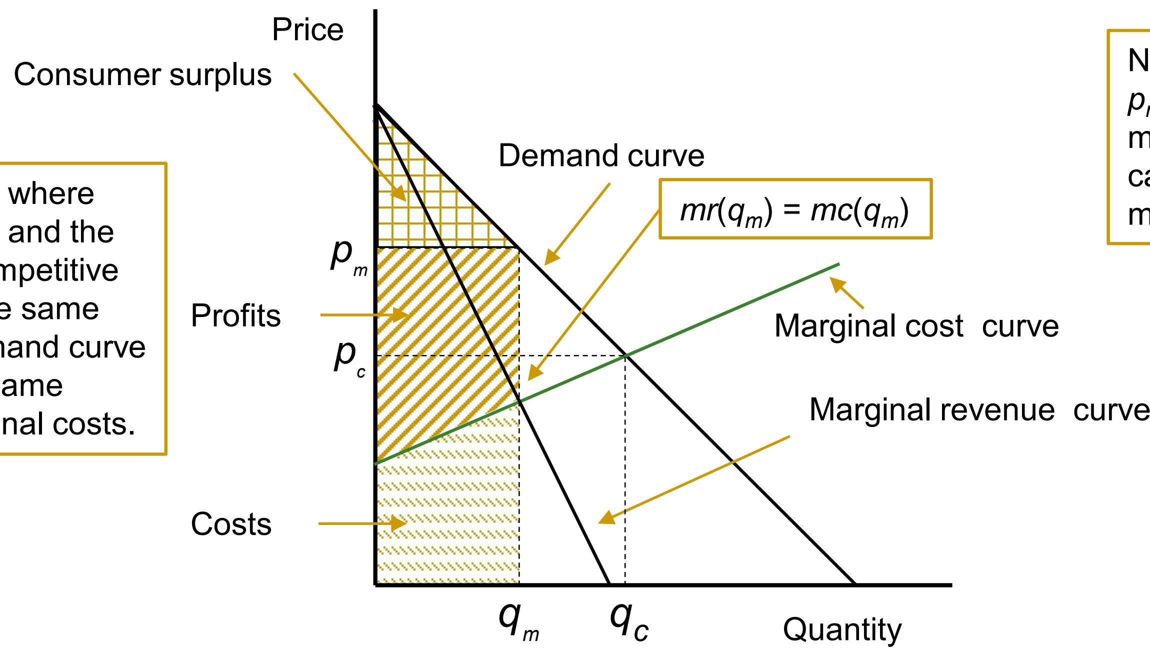
Monopolist firm

- A monopolist's chooses output q_m so that $mr(q_m) = mc(q_m)$
  - A monopolist charges a higher price than a competitive firm

$$p_m > mr(q_m) = mc(q_m) = p_c$$

  - A monopolist produces a lower output than would a competitive firm facing the same residual demand curve ($q_m < q_c$)

NB: $q_m = ½\, q_c$, where the monopolist and the firms in the competitive market face the same aggregate demand curve and have the same constant marginal costs.



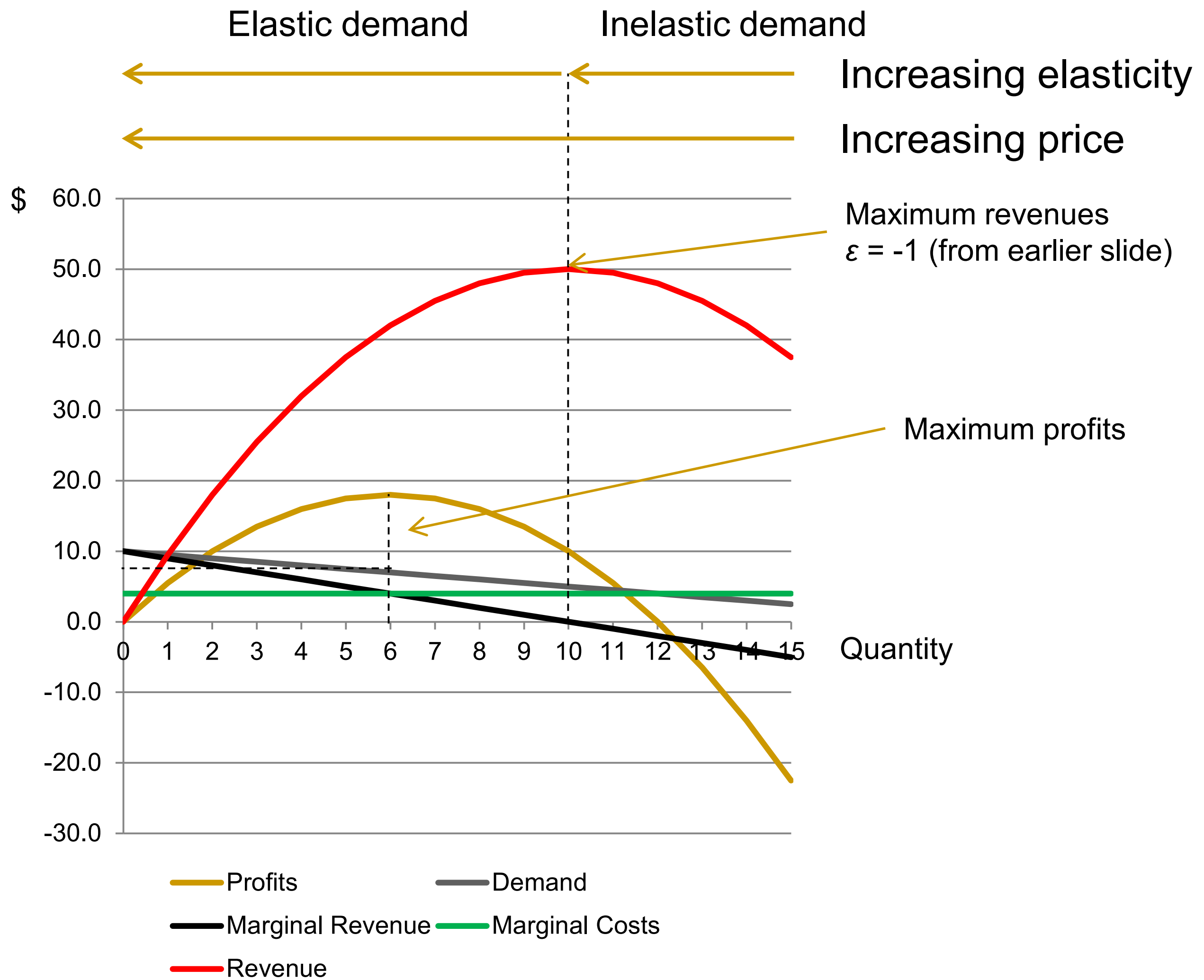
NB: The monopolist price p_m is the price at which the maximum available profits can be drawn from the market.

Monopolists and elasticities

- Proposition
  - A monopolist will not operate in the inelastic portion of its demand curve

Remember:

$$\varepsilon = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$



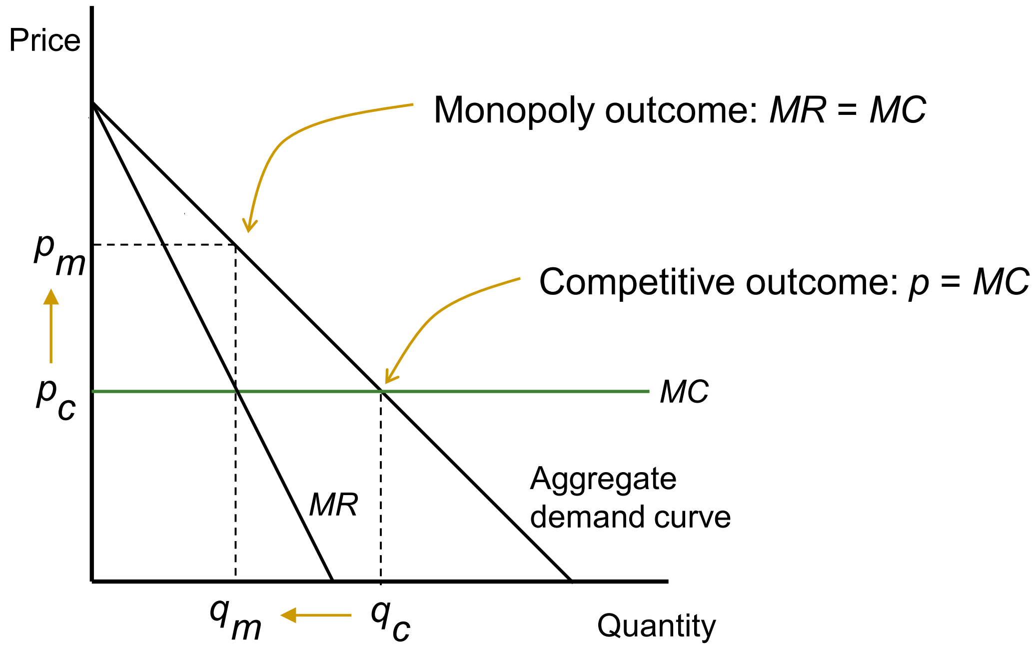
Review: Public policy on monopolies

- Modern view on why monopolies are bad:
  - Increase price and decrease output
  - Shift wealth from consumers to producers
  - Create economic inefficiency ("deadweight loss")

  - May (or may not) have other socially adverse effects
    - Decrease product or service quality
    - Decrease the rate of technological innovation or product improvement
    - Decrease product choice

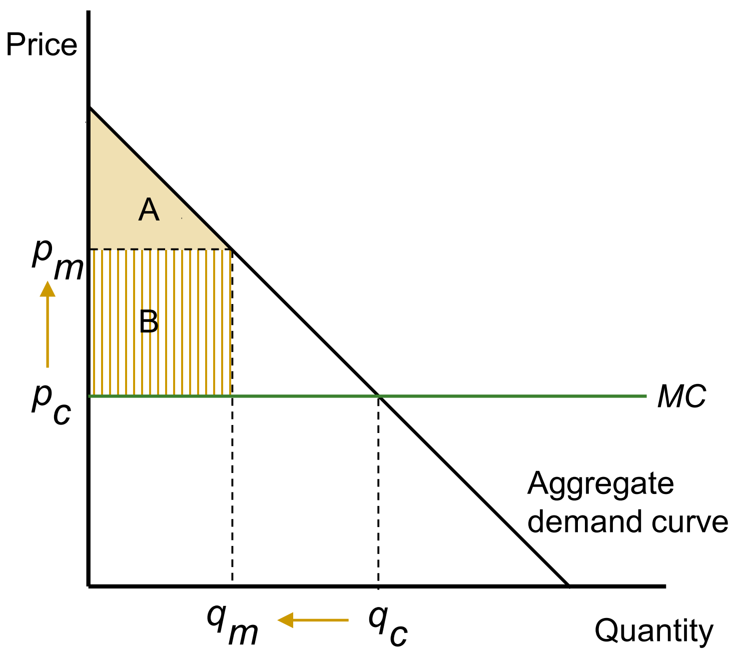
Review: Public policy on monopolies

- Output decreases: $q_c > q_m$
- Prices increase: $p_c < p_m$



Review: Public policy on monopolies

- Shift in wealth from inframarginal consumers to producers*
  - Total wealth created ("surplus"): A + B
  - Sometimes called a "rent redistribution"

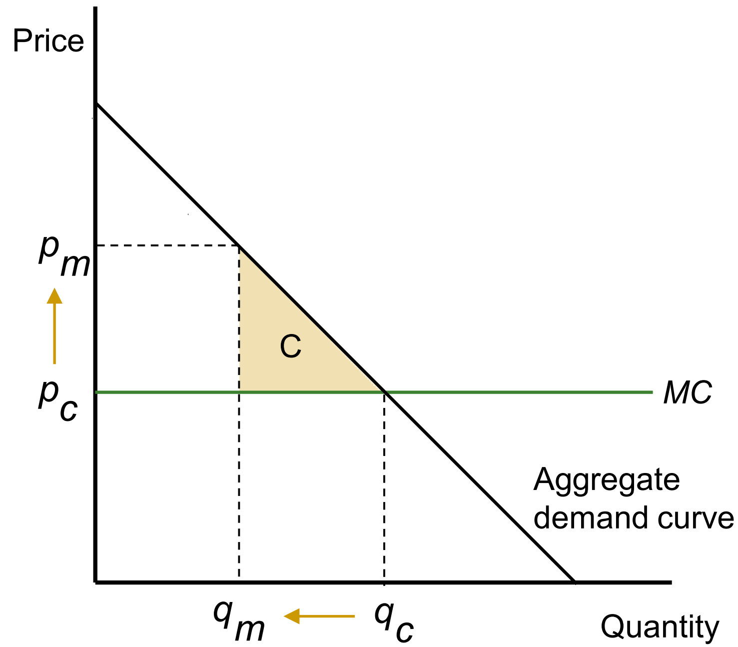


	Competitive	Monopoly
Consumers	A + B	A
Producers	0	B

* Inframarginal customers here means customers that would purchase at both the competitive price and the monopoly price

Review: Public policy on monopolies

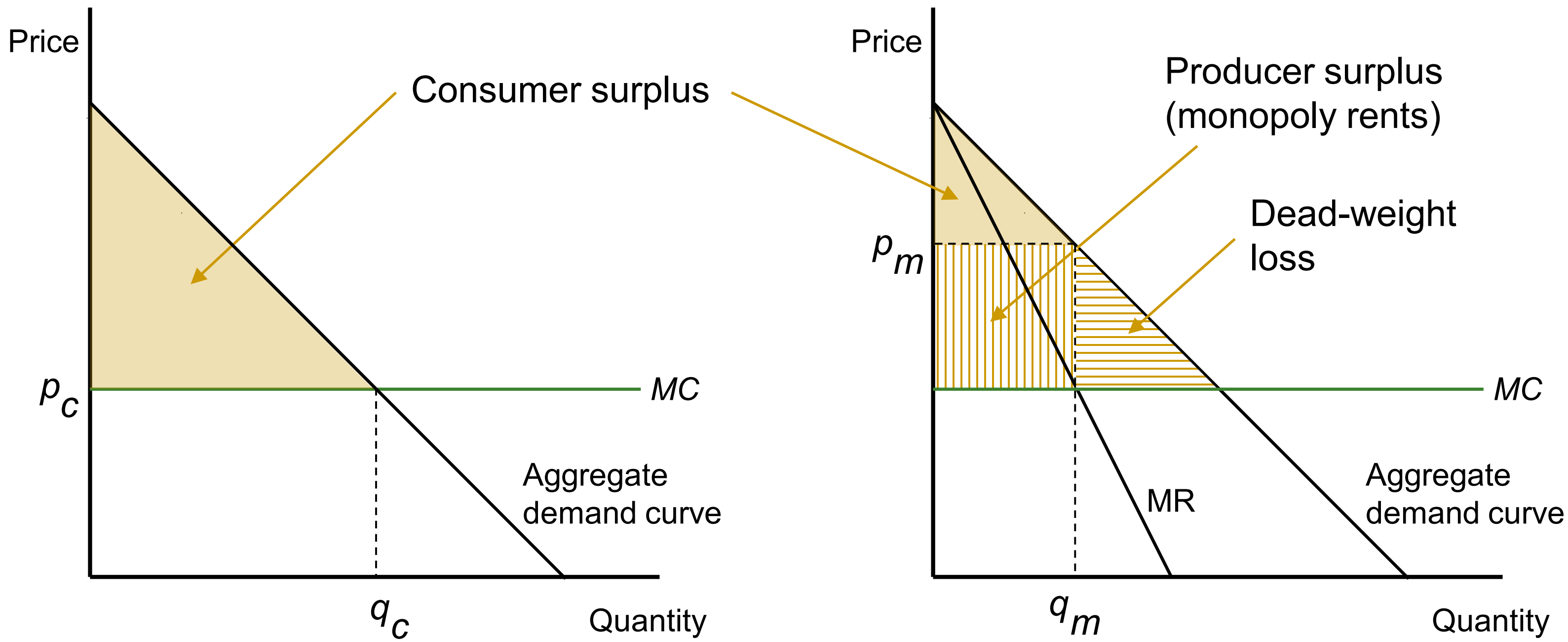
- "Deadweight loss" of surplus of marginal customers*
  - Surplus C just disappears from the economy
  - Creates "allocative inefficiency" because it does not exhaust all gains from trade



* Marginal customers here means customers that would purchase at both the competitive price and the monopoly price

Review: Public policy on monopolies

1. Shift in wealth from consumers to producers
2. Deadweight loss
3. May retard innovation



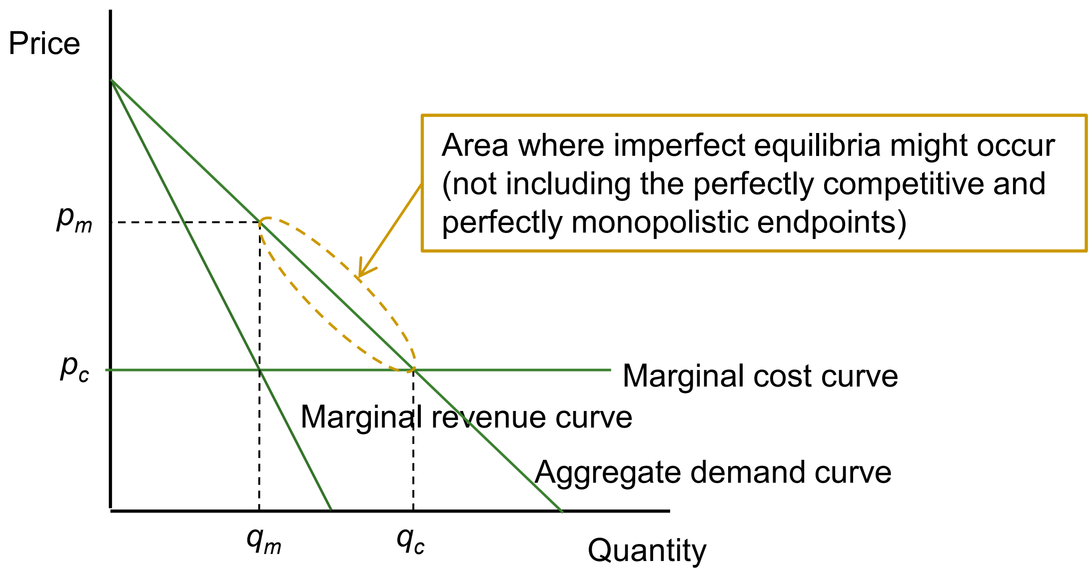
Perfectly Competitive Market

Perfect Monopoly Market

Imperfectly Competitive Markets

Imperfectly Competitive Markets

- Range of imperfect equilibria
  - An imperfectly competitive equilibrium occurs when the equilibrium price and output on the demand curve falls strictly between the perfect monopoly equilibrium and the perfectly competitive equilibrium

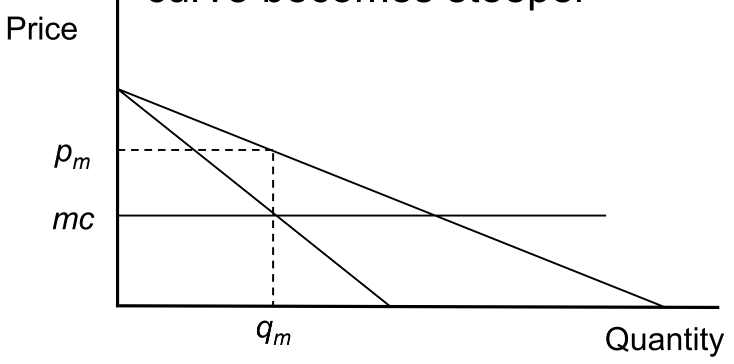


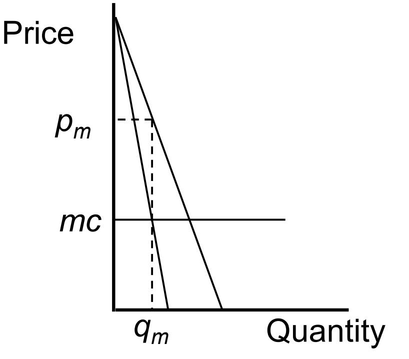
Market power

- Measuring market power
  - Economically, market power is the power of the firm to affect the market-clearing price through its choice of output level
  - The traditional economic measure of market power is the *price-cost margin* or *Lerner index L*, which is a measure of how much price has been marked up as a percentage of price:

$$L = \frac{p - mc}{p}$$

    - In a competitive market, $L = 0$ since because $p = mc$
    - In a perfectly monopolized market, L increases as the aggregate demand curve becomes steeper





Homogeneous product models

- Homogeneous product models
  - Assumes that products are undifferentiated (that is, *fungible* or *homogeneous*) in the eyes of the customer
  - Common examples:
    - Ready-mix concrete
    - Winter wheat
    - West Texas Intermediate (WTI) crude oil
    - Wood pulp
  - Two properties of homogeneous products
    - Customers purchase from the lowest cost supplier → This forces all suppliers in the market to charge the same price
    - Since the goods are identical, their quantities can be added

$$Q(p) = \sum q_i(p)$$

      - Adding all individual consumer demands at price p gives aggregate demand
      - Adding all individual firm outputs at price p gives aggregate supply

Cournot oligopoly models

- The setup
  - The standard homogenous product model is the *Cournot model*
  - In a Cournot model the firm's control variable is *quantity*
    - The (download-sloping) demand curve gives the relationship between the aggregate quantity produced *Q* and the market-clearing price *p*

$$p = p(Q), \text{ where } Q = \sum_{i=1}^{N} q_i,$$

  - The profit equation for firm *i* is:

$$\pi_i = p(Q)q_i - c_i(q_i), \quad i = 1,2,\ldots,N$$

Each firm *i* choses its level of output q_i, but the aggregate level of output determines the market prices

  - First order condition:

$$m\pi_i(q_i) = mr_i(q_i) - mc_i(q_i) = 0$$

This generates *N* equations in *N* unknows and can be solved for each q_i

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example
    - Compare the competitive, Cournot, and monopoly outcomes in this example
      Demand curve: $Q = 100 - 2p$

	Price	Quantity
Perfectly competitive	5 (= mc)	90
Cournot (n=2)	20	60
Perfect monopoly	27.5	45

  - When demand is linear and there are n identical firms in a Cournot model, then:

$$Q_{Cournot} = \frac{n}{n+1} Q_{Competitive}$$

$q_{competitive}$	90	90	90	90	90	90	90	90	90
n	9	8	7	6	5	4	3	2	1
$q_{cournot}$	81	80	78.8	77.1	75	72	67.5	60	45

Cournot oligopoly models (optional)

- Two important results
  - The firm's Lerner index

$$\lambda_i = \frac{p - mc_i}{p} = \frac{s_i}{\varepsilon}$$

where s_i is the market share of firm *I* and ε is the own-elasiticity of demand of the aggregate demand curve and the market equilibrium price

  - So the market Lerner index is:

$$\lambda = \sum_{i=1}^{N} \frac{p - mc_i}{p} s_i = \sum_{i=1}^{N} \frac{s_i^2}{\varepsilon} = \frac{HHI}{\varepsilon}$$

where λ is the market-share weighted sum of the λ_I of the individual firms in the market

- The Herfindahl-Hirschman Index (HHI), which is the principal measure of market concentration, is the sum of the squares of the markets shares of the firms in the market. That is,

$$HHI = s_1^2 + s_2^2 + \cdots + s_N^2 = \sum_{i=1}^{N} s_i^2$$

Bertrand oligopoly models

- The setup
  - In a Bertrand model the firm's control variable is *price*
    - Compare with the Cournot model, where the firm's control variable is quantity
    - The (download-sloping) residual demand curve gives the relationship between the firms choice of price and the quantity consumers will demand from the firm at that price
  - The profit equation for firm i is:

This is the demand function

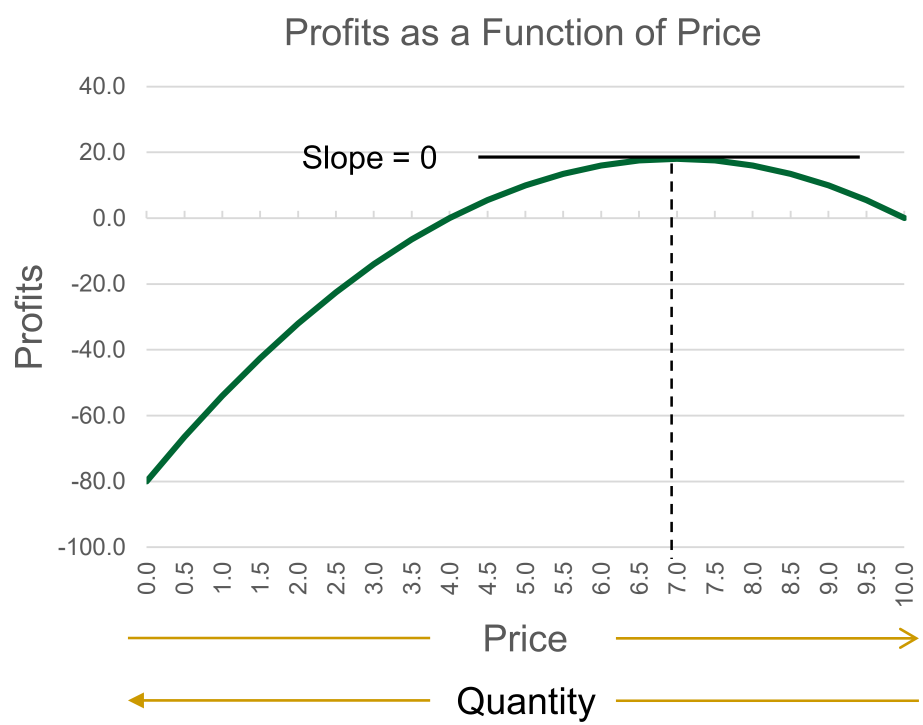
$$\pi_i(p_i) = p_i \underbrace{q_i(p_i)} - C_i(\underbrace{q_i(p_i)}), \quad i = 1,2,\ldots,N$$

Bertrand oligopoly models

- Profits as a function of price: Example

Demand: $q = 20 – 2p$
Fixed costs = 0
Marginal costs = 4

Price p	Quantity q	Revenues r	Costs C	Profits Π
0.0	20	0.0	80	-80.0
0.5	19	9.5	76	-66.5
1.0	18	18.0	72	-54.0
1.5	17	25.5	68	-42.5
2.0	16	32.0	64	-32.0
2.5	15	37.5	60	-22.5
3.0	14	42.0	56	-14.0
3.5	13	45.5	52	-6.5
4.0	12	48.0	48	0.0
4.5	11	49.5	44	5.5
5.0	10	50.0	40	10.0
5.5	9	49.5	36	13.5
6.0	8	48.0	32	16.0
6.5	7	45.5	28	17.5
7.0	6	42.0	24	18.0
7.5	5	37.5	20	17.5
8.0	4	32.0	16	16.0



Bertrand oligopoly models

- Observations
  - The profit curve as a function of price is a parabola
    - Although different in shape than the profit curve as a function of quantity
  - The profit maximum is when the slope of the profit curve is zero
  - So:
    - Marginal profits (as a function of price)
    - = Marginal revenues (as a function of price)
      minus marginal costs (as a function of price)
    - = 0

Bertrand oligopoly models

- Example

  Demand: $q = 20 - 2p$
  Fixed costs = 0
  Marginal costs = 4

  - Revenues:

$$\begin{aligned}\pi(p) &= pq(p) - C(q(p)) \\ &= p(20-2p)\ \text{-}4(20-2p) \\ &= 20p - 2p^2 - 80 + 8p \\ &= -2p^2 - 28p - 80\end{aligned}$$

This describes the parabola on the prior slide

  - Marginal revenues: $mr(p) = 20 - 4p$

Remember, if y = *a* + *bx* is the function, then the marginal function of *xy* is *a* + 2*bx*

  - Cost

$$\begin{aligned}C(q(p)) &= C(20-2p) \\ &= 4(20-2p) \\ &= 80 - 8p\end{aligned}$$

  - Marginal cost: $mc(p) = -8$
  - FOC:

$$mr(p) = mc(p)$$

$$20 - 4p = -8$$

So $p^* = 7$

Bertrand oligopoly models

- Homogeneous products case with equal cost functions
  - Consider two firms producing homogeneous (identical) products at constant marginal cost *c* and use price as their control variable
  - Consumers also purchase from the lower priced firm; if both firms charge the same price, they split equally consumer demand
  - Profit function for firm *i*:

$$\pi(p_i) \begin{cases} = p_i q_i(p_i) - c\big(q_i(p_i)\big) & \text{if } p_i < p_j \\ = \dfrac{p_i q_i(p_i) - c\big(q_i(p_i)\big)}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

  - That is, firm *i* gets 100% of market demand at price p_i if p_i is the lower price of the two firms, the two firms split the market demand if their prices are equal, and firm *i* gets nothing if it has the higher price
  - *Equilibrium*: $p_1 = p_2 = mc$, so that both firms price at marginal cost (i.e., the competitive price) and split equally market demand and total market profits

Bertrand oligopoly models

- Homogeneous products case with asymmetric cost functions
  - Now consider two firms producing homogeneous (identical) products but with different cost functions costs, with firm 1 have lower marginal costs than firm 2 (i.e., $mc(q(p(1)) < mc(q(p2)))$
  - The profit function is the same as before:

$$\pi(p_i) \begin{cases} = p_i q_i(p_i) - c(q_i(p_i)) & \text{if } p_i < p_j \\ = \dfrac{p_i q_i(p_i) - c(q_i(p_i))}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

  - *Equilibrium*: Firm 1 prices just below firm 2 and captures 100% of market demand
    - *Idea*: firm 1 and firm 2 compete the price down to firm 2's marginal cost as in the symmetric cost case. Then firm 1 just underprices firm 2 and captures 100% of the market demand

Bertrand oligopoly models

- Differentiated products case
  - When products are differentiated, a lower price charged by one firm will not necessarily move all of the market demand to that firm
    - Consider a market with only red cars and blue cars.
    - Some consumers like blue cars so much that even if the price of red cars is lower than the price of blue cars there will still be positive demand for blue cars
    - Moreover, if the price of blue cars increases, some (inframarginal) blue car customers will purchase blue cars at the higher price while some (marginal) customers will switch to red cars
    - This means that the demand for red cars (and separately for blue cars) is a function both of the price of red cars and the price of blue cars
    - It also means that the price of blue cars may not equal the price of red cars in equilibrium

Bertrand oligopoly models

- Differentiated products case
  - Simple linear model
    - Firms 1 and 2 produce differentiated products and face the following residual demand curves:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

NB: Each firm's demand decreases with increase in its own price and increases with increases in the price of the other firm

Assume that $b_1 > b_2$, so that each firm's residual demand is more sensitive to its own price than to the other firm's price

    - Assume each firm has a cost function with no fixed costs and constant marginal costs:

$$c_i(q_i) = cq_i$$

    - Firm 1's profit-maximization problem:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2)$$

NB: This formulation does not take into account firm 2's reaction to a change in firm 1's price

    - Bertrand equilibrium:

$$p_1^* = p_2^* = \frac{a + cb_1}{2b_1 - b_2}$$

Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with
    - a dominant firm, which sees its output decisions as affecting price and so sets output so that $mr = mc$, and
    - a fringe of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., $p = mc$)
  - Choice question for the dominant firm: Pick the profit-maximizing level for its output given the competitive fringe
    - The model requires some constraint on the ability of the competitive fringe to expand it output. Otherwise, the competitive fringe will take over the market.
    - The constraint usually is either limited production capacity or increasing marginal costs

Dominant firm with a competitive fringe

- The model
  - At market price p, let $Q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe. Then the residual demand $q_d(p)$ for the dominant firm is $Q(p) - q_f(p)$.
  - The dominant firm's profit maximization problem:

$$\max_{p} \pi_D = p \times \left[Q(p) - q_f(p) \right] - C\left(q(p)\right)$$

The dominant firm does not control market price directly, it in this model it can determine the price at which it would maximize its profits, and then back out the quantity it should produce using the aggregate demand function

Dominant firm with a competitive fringe

- Dominant oligopolies
  - The model can be extended to the case where the dominant firm is replaced by a dominant oligopoly
    - The key is to specify the solution concept for the choice of output by the firms in the oligopoly (e.g., Cournot). You then create a residual demand curve for the oligopoly and apply the solution concept to that demand curve.
- Fringe firms
  - As we saw in Unit 2, the DOJ and the FTC typically ignore fringe firms. The dominant oligopoly model with a competitive fringe provides a theoretical justification.