# Basic Competition Economics

Merger Antitrust Law

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# Topics

- Introduction
- Consumer demand and the aggregate consumer demand curve
- Producer profit maximization and the aggregate supply curve
- Perfect market equilibria
  - Perfectly competitive markets
  - Perfect monopoly
  - Incentives for coordination
  - Public policy re monopoly
- Imperfect market equilibria
  - Cournot equilibria
  - Bertrand equilibria
  - Dominant firm with a competitive fringe
- Merger typology, substitutes and complements, and elasticities

# Introduction

# Basic competition economics

- Questions
  - How does a firm chose its production levels and prices in order to maximize its profits absent a price-fixing agreement?
  - How can firms coordinate their behavior—through a price-fixing agreement or a merger—to increase their aggregate profits?

# Basic competition economics

- Two asides
  - A plea

> Do not be put off by the mathematical notation in the slides that follow. All of the notation can be ignored without losing any substance. However, economics is an essential language in modern antitrust law in general and merger antitrust law in particular. As you will see, economists love to use mathematical notation to make things look complicated, but with a small investment of effort you will see that all of this is very simple. Learning the basic economics is an investment that will give you a significant comparative advantage against many other antitrust attorneys.

  - An observation by Dave Berry

> Later on, Newton also invented calculus, which is defined as “the branch of mathematics that is so scary it causes everybody to stop studying mathematics.” That's the whole point of calculus. At colleges and universities, on the first day of calculus, professors go to the board and write huge, incomprehensible “equations” that they make up right on the spot, knowing that this will cause all the students to drop the course and never return to the mathematics building. This frees the professors to spend the rest of the semester playing cards and regaling one another with stories about the “mathematical symbols” they've invented over the years. (“Remember the time Professor Hinkwattle drew a ‘cosine derivative’ that was actually a picture of a squid?” “Yes! Students were diving out the windows! From the fourth floor!”)[1]

[1] Dave Berry, *Up in the Air on the Question of Gravity*, Baltimore Sun, Mar. 16, 1997, at 3J.

# Price formation models

- Standard assumptions (neo-classical model)
  - Consumers
    - Individually maximize preferences (utility) subject to their individual budget constraints
    - Yields a consumer demand function, which gives the quantity demanded $q_i^{demanded}$ by consumer *i* for a given market price *p*
  - Firms
    - Individually maximize profits subject to their available production technology (production possibility sets)
    - Yields a production function that gives the quantity produced $q_j^{produced}$ by firm *j* for a given market price *p*
  - Equilibrium condition
    - No price discrimination (all purchases are made at the single market price)
    - Market clears at the market price (i.e., demand equals supply):

$$\sum_i q_i^{demanded} = \sum_j q_j^{produced}$$

$\Sigma$ simply means to add up the *q*'s

# Consumers

# Consumers

- *Assumption*: Consumers maximize their preference (utility) subject to their individual budget constraints
  - An individual consumer's demand for a product is a function of:
    - The consumer's preferences
    - The price the consumer pays for product
    - Other products and services the consumer may purchase and their respective prices
    - The consumer's budget constraint
  - The relationship between quantity and price is known as the *consumer demand function* or *consumer demand curve*
    - Typically, the consumer will purchase a larger quantity of the product as the price decreases
    - If so, then the consumer demand curve is *downward sloping*
  - The sum of consumer demand functions is known as the *industry (aggregate) demand function*

Almost all antitrust economic analysis takes this as the point of departure. It is a critical assumption.

# Demand curves

- Demand curves and inverse demand curves
  - A *demand curve* gives quantity as a function of price
    - *Example:* Given my budget constraint, if the price is $4.00, I will buy 12 units, but if the price is $5.00 I will buy only 10 units
  - Linear demand curves
    - Linear demand curves are straight lines.
      - Although demand curves need not be straight lines, all of the principles in which we will be interested may be illustrated using linear demand curves
    - A *linear demand curve* has the form $q = a + bp$, where $q$ is the quantity demanded at price $p$, $a$ is the quantity when $p = 0$, and $b$ gives the change in $q$ for a change in price
      - $q$ and $p$ are called *variables* and are the numbers of interest to us
        - They are related in pairs $(p_i, q_i)$ by the demand curve, that is, each $(p_i, q_i)$ lies on the demand curve so that $q_i = a + bp_i$ for each observation $i$
      - $a$ and $b$ are constants called *parameters*
        - The parameter $a$ is the quantity demanded when the price is equal to zero
        - The parameter $b$ is the *slope* of the demand curve: it gives the decrease in the quantity demanded for an increase of one unit in price
        - Since demand curves are downward sloping, b will be a negative number (i.e., $b < 0$)

# Demand curves

- Demand curves and inverse demand curves
  - Graphing demand curves
    - *Example:* Given my budget constraint, if the price is \$4.00 I will buy 12 units, but if the price is \$5.00 I will buy only 10 units
    - A linear demand curve has the form $q = a + bp$, where $q$ is the quantity demanded at price $p$, $a$ is the quantity when $p$ =0, and $b$ gives the change in $q$ for a change in price
      - $b$ is also the *slope* of the demand curve
    - If we know two points on a linear demand curve, we can derive the demand function

$$b = \frac{\text{Change in quantity}}{\text{Change in price}} \equiv \frac{\Delta q}{\Delta p} = \frac{-2}{1} = -2$$

The change $\Delta q$ is negative because demand declines as price increases

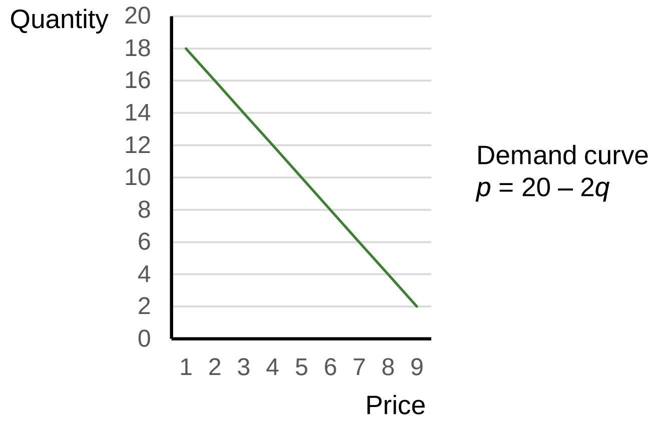
The symbol "≡" means a definition

Use one point to solve for $a$ (say, p =4; q = 12):

$$12 = a - 2 \bullet 4 \Rightarrow a = 20$$

So the demand curve is:

$$q = 20 - 2p$$

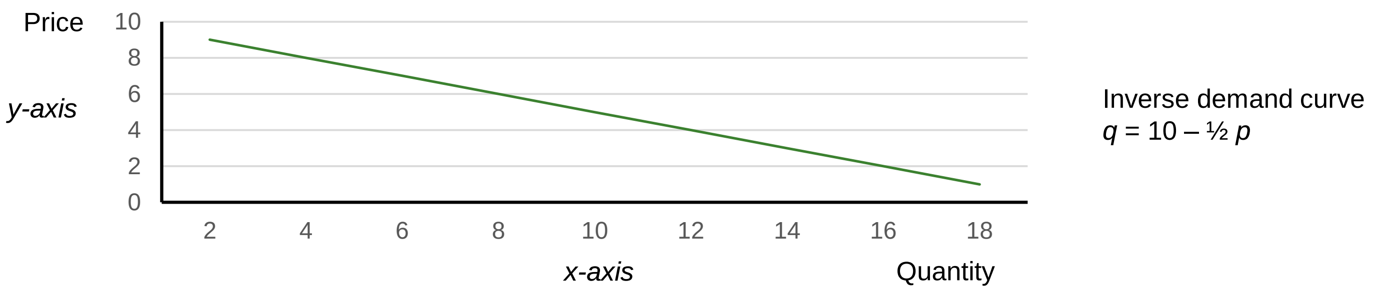


# Demand curves

- Demand curves and inverse demand curves
  - An *inverse demand curve* gives price as a function of quantity
  - So if the demand curve is $q = a + bp$, the inverse demand curve can be derived by solving for $p$:
    - *Example*: If the demand  curve is $q = 20 - 2p$, the inverse demand curve is:

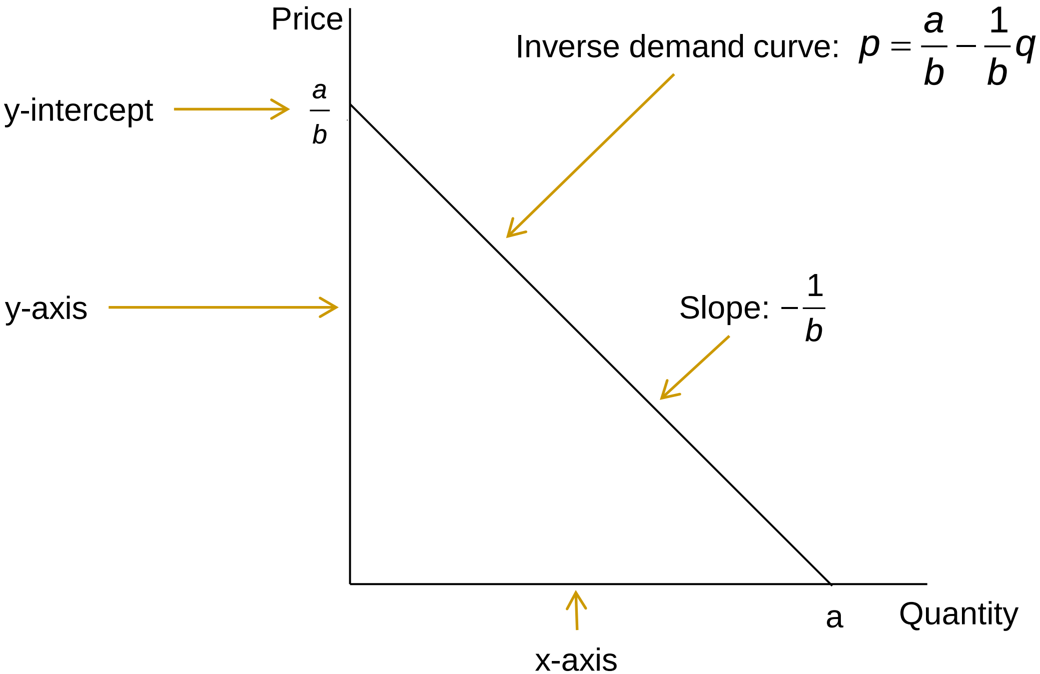
$$p = \frac{20 - 2q}{2} = 10 - \frac{1}{2}q$$

  - Think about the inverse demand curve as the price necessary to *clear the market* given production $q$
    - "Clear the market" means that consumers demand no more and no less that $q$ at price $p$
  - Inverse demand functions put price on the y-axis and quantity on the x-axis of a graph
    - The demand curve has the axes reversed



# Demand curves

- Linear (inverse) demand curve




The slope of the demand curve gives the required change in the price to sell one additional unit of the product. So the price needs to drop by -1/*b* to sell one additional unit:.

$$p_1 = \frac{a}{b} - \frac{1}{b}(q+1)$$

$$p_0 = \frac{a}{b} - \frac{1}{b}(q)$$

$$\Delta p = p_1 - p_0 = -\frac{1}{b}$$

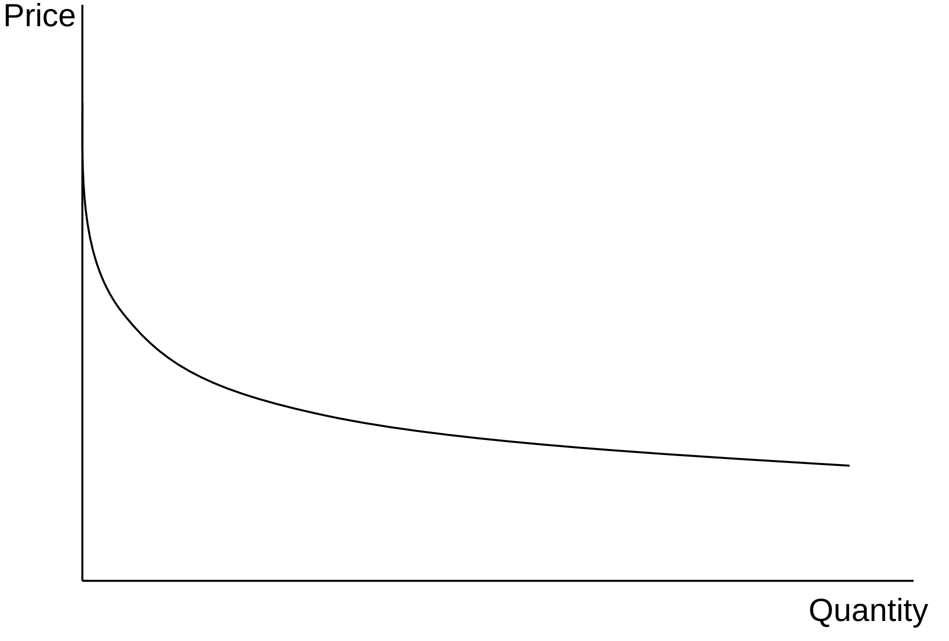
Notes: The y-intercept *a*/*b* is the price above which there is zero demand.

The x-intercept *a* is the quantity demanded when the price is zero.

For linear demand, unless the demand curve is strictly vertical, the x- and y-intercepts will be finite. Because they can be very large, this usually does not result in any loss of generality.

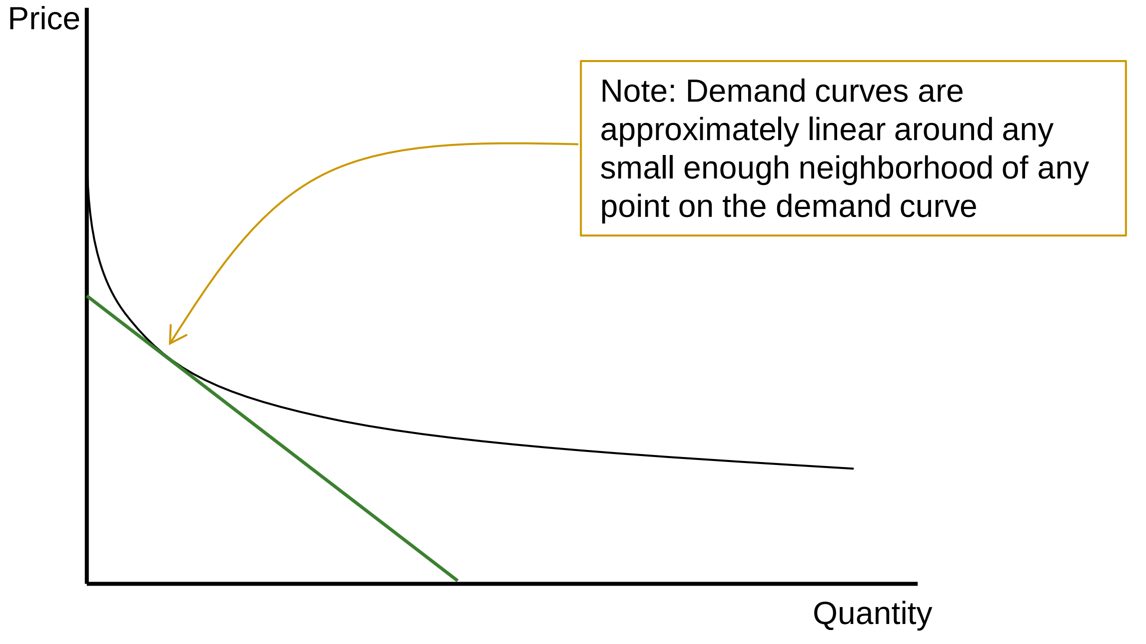
# Demand curves

- Example: Nonlinear inverse demand curve with no x-axis intercept



# Demand curves

- Example: Nonlinear inverse demand curve with no x-axis intercept



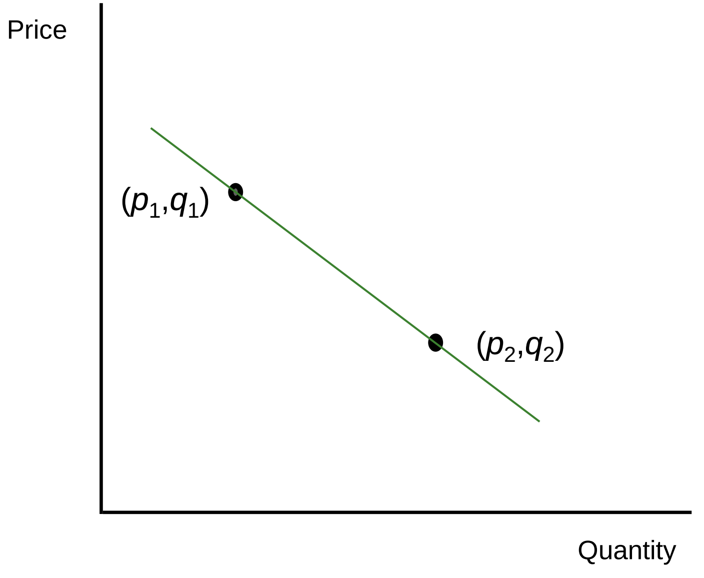
# Demand curves

- Some technical points about demand curves and inverse demand curves
  - When economists and antitrust lawyers refer to the demand curve, they almost always mean the inverse demand curve
    - You can tell the difference in context by whether:
      - *Demand curve*: Quantity is a function of price and on the y-axis
      - *Inverse demand curve*: Price is a function of quantity and on the y-axis
    - We will follow convention and not draw the distinction
  - What I have called the "demand curve" is really the "demand function"
    - The demand curve is the *graph* of the demand function
    - Distinguishing between the two will qualify you as an irredeemable geek
  - Demand curves are for aggregate demand in the marketplace, that is, the sum of demands by consumers on all firms in the marketplace
    - The demand curve for a single firm in the market is called the firm's *residual demand curve*
    - The residual demand curve is a critical concept in antitrust economics

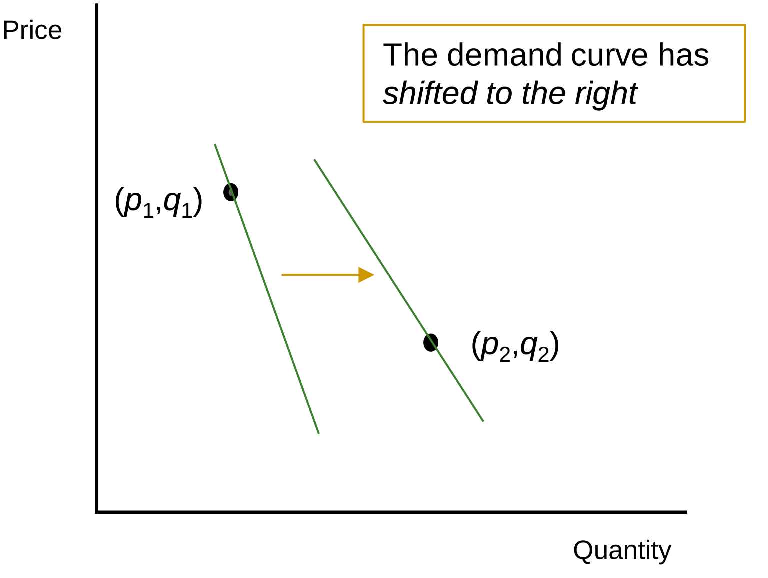
# Demand curves

- Some technical points about demand curves and inverse demand curves
  - Even if we assume linear demand, two observations of prices and quantities demanded $(p_1,q_1)$ and $(p_2,q_2)$ may either be—
    - On the same demand curve, or
    - On different demand curves (when demand has *shifted*)

Two points on the same demand curve




Two points on different demand curves



# Consumer demand functions

OPTIONAL

- Deriving the consumer demand function
  - Consider a world with two products offered at prices $p_1$ and $p_2$, respectively
  - If the consumer has a budget constraint $B$, then

This *inequality* "≥" simply says that the consumer's expenditure on product 1 ($p_1q_1$) and product 2 ($p_2q_2$) cannot exceed her budget

$$B \geq p_1q_1 + p_2q_2,$$

This means the price $p_1$ of product 1 times the quantity $q_1$ of product 1 that the consumer purchased, or the total expenditure by the consumer for product 1

where $q_1$ and $q_2$ are the quantities the consumer purchases of products 1 and 2

  - The inequality requires that the consumer cannot spend more than her budget on the two products
  - If the consumer always prefers more of each product to less, then she will always spend all of her budget (there are no savings in this model):

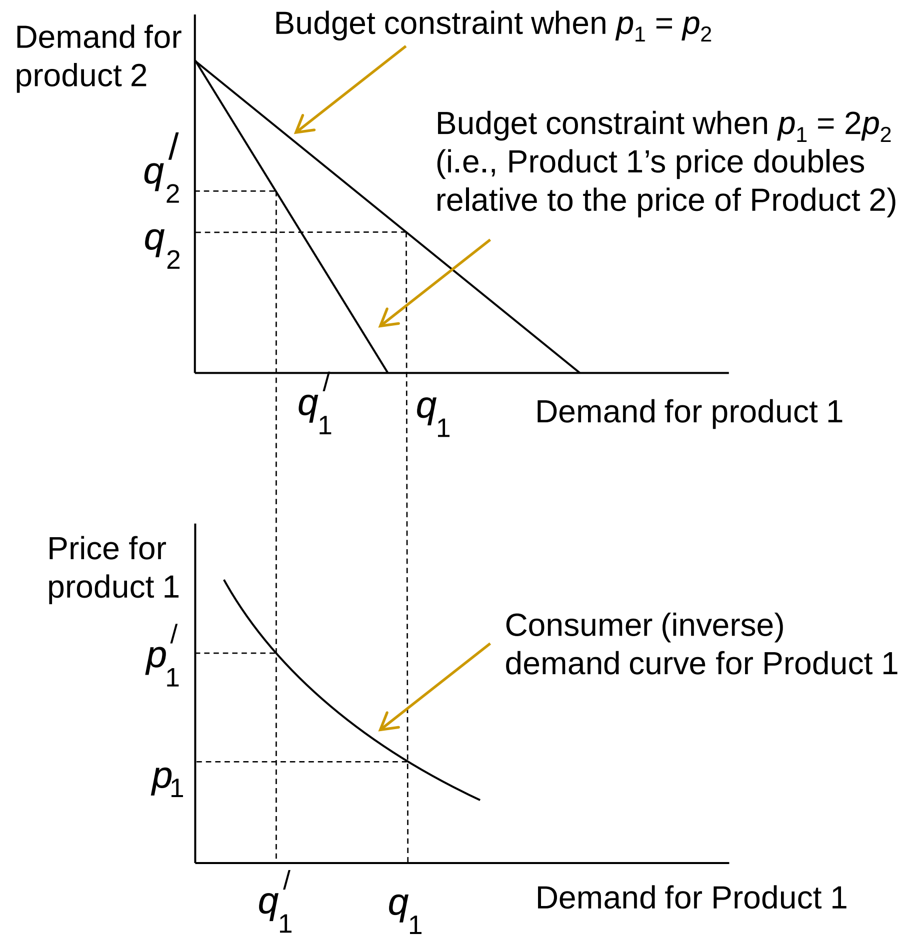
$$B = p_1q_1 + p_2q_2$$

This is the usual assumption

# Consumer demand functions

OPTIONAL

- Deriving the consumer demand function
  - At prices $p_1$ and $p_2$, the consumer purchases quantities $q_1$ and $q_2$
  - When price $p_1$ doubles relative to $p_2$, the consumer decreases its purchases of Product 1 to $q_1^{/}$ and increases its purchases of Product 2 to $q_2^{/}$
  - By holding the budget constant and varying $p_1$ relative to $p_2$ and observing the resulting quantities of Product 1 purchases produces the consumer demand function for that budget constraint
  - In a similar way, the demand function can be made a function of the budget constraint by observing purchases at different prices and different budgets

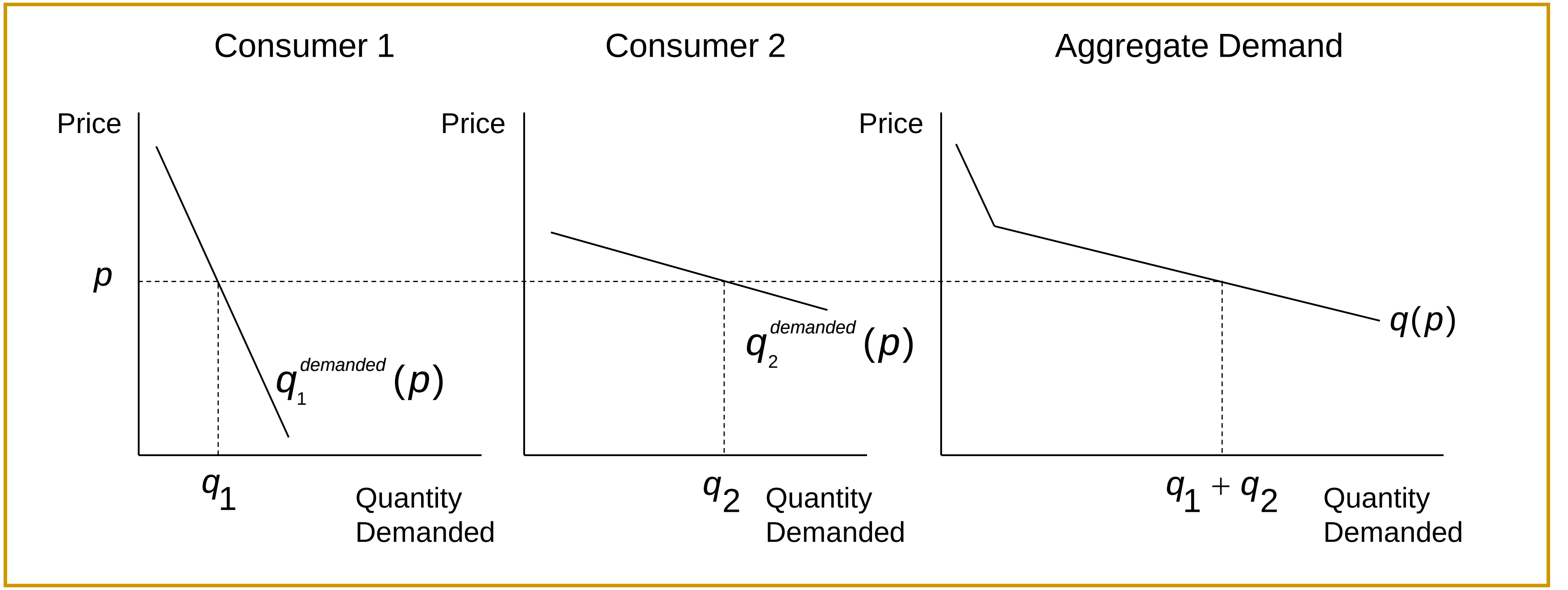


# Aggregate consumer demand

- Aggregate consumer demand
  - Sum of individual consumer demands = Aggregate consumer demand (by definition)

$$\sum_i q_i^{demanded}(p) \equiv q(p),$$

where $q(p)$ is aggregate demand at price $p$



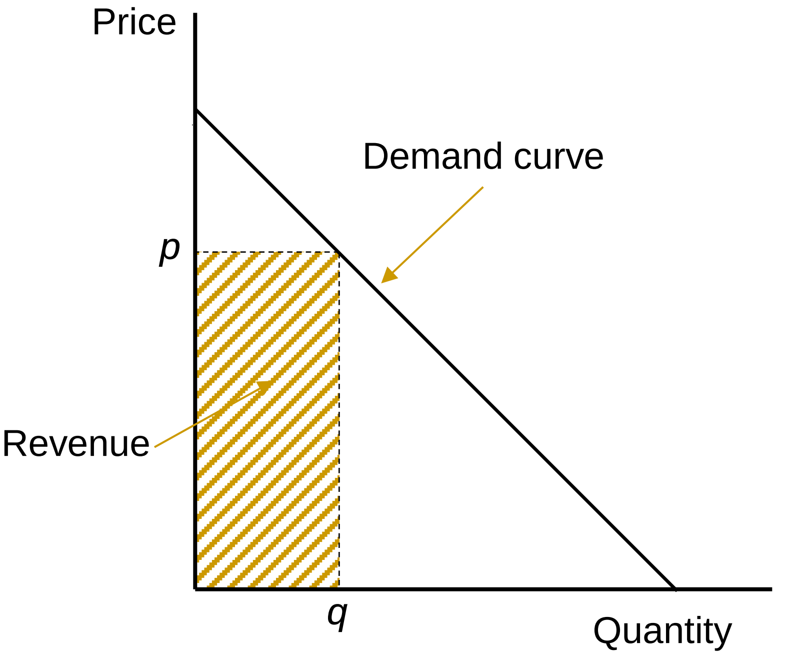
# Producers

# Producers

- *Assumption*: Firms maximize their profits subject to the technology available to them
  - Profits ($\pi$) = Revenues ($r$) – Costs ($c$)
- To analyze the conditions under which a firm maximizes it profit, need to look at:
  - Revenues and revenue functions
  - Costs and cost functions
  - The relationship between revenues and costs when the firm maximizes its profit

# Revenues

*Revenue* = *p* times *q* (= *pq*)



# Revenues

*Marginal revenue (mr)* = Revenue gain from incremental sales (the sale of one additional unit) – revenue loss from lower price on preexisting sales

$$\equiv \frac{\Delta r}{\Delta q} = \underbrace{(p + \Delta p)\Delta q}_{\text{Gain}} + \underbrace{\Delta p q}_{\text{Loss}}$$

NB: If $\Delta q$ is positive, the $\Delta p$ will be negative since demand curves are downward sloping. So $\Delta pq$ is a negative number, reflecting a loss.

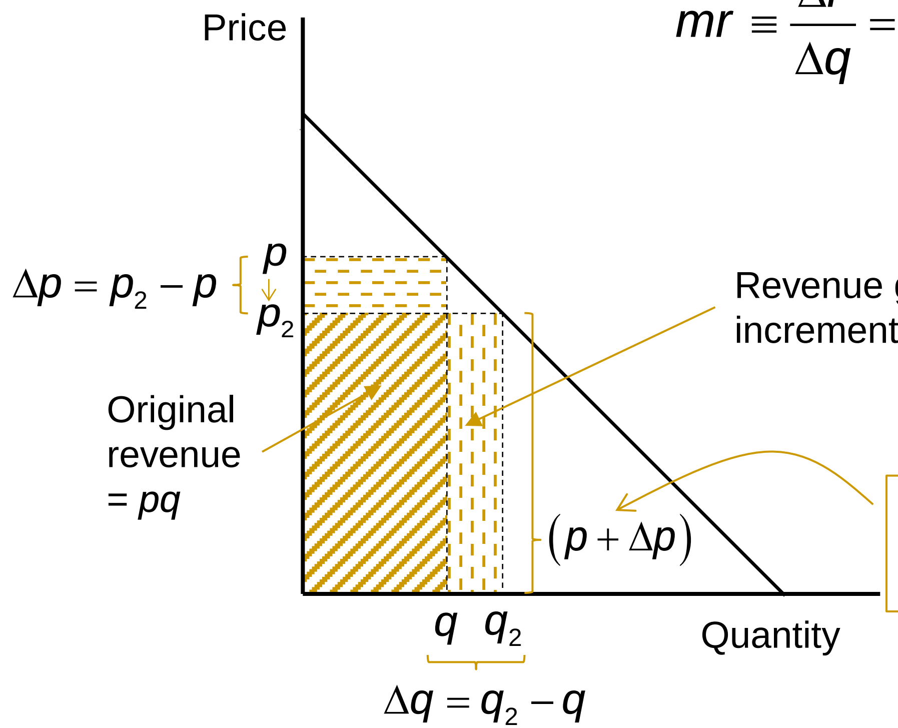
The next three slides demonstrate this

# Revenues

*Marginal revenue (mr)* = Revenue gain from incremental sales (the sale of one additional unit)
– revenue loss from lower price on preexisting sales

Revenue gain:

$$mr \equiv \frac{\Delta r}{\Delta q} = (p + \Delta p)\Delta q + \Delta pq$$




Revenue gain from lower price on incremental sales $= (p + \Delta p)\Delta q$

Revenues from each incremental sale. (Remember, $\Delta p$ is negative, meaning that price is decreasing)

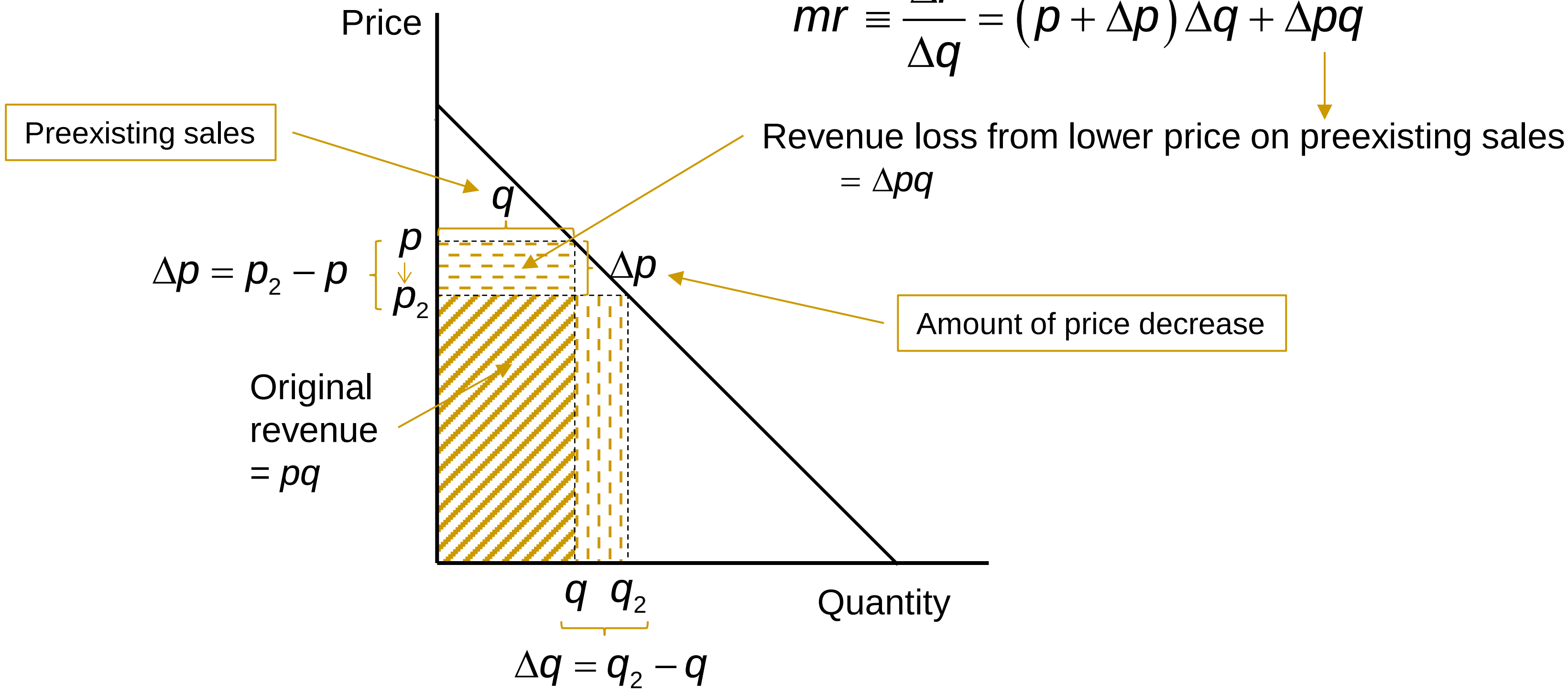
Incremental sales

# Revenues

*Marginal revenue (mr)* = Revenue gain from incremental sales (the sale of one additional unit) – revenue from lower price on preexisting sales

Revenue loss:

$$mr \equiv \frac{\Delta r}{\Delta q} = (p + \Delta p)\Delta q + \Delta pq$$

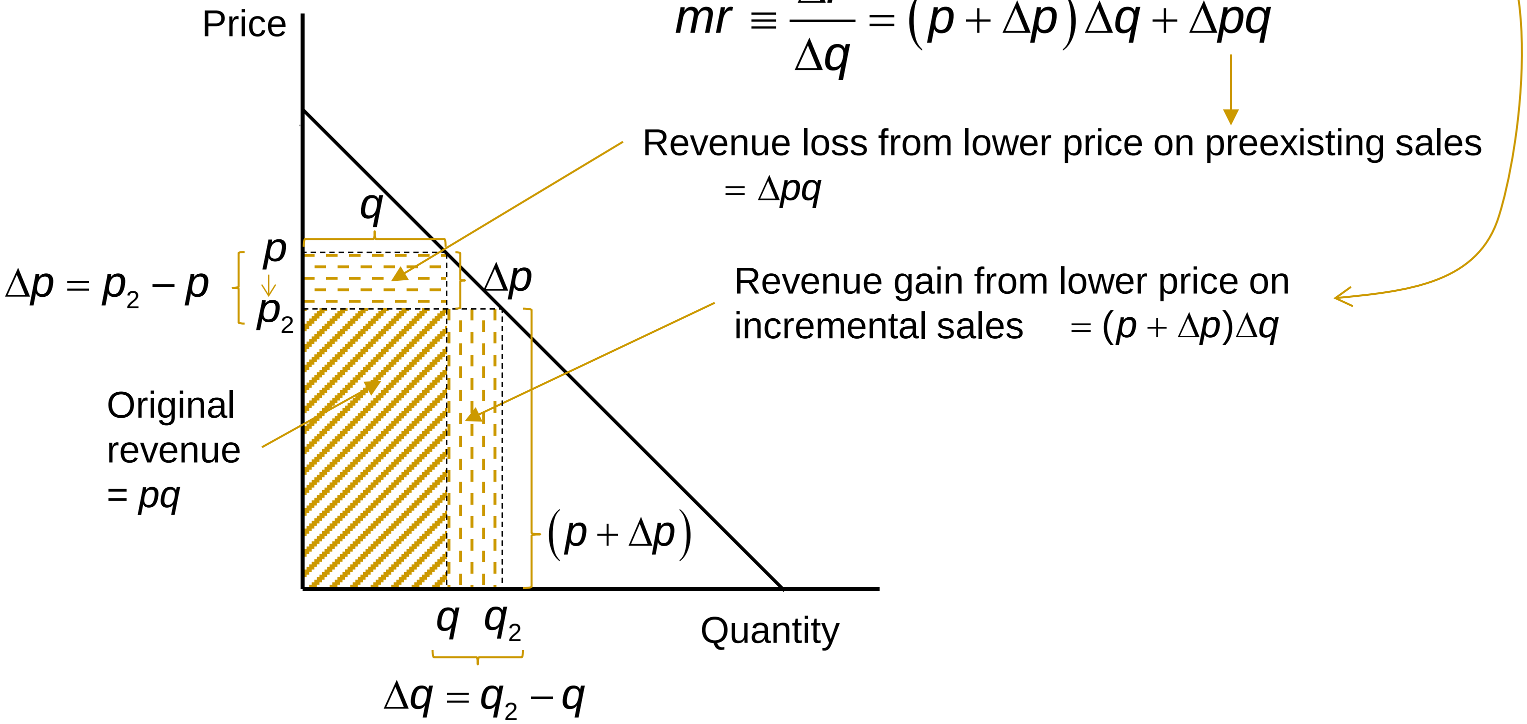


# Revenues

*Marginal revenue (mr)* = Revenue gain from incremental sales (the sale of one additional unit) – revenue loss from lower price on preexisting sales

Putting it together:

$$mr \equiv \frac{\Delta r}{\Delta q} = (p + \Delta p)\Delta q + \Delta pq$$



# Revenues

- Relationship between revenues and marginal revenue (discrete case)

Read this " *r* of *q*": This is the revenues at production level *q*.

$$r(q) = \sum_{i=1}^{q} mr_i$$

  - That is, total revenues for a production level *q* is equal to the sum of the marginal revenues for units 1 to *q*

# Revenues

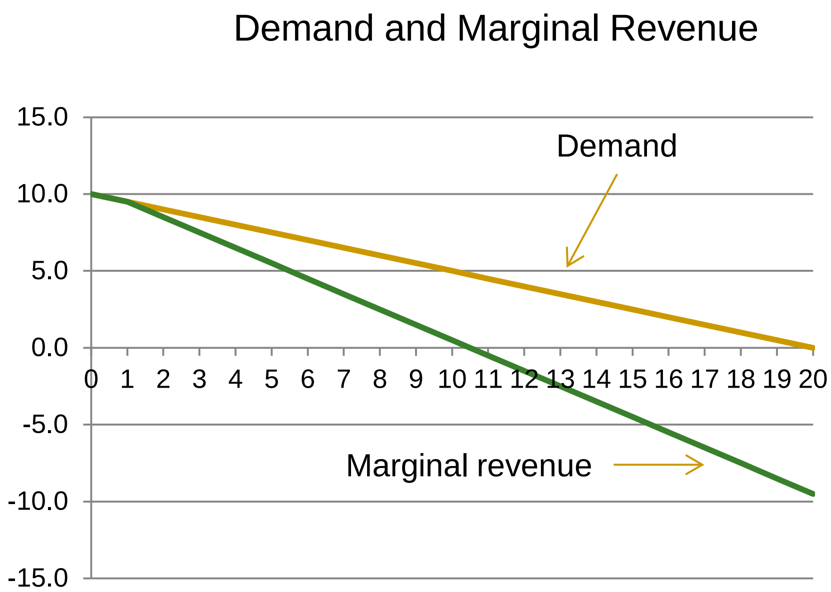
- Numerical example
  - Demand: $p = 10 - \frac{1}{2}q$

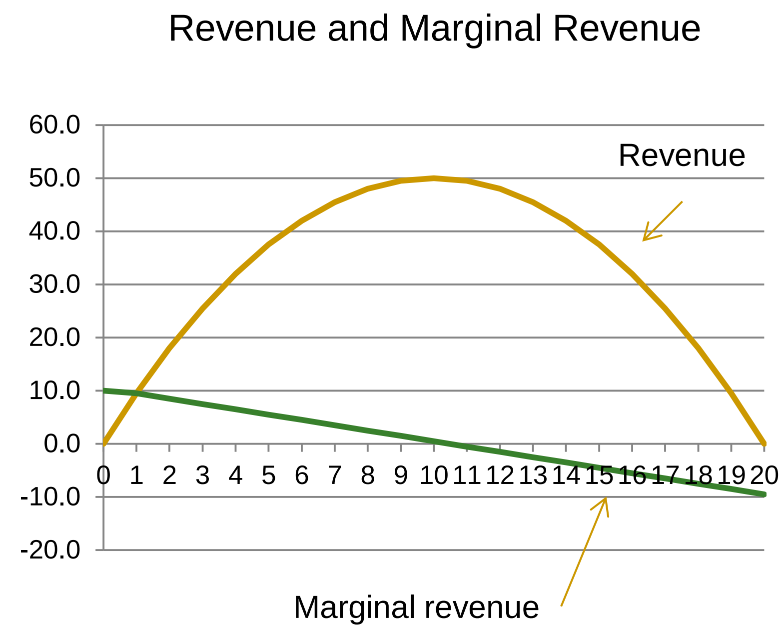
$$r(q) = \sum_{i=1}^{q} mr_i$$

| | | | Marginal | Change | Marginal | Marginal | Marginal | |
|---|---|---|---|---|---|---|---|---|
| Quantity | Price | Revenue | Quantity | Price | Gain | Loss | Revenue | Sum |
| *q* | *p* | *r* | Δq | Δp | (p+Δp)Δq | Δpq | (p+Δp)Δq+Δpq | *mr* |
| 0 | 10.0 | 0.0 | | | | | 10.0 | |
| 1 | 9.5 | 9.5 | 1 | -0.5 | 10.0 | -0.5 | 9.5 | 9.5 |
| 2 | 9.0 | 18.0 | 1 | -0.5 | 9.5 | -1.0 | 8.5 | 18.0 |
| 3 | 8.5 | 25.5 | 1 | -0.5 | 9.0 | -1.5 | 7.5 | 25.5 |
| 4 | 8.0 | 32.0 | 1 | -0.5 | 8.5 | -2.0 | 6.5 | 32.0 |
| 5 | 7.5 | 37.5 | 1 | -0.5 | 8.0 | -2.5 | 5.5 | 37.5 |
| 6 | 7.0 | 42.0 | 1 | -0.5 | 7.5 | -3.0 | 4.5 | 42.0 |
| 7 | 6.5 | 45.5 | 1 | -0.5 | 7.0 | -3.5 | 3.5 | 45.5 |
| 8 | 6.0 | 48.0 | 1 | -0.5 | 6.5 | -4.0 | 2.5 | 48.0 |
| 9 | 5.5 | 49.5 | 1 | -0.5 | 6.0 | -4.5 | 1.5 | 49.5 |
| 10 | 5.0 | 50.0 | 1 | -0.5 | 5.5 | -5.0 | 0.5 | 50.0 |
| 11 | 4.5 | 49.5 | 1 | -0.5 | 5.0 | -5.5 | -0.5 | 49.5 |
| 12 | 4.0 | 48.0 | 1 | -0.5 | 4.5 | -6.0 | -1.5 | 48.0 |
| 13 | 3.5 | 45.5 | 1 | -0.5 | 4.0 | -6.5 | -2.5 | 45.5 |
| 14 | 3.0 | 42.0 | 1 | -0.5 | 3.5 | -7.0 | -3.5 | 42.0 |
| 15 | 2.5 | 37.5 | 1 | -0.5 | 3.0 | -7.5 | -4.5 | 37.5 |
| 16 | 2.0 | 32.0 | 1 | -0.5 | 2.5 | -8.0 | -5.5 | 32.0 |
| 17 | 1.5 | 25.5 | 1 | -0.5 | 2.0 | -8.5 | -6.5 | 25.5 |
| 18 | 1.0 | 18.0 | 1 | -0.5 | 1.5 | -9.0 | -7.5 | 18.0 |
| 19 | 0.5 | 9.5 | 1 | -0.5 | 1.0 | -9.5 | -8.5 | 9.5 |
| 20 | 0.0 | 0.0 | 1 | -0.5 | 0.5 | -10.0 | -9.5 | 0.0 |

# Revenues

- Graphing revenue and marginal revenue curves
  - Demand: $$p = 10 - \frac{1}{2}q$$




Notes:

1. When demand is linear, the slope of the marginal revenue curve is twice as steep as the demand curve.
2. When marginal revenue equals zero (here, a $q = 10$), revenues are at their maximum.

# Revenues

OPTIONAL

- Marginal revenues
  - Discrete version

$$mr \equiv \frac{\Delta r}{\Delta q} = (p - \Delta p)\Delta q - \Delta pq$$

  - Calculus version

Taking the limit: $$mr = \lim_{\Delta q \to 0} \frac{(p - \Delta p)\Delta q - \Delta pq}{\Delta q} = \frac{dr}{dq}$$

$$mr = p + q\frac{dp}{dq}$$

# Costs

- Cost function
  - The cost to produce output *q* depends on the costs of the inputs to produce quantity *q*
  - The *technology* available to the firm provides the relationship between the inputs (including labor and capital) the firm purchases and the output the firm can produce with those inputs
  - The firm's *cost function c*(*q*) is the minimum cost to the firm of producing quantity *q* given the firm's technology
    - The firm's cost function *c* may change as the technology changes

# Costs

- Cost function—Some useful definitions
  - *Total cost* (TC) is the sum of all costs incurred by the firm to produce output *q*. Total cost is equal to the sum of fixed cost plus variable cost.
  - *Fixed cost* (FC) is that cost incurred by the firm that do not depend on the firm's level of production (e.g., the cost of the factory)
  - *Variable cost* (VC) is the cost incurred by the firm that depends on the firm's level of production
  - *Average total cost* (ATC) is total cost divided by output
  - *Average variable cost* (AVC) is variable cost divided by output
  - *Marginal cost* is the cost to the firm of producing one incremental unit of output

$$TC(q) = FC + VC(q)$$

$$ATC(q) = \frac{TC(q)}{q}$$

$$AVC(q) = \frac{AC(q)}{q}$$

$$MC(q) = C(q) - C(q-1)$$

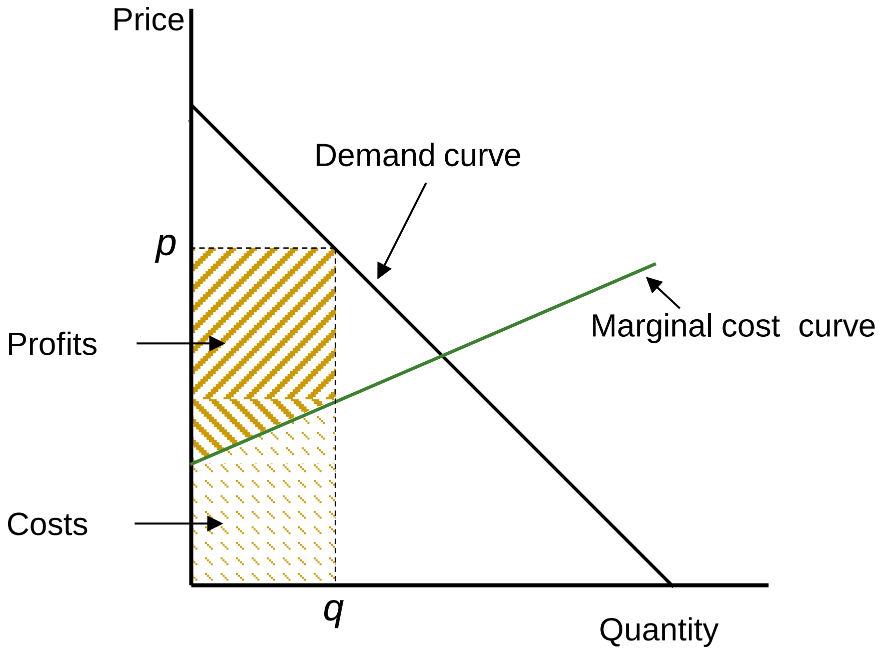
$$= \frac{\Delta C}{\Delta q} \text{ where } \Delta q = 1$$

$$= \frac{dC}{dq}$$

$\Delta q$ means the change in *q* is read "delta *q*"

# Marginal costs

*Marginal cost* (*mc*): The cost *mc* of producing the (*n* + 1)th unit after producing *n* units
*Marginal cost curve*: Traces the relationship between *n* and *mc*




*Query*: The marginal cost curve is shown upward sloping. Why might that be?
Can the marginal cost curve be flat or even downward sloping?

# Profit maximization

- Profit maximization
  - Firm's objective function in revenues (with quantity *q* as the control variable):

$$\max_q \text{ Profits} = \text{Revenues} - \text{Costs}$$
$$= r(q) - c(q)$$

This equation says pick production level *q* to maximize profits, that is, the difference between the revenues the firms earns when it sells quantity *q* and the costs it incurs to produce quantity *q.*

In this maximization problem, the *objective function* is the function that we are trying to maximize, in this case *r*(*q*) – *c*(*q*).

The *control variable* is the variable the firm gets to pick. In this simple model, the firm can control its production level *q*, but market conditions determine the price at which the sells.  Variables that the firm does not control are called as *parameters*.

# Profit maximization

- Profit maximization
  - The profit function looks like a hill

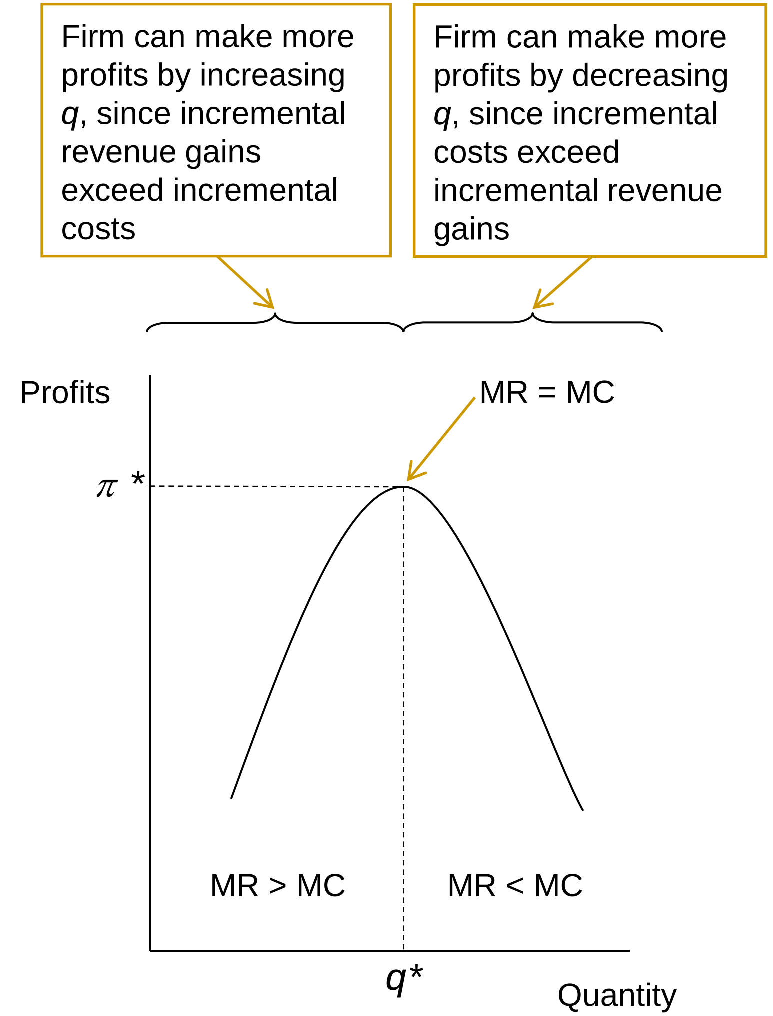
    Think about it this way: When price equal zero, there are zero profits. At some high price point, no one will be willing to buy, so there are zero profits. In between, there are positive profits.

  - The profit-maximizing quantity *q** is the quantity at the peak of the profit curve

    Economists typically use an asterisk to denote an optimum, so that *q** is the profit-maximizing level of output and $\pi$* is the maximum level of profits.

  - The profit-maximizing quantity *q** occurs when MR = MC

    MR = MC is called the *first order condition* for a profit maximum. This attribute of a profit maximum is invoked frequently in antitrust analysis.



# Profit maximization

- Profit maximization (this time with a little calculus)
  - At its peak, the slope of the profit curve is zero, that is, where

$$\frac{\Delta \pi}{\Delta q} = 0$$

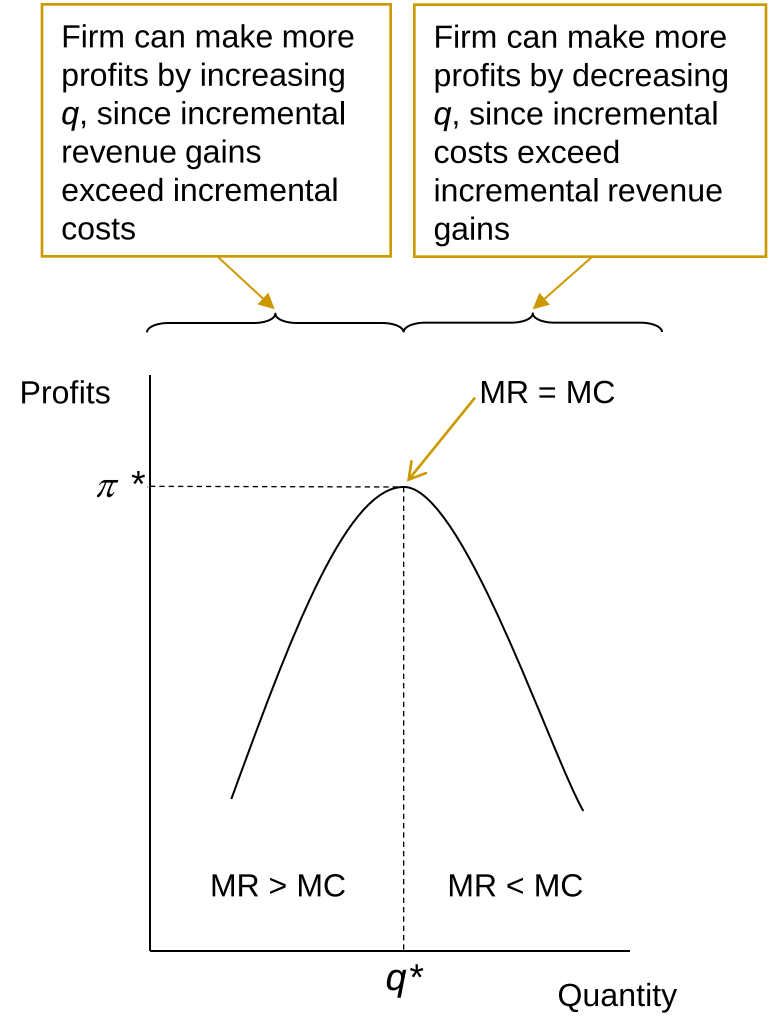
  - We get the same result by setting the derivative of the profit function to zero:

$$\frac{d\pi}{dq} = \frac{dr}{dq} - \frac{dc}{dq} = 0$$

  - Rearranging terms yields:

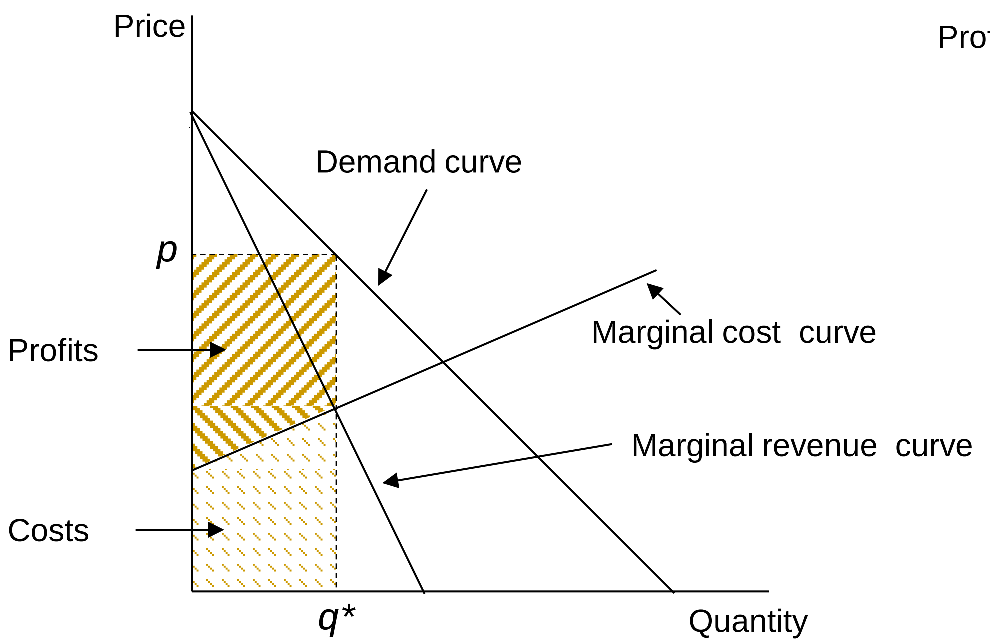
Marginal revenue → $$\frac{dr}{dq} = \frac{dc}{dq}$$ ← Marginal cost

which is just another way of saying marginal revenue equal marginal cost



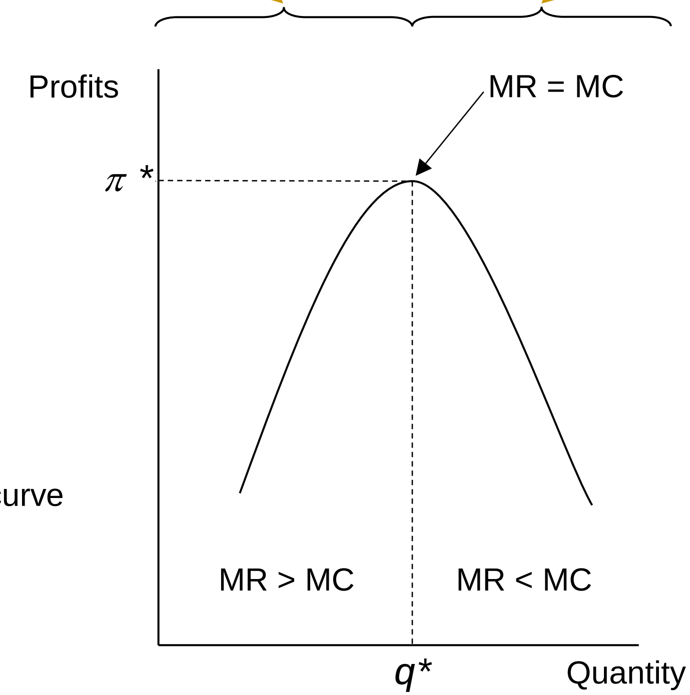
# Profit maximization

- Firms maximize profits when *mr = mc*




Firm can make more profits by increasing *q*, since incremental revenue gains exceed incremental costs

Firm can make more profits by decreasing *q*, since incremental costs exceed incremental revenue gains



# Profit maximization

- Profit maximization for the individual firm—Example

Assume $q = 20 - 2p$ (firm's residual demand curve)

so $p = 10 - \frac{1}{2}q$ (inverse demand curve)

Revenue $(r) = pq = (10 - \frac{1}{2}q)q = 10q - \frac{1}{2}q^2$

Marginal revenue $(MR) = \frac{dr}{dq} = 10 - q$

Constant marginal cost $(MC) = 4$

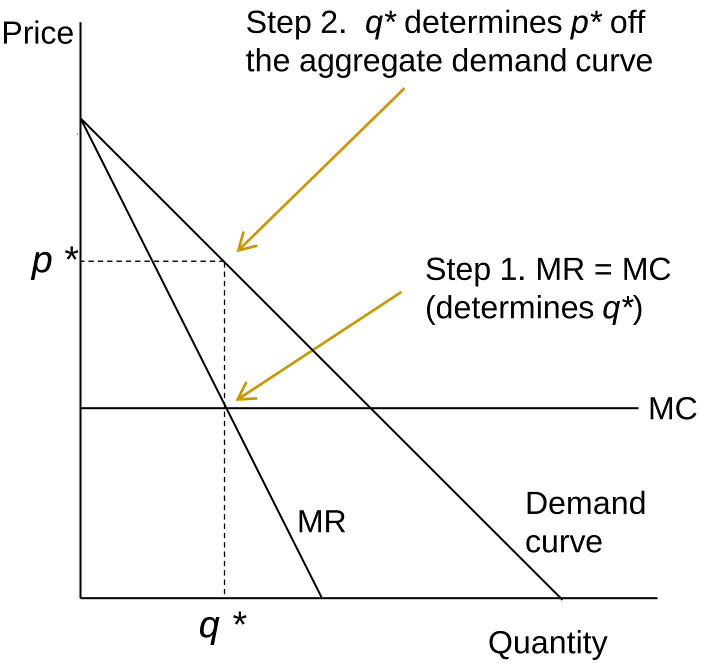
Equating marginal revenue and marginal cost

for a profit maximum: $10 - q = 6$

So $q^* = 6$ is the firm's profit-maximizing quantity

Plugging $q^*$ into the inverse demand function to obtain

$p^* = 7$ as the firm's profit-maximizing price



# Profit maximization

- Calculus version

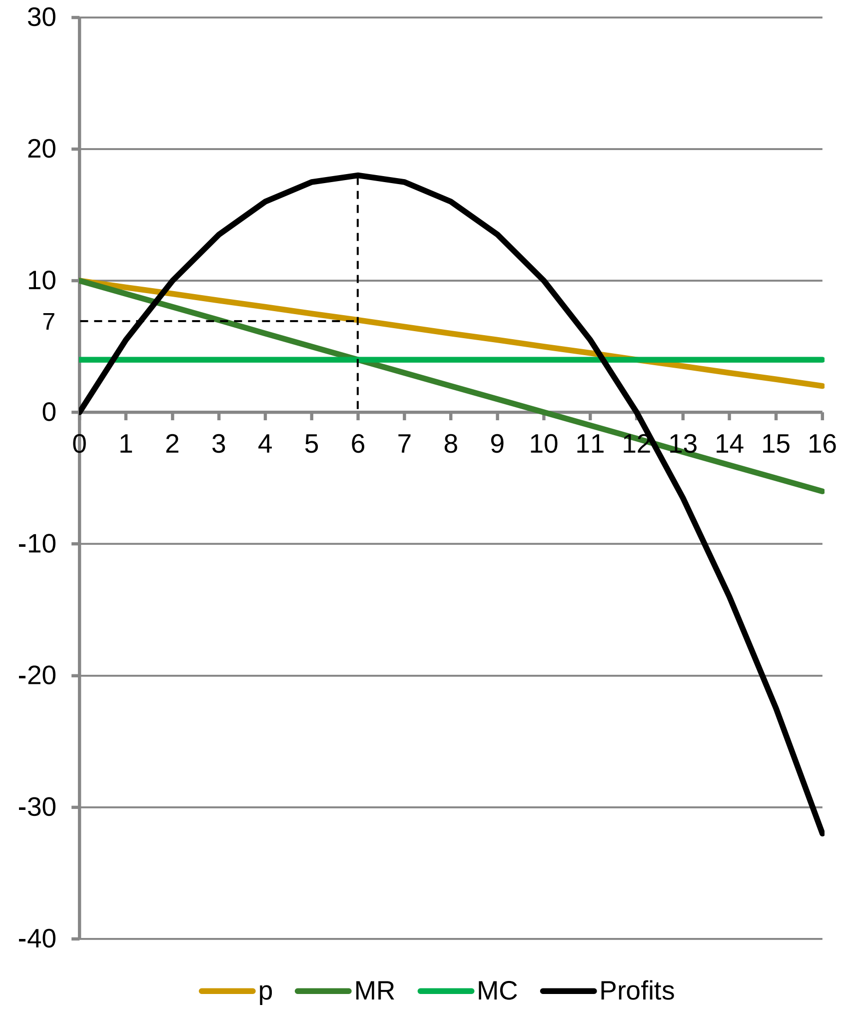
Demand: $p = 10 - \frac{1}{2}q$

Revenue: $r = pq = 10q - \frac{1}{2}q^2$

Marginal revenue: $mr = \frac{dr}{dq} = 10 - q$

Marginal cost: $mc = 4$

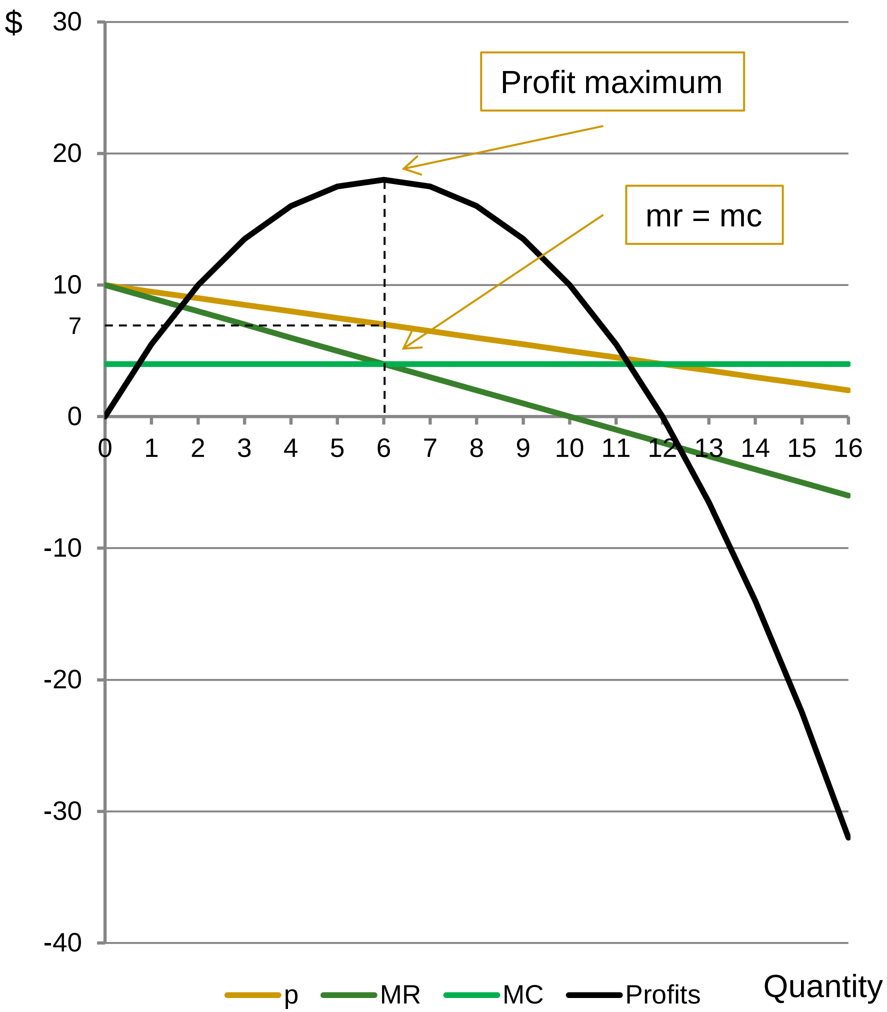
Profit max:
$$mr = mc$$
$$10 - q = 4$$
$$q = 6$$
$$p = 7$$



# Profit maximization

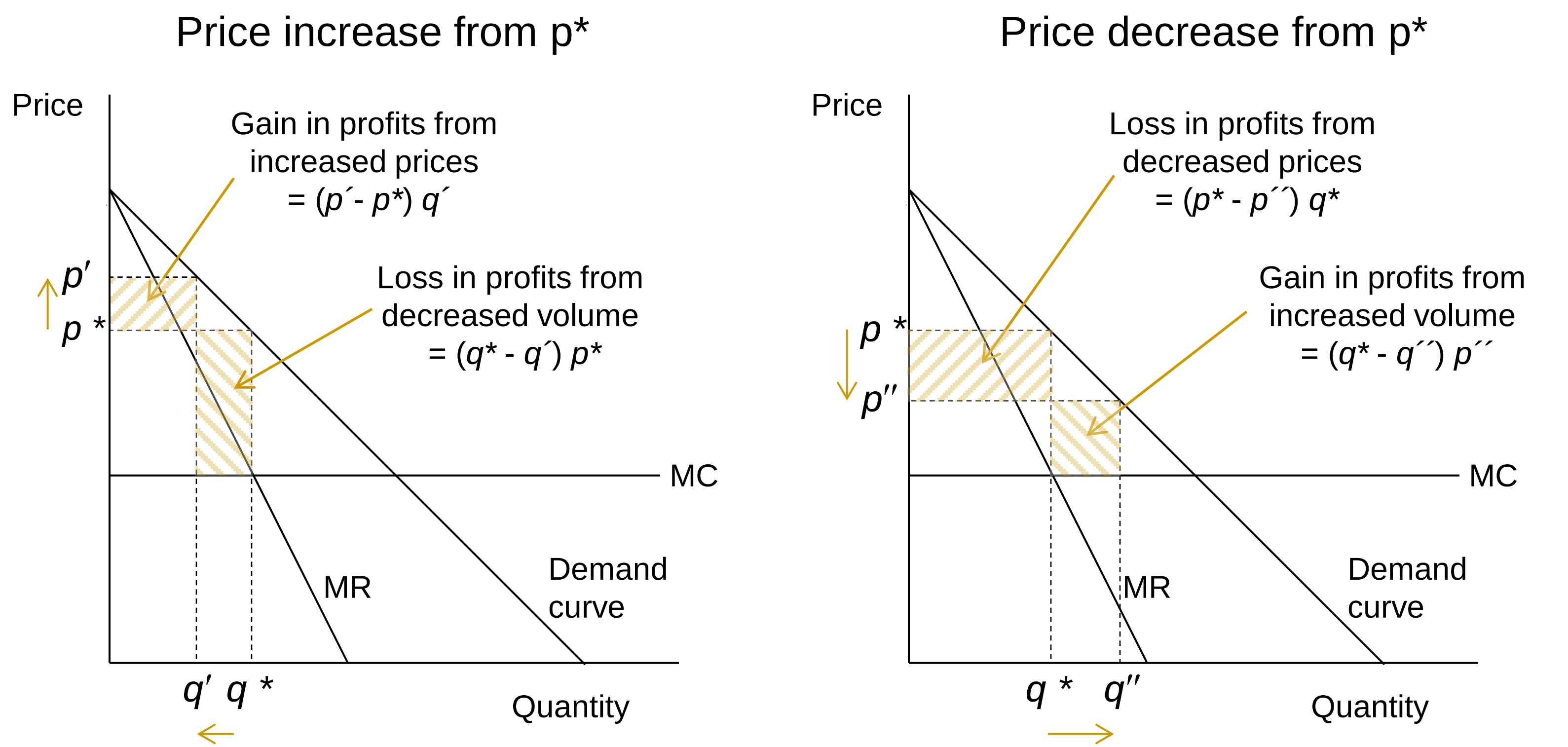
- Numerical version

| Quantity q | Price p | Revenue r | Marginal Revenue mr | Marginal Costs mc | Total Costs c | Profits Π |
|---|---|---|---|---|---|---|
| 0 | 10.0 | 0.0 | | | 0.0 | 0.0 |
| 1 | 9.5 | 9.5 | 9.5 | 4.0 | 4.0 | 5.5 |
| 2 | 9.0 | 18.0 | 8.5 | 4.0 | 8.0 | 10.0 |
| 3 | 8.5 | 25.5 | 7.5 | 4.0 | 12.0 | 13.5 |
| 4 | 8.0 | 32.0 | 6.5 | 4.0 | 16.0 | 16.0 |
| 5 | 7.5 | 37.5 | 5.5 | 4.0 | 20.0 | 17.5 |
| 6 | 7.0 | 42.0 | 4.5 | 4.0 | 24.0 | 18.0 |
| 7 | 6.5 | 45.5 | 3.5 | 4.0 | 28.0 | 17.5 |
| 8 | 6.0 | 48.0 | 2.5 | 4.0 | 32.0 | 16.0 |
| 9 | 5.5 | 49.5 | 1.5 | 4.0 | 36.0 | 13.5 |
| 10 | 5.0 | 50.0 | 0.5 | 4.0 | 40.0 | 10.0 |
| 11 | 4.5 | 49.5 | -0.5 | 4.0 | 44.0 | 5.5 |
| 12 | 4.0 | 48.0 | -1.5 | 4.0 | 48.0 | 0.0 |
| 13 | 3.5 | 45.5 | -2.5 | 4.0 | 52.0 | -6.5 |
| 14 | 3.0 | 42.0 | -3.5 | 4.0 | 56.0 | -14.0 |
| 15 | 2.5 | 37.5 | -4.5 | 4.0 | 60.0 | -22.5 |
| 16 | 2.0 | 32.0 | -5.5 | 4.0 | 64.0 | -32.0 |
| 17 | 1.5 | 25.5 | -6.5 | 4.0 | 68.0 | -42.5 |
| 18 | 1.0 | 18.0 | -7.5 | 4.0 | 72.0 | -54.0 |
| 19 | 0.5 | 9.5 | -8.5 | 4.0 | 76.0 | -66.5 |
| 20 | 0.0 | 0.0 | -9.5 | 4.0 | 80.0 | -80.0 |



# Profit maximization

- Illustration of profit loss from price changes from *p**
  - Assuming no fixed costs




In each case, the loss from the price change exceeds the gain, so that moving away from *p** decreases profits.

# Cournot competition

- Consider a firm's profit-maximizing function when it competes in *quantities*:

$$\max_q \pi = p(q)q - c(q)$$

Here, the production quantity is the control variable. Economists call this *Cournot competition*.

  - First order condition ("FOC") for a profit maximum:

$$\frac{d\pi}{dq} = \underbrace{p + q\frac{dp}{dq}}_{\text{Marginal revenue}} - \underbrace{\frac{dc}{dq}}_{\text{Marginal cost}} = 0$$

So marginal revenue equals marginal cost at a profit maximum

    - *dp*/*dq* is the slope of the firm's (inverse) demand curve. It indicates the degree to which the firm can influence price by changing its level of production. But think about it here as the *decrease* in price that is required to clear the market when an additional unit is added to market supply.
    - So the gross loss in revenues that comes with the introduction of an additional unit of supply is the original quantity *q* times the reduction in price *dp/dq* necessary to clear the market. Marginal revenue is then *p* (the revenue earned by selling an additional unit minus this loss).

# Cournot competition

- With a little mathematical manipulation:

$$\frac{d\pi}{dq} = \left(p - \frac{dc}{dq}\right) + q\frac{dp}{dq} = 0$$

or

$$\underbrace{\left(p - \frac{dc}{dq}\right)} + q\underbrace{\frac{dp}{dq}} = 0$$

Gross margin
(*p-mc*)

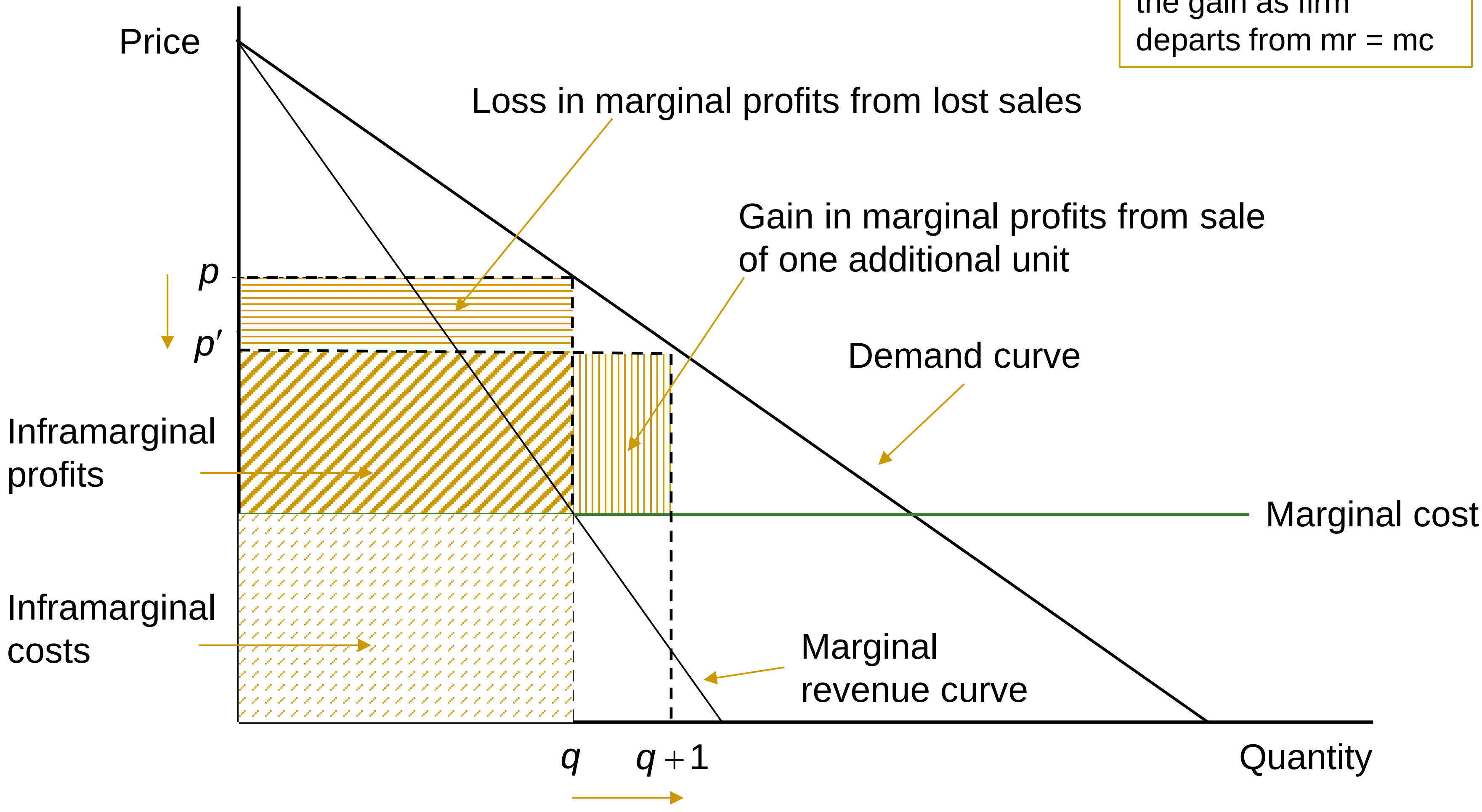
Margin loss on
preexisting sales from
decrease in price

So, at a profit maximum, the gain of the gross margin of an incremental sale is equal to the margin loss on preexisting sales *q* resulting from the price decrease *dp*/*pq* necessary to clear the market

# Cournot competition

- Profit maximization under Cournot competition

Loss is greater than the gain as firm departs from mr = mc



# Bertrand competition

- Consider a firm's profit-maximizing function when it competes in *price*:

$$\max_p \pi = pq(p) - c(q(p))$$

Here, firms compete using firm price as the control variable. Economists call this *Bertrand competition*.

  - First order condition for a profit maximum:

$$\frac{d\pi}{dp} = q + p\frac{dq}{dp} - \frac{dc}{dq}\frac{dq}{dp} = 0$$

$$= q + \left(p - \frac{dc}{dq}\right)\frac{dq}{dp} = 0$$

Price minus marginal cost = Gross margin

Change in market-clearing quantity with an increase in price (i.e., loss of sales due to a price increase)

Gross revenue gain from selling *q* units when the price increases by 1

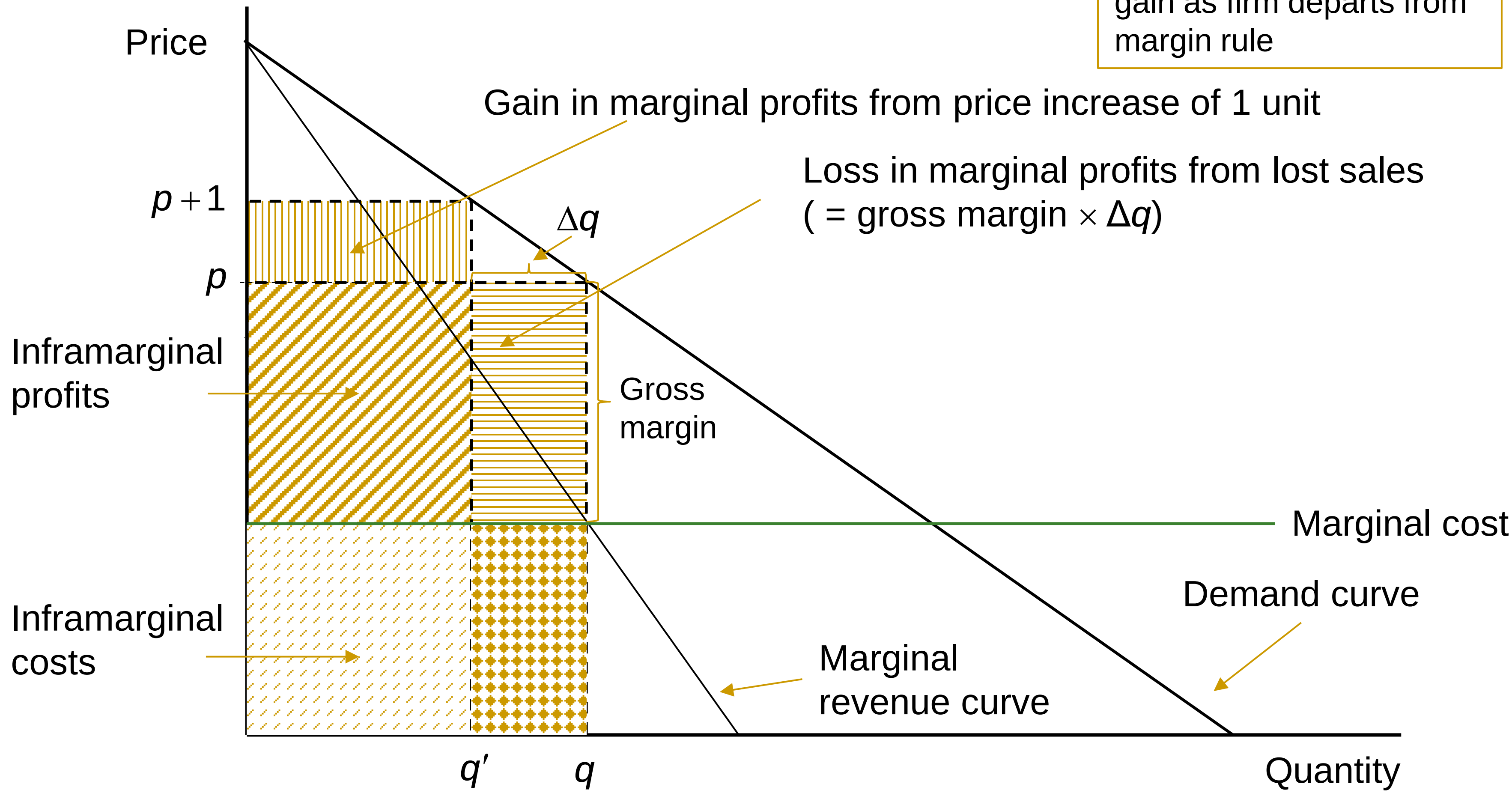
Gross margin times the loss of sales = gross revenue loss from lost sales

  - So at a profit maximum, the gross revenue gain from increased prices on retained sales equals the gross revenue loss from losing the entire gross margin on lost sales

# Bertrand competition

- Profit maximization under Bertrand competition

Loss is greater than the gain as firm departs from margin rule



# Summary: Cournot and Bertrand competition

- Cournot competition
  - Firms compete in quantities
  - Most useful in homogenous product markets
  - *FOC*: Marginal revenue = marginal cost

$$mr = mc$$

$$p + q\frac{dp}{dq} = \frac{dc}{dq}$$

- Bertrand competition
  - Firms compete in prices
  - Most useful in differentiated product markets
  - FOC: Gross revenue gain from selling $q$ units when the price increases by 1 = gross margin times the loss of sales

$$\text{Gross revenue gain} = \text{Gross revenue loss}$$

$$q = \left(p - \frac{dc}{dq}\right)\frac{dq}{dp}$$

# Perfect Market Equilibria

# Perfectly Competitive Markets

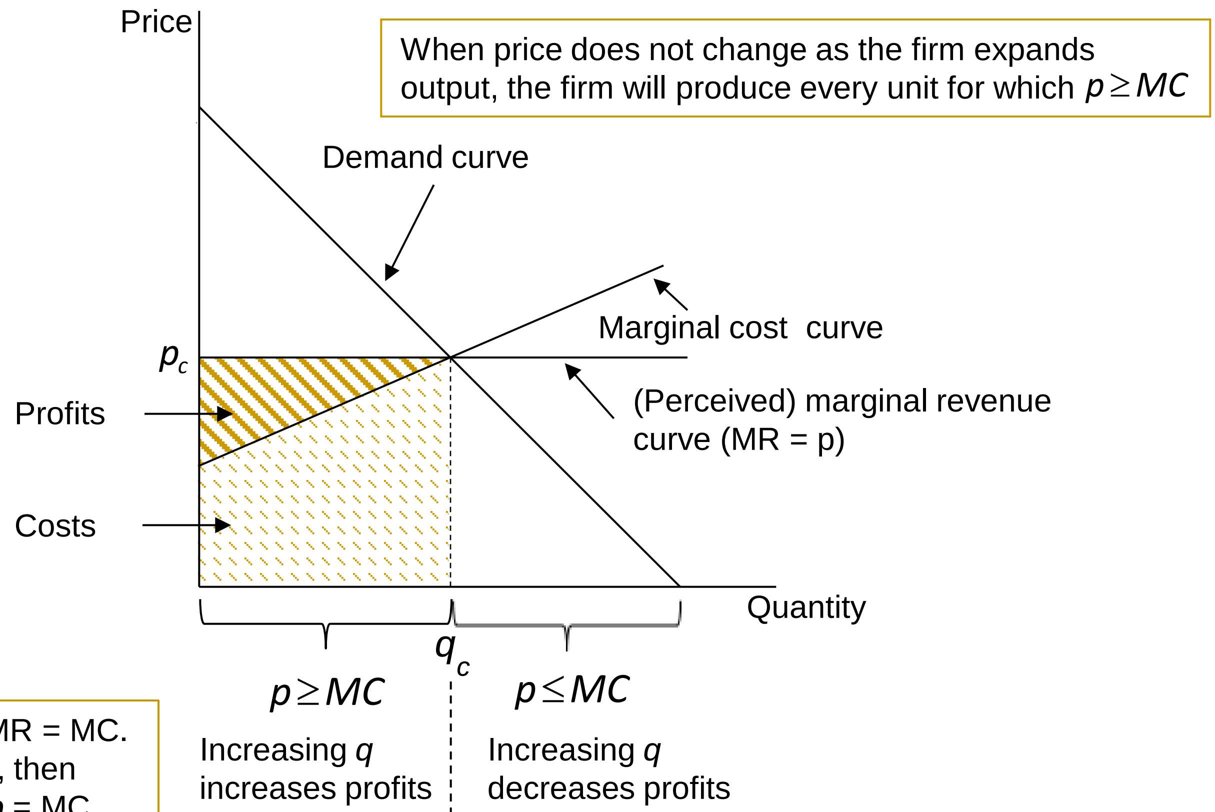
- *Definition*: A market in which no single firm can effect price, meaning:
  - The firm's residual demand curve is horizontal,
  - The firm can sell any amount of product without affecting the market price,
  - $\frac{dp}{dq} = 0$, or
  - $p = \frac{dc}{dq}$ (i.e., price = marginal cost)

These four bullets are just different ways of saying exactly the same thing.

- What could cause a market to be perfectly competitive?
  - *Traditional theory*: Each individual firm's production is very small compared to aggregate demand at any price, so that individual production changes cannot move significantly along the aggregate demand curve
    - This implies that there are a very large number of firms in the market
  - *Modern theory*: Competitors in the market place react strategically but non-collusively to price or quantity changes by a firm in ways that maintain the competitive equilibrium

# Competitive firms

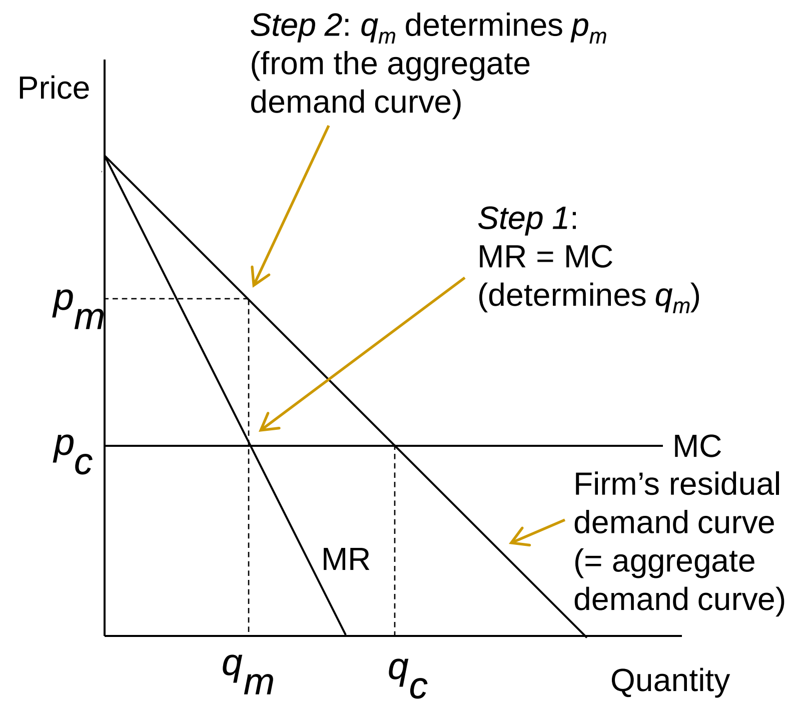
- Competitive firms take prices as given
  - → Individual output decisions do not affect the market-clearing price




As always, the FOC is MR = MC. If the firm is competitive, then MR = *p and so* FOC is *p* = MC

# Perfect Monopoly

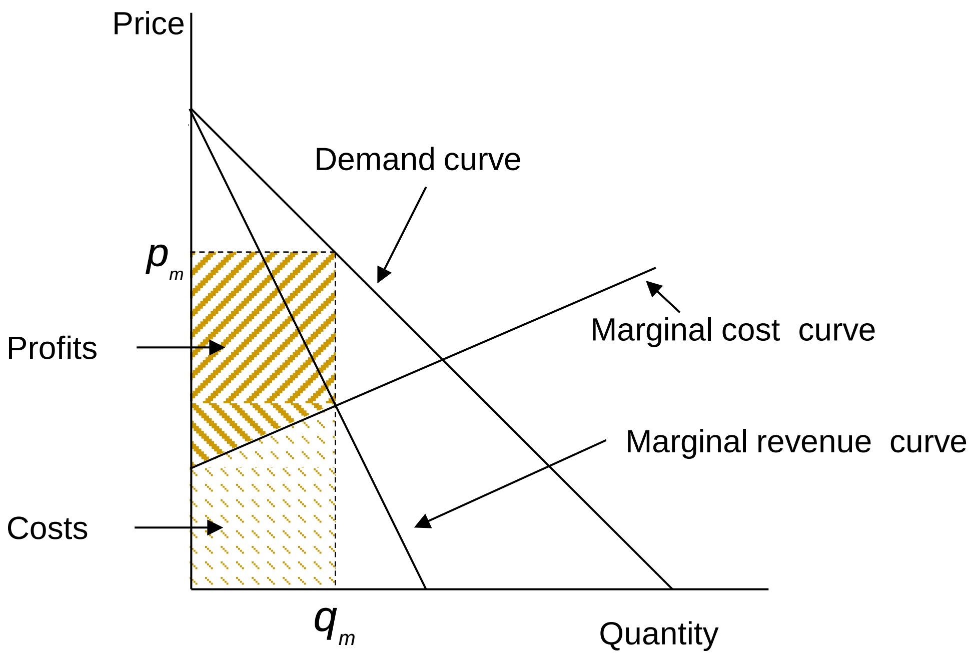
- *Definition*: A market in which only one firm operates*
  - In this case, the firm's residual demand curve is the same as the aggregate demand curve
  - As always, the firm's chooses production so that its marginal cost is equal to marginal revenue




* Keep in mind that this is the way economists define "perfect monopoly." As we shall see later, the legal definition of "monopoly" is quite different.

# Monopolist Firm

- A monopolist choice of output *q* affects the market-clearing price *p*




*Rule*: Monopolists price at MR = MC, where marginal revenue is determined by the aggregate demand curve

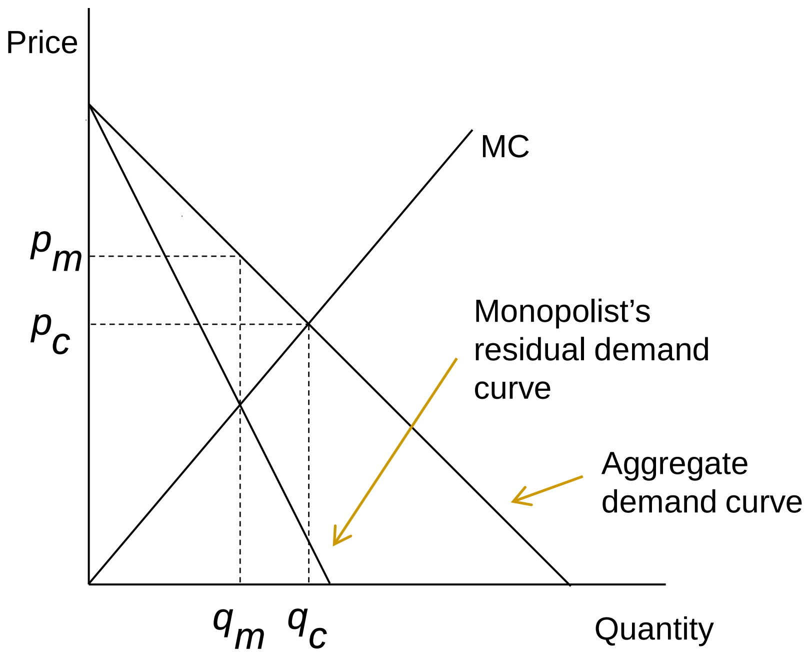
# Summary of Key Results So Far

- Profit-maximizing firms choose production levels so that marginal revenue equals marginal cost (MR = MC) (in Cournot competition)
  - Step 1: MR = MC determines the firm's profit-maximizing production level $q^*$
  - Step 2: The firm's residual demand curve determines the firm's profit-maximizing price $p^*$ given $q^*$
- In a perfectly competitive market, a firm's choice of production level cannot effect market price, so:
  - Marginal revenue is equal to the market unit price (MR = $p_{market}$),
  - MR = MC implies that MC = $p_{market}$, so firm picks $q_{comp}$ to satisfy this condition
    - We have not discussed how the market price is determined in a perfectly competitive market. For our purpose, just take market price as a given.
  - By definition, firm cannot affect market price, so $p_{comp} = p_{market}$
- In a perfect monopoly market, consumers can only purchase from the monopolist, so:
  - The firm's residual demand curve is the same as the aggregate demand curve
  - MR = MC determines the monopolist's profit-maximizing quantity $q_m$
  - The aggregate demand curve determines $p_m$ given $q_m$

# Coordination Among Firms

# Incentives for Coordination

- Consider the difference between a firm's profits under perfect competition and perfect monopoly
  - Example:




Aggregate demand: $p = 30 - 2q$

Marginal cost: $MC = q$

Total cost: $TC = \frac{1}{2}q^2$

Competitive market: $p = MC$

Competitive quantity: $q_c = 10$ (5 per firm)

Competitive price: $p_c = 10$

Competitive profits: $\pi_c = 50$ (25 per firm)

Monopoly market: $MR = MC$

Monopoly quantity: $q_m = 6$

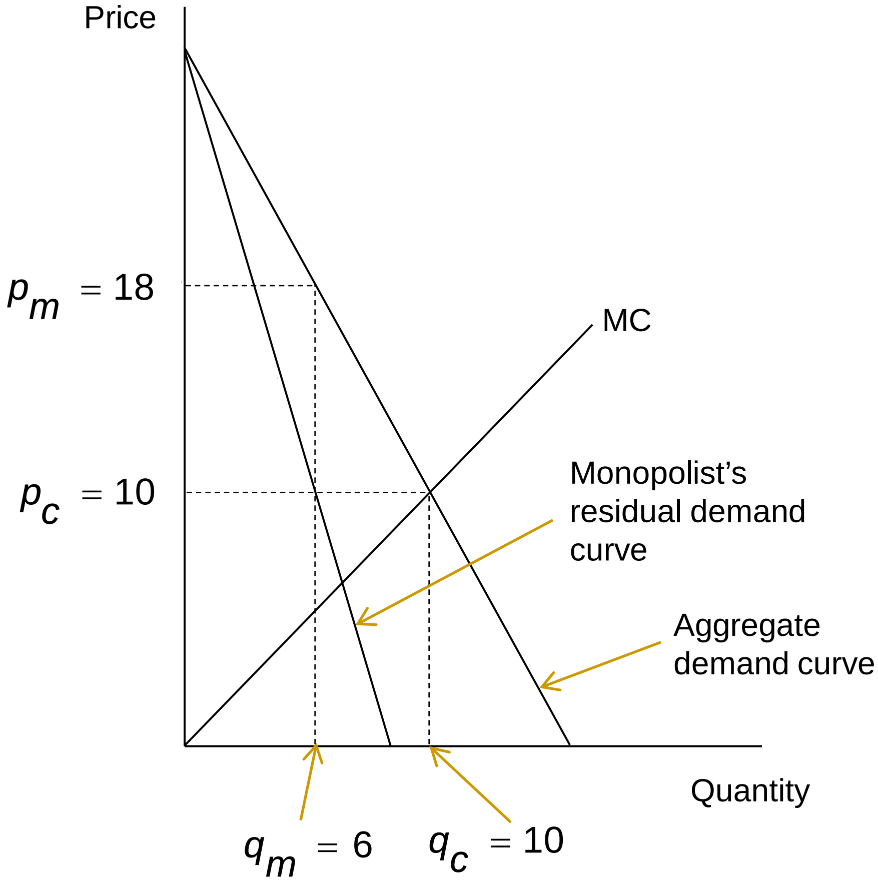
Monopoly price: $p_m = 18$

Monopoly profits: $\pi_m = 90$

In this example, the monopoly profits are almost twice the total competitive profits

# Gains from Coordination[1]

- Example
  - Two symmetric firms in the market
- If the firms coordinate their activities, they can—
  - Collectively produce the monopoly output of 6 units
  - Split the monopoly profits of $90, with $45 going to each
  - Each earns $45 instead of the competitive profit of $25, a gain of 80%




[1] You may consider a merger to be an extreme form of coordination between two firms.

# Gains from Coordination

- Monopoly rents
  - The difference in profits between the monopoly and competitive equilibria is called the *monopoly rent*
    - In economic terms, a *rent* is the return due to some scarcity in supply
  - Monopolies earn profits above the competitive level because:
    - They restrict their output and so create an artificial scarcity in supply,
    - This causes *inframarginal customers*—that is, those who value the product at levels above the competitive price—to bid up the market-clearing price
      - This is sometimes called "riding up the demand curve"

> The idea that firms restrict output in order to create an artificial scarcity in supply and thereby increase the market-clearing price is fundamental to many theories of anticompetitive harm in antitrust law.

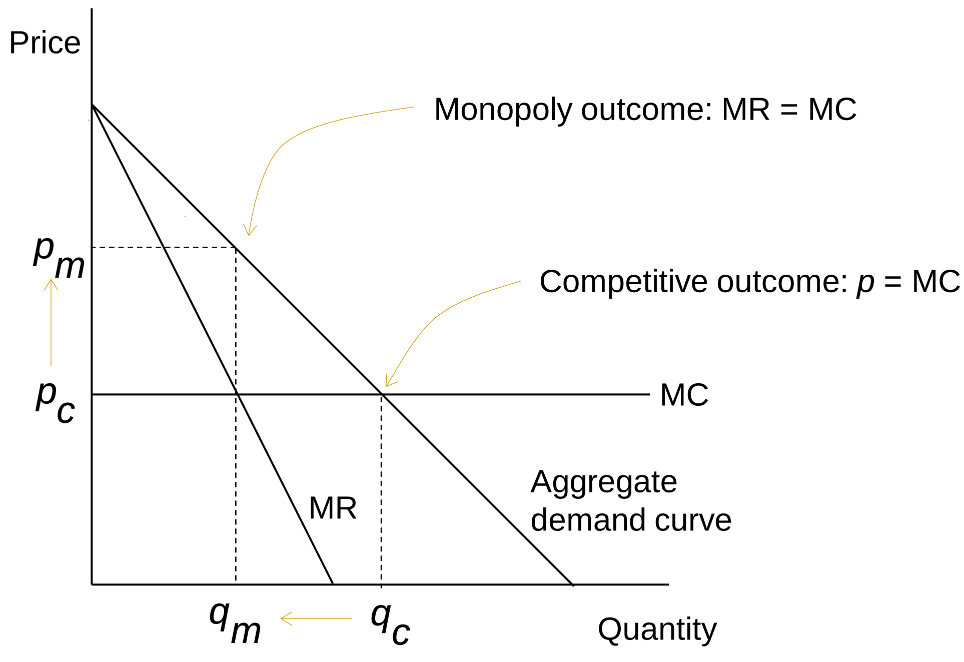
# Public Policy on Coordination

- Modern view on why monopolies are bad:
  - Increase price and decrease output
  - Shift wealth from consumers to producers
  - Create economic inefficiency (“deadweight loss”)

  - May (or may not) have other socially adverse effects
    - Decrease product or service quality
    - Decrease the rate of technological innovation or product improvement
    - Decrease product choice

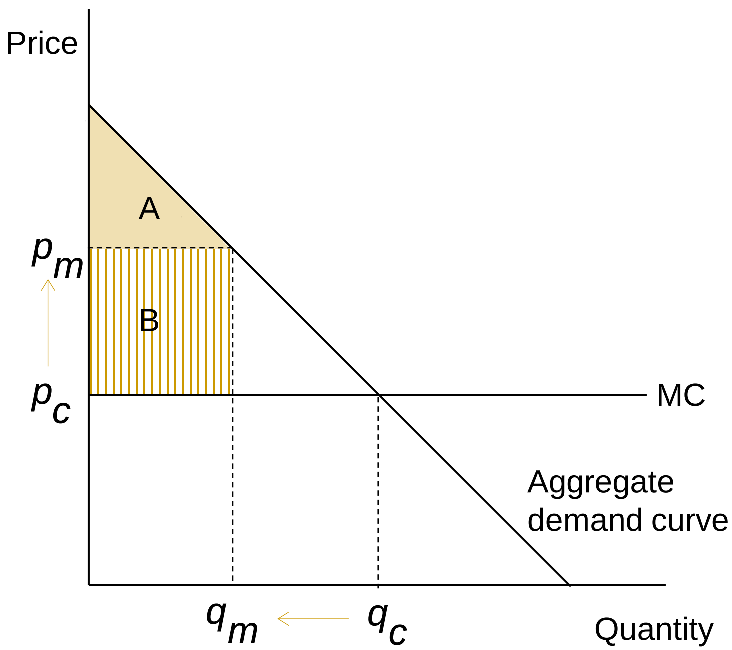
# Public Policy on Coordination

- Output decreases: $q_c \searrow q_m$
- Prices increase: $p_c \nearrow p_m$



# Public Policy on Coordination

- Shift in wealth from inframarginal consumers to producers*
  - Total wealth created ("surplus"): A + B
  - Sometimes called a "rent redistribution"

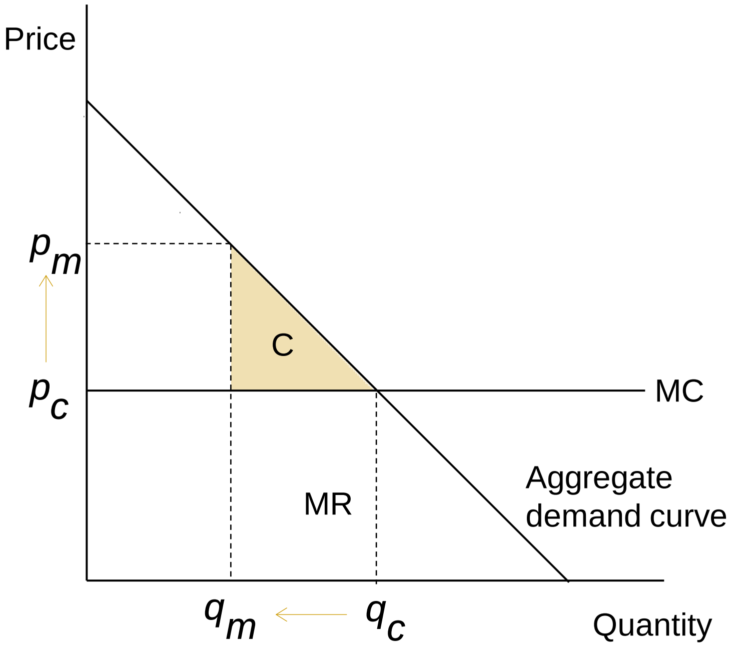



| | Competitive | Monopoly |
|---|---|---|
| Consumers | A + B | A |
| Producers | 0 | B |

* Inframarginal customers here means customers that would purchase at both the competitive price and the supracompetitive (monopoly) price

# Public Policy on Coordination

- "Deadweight loss" of surplus from marginal customers*
  - Surplus C just disappears from the economy
  - Creates "allocative inefficiency" because it does not exhaust all gains from trade




* Marginal customers here means customers that would purchase at the competitive price but not at the monopoly price

# Imperfectly Competitive Market Equilibria

# Introduction

- The basic concept
  - An imperfectly competitive market exhibits—
    - A degree of competition, but not to the extent of a perfectly competitive market
    - A degree of market power, but not to the extent of perfect monopoly market
  - Role in merger antitrust analysis
    - Almost all mergers of interest occur in markets that are imperfectly competitive
  - Characteristics of imperfectly competitive markets
    - Some or all firms exercise some degree of market power
      - One way of things about this is that they each face a downward-sloping residual demand curve, so that changes in a firm's output level will have an effect on the firm's market-clearing price
    - Multiple firms, but few enough that each firm recognizes its optimal control variables (e.g., price, output, quality) depends on the choices
    - Firms are differentiated
      - Product differentiation (e.g., various makes and models of automobiles)
      - Spatial differentiation (e.g., gasoline stations located at different geographies)

      NB: In both cases, firms are "close enough" to one another to exhibit significant cross-elasticities of demand

# Market power

- Some definitions
  - Market power
    - "As an economic matter, market power exists whenever prices can be raised above the levels that would be charged in a competitive market."[1]
    - "Market power is usually stated to be the ability of a single seller to raise price and restrict output, for reduced output is the almost inevitable result of higher prices."[2]
    - "Market power generally is defined as the power of a firm to restrict output and thereby increase the selling price of its goods in the market."[3]
    - Market power means "by definition, means that the defendant can produce anticompetitive effects."[4]
    - "A merger enhances market power if it is likely to encourage one or more firms to raise price, reduce output, diminish innovation, or otherwise harm customers as a result of diminished competitive constraints or incentives."[5]

[1] Jefferson Parish Hosp. Dist. No. 2 v. Hyde, 466 U.S. 2, 27 n.46 (1984); *accord* NCAA v. Board of Regents, 468 U.S. 85, 109 n.38 (1984); Copperweld Corp. v. Independence Tube Corp., 467 U.S. 752, 789 n.19 (1984).
[2] Fortner Enters., Inc. v. United States Steel Corp., 394 U.S. 495, 503 (1969)
[3] Ryko Mfg. Co. v. Eden Servs., 823 F.2d 1215, 1232 (8th Cir. 1987).
[4] Agnew v. National Collegiate Athletic Ass'n 683 F.3d 328, 337 (7th Cir. 2012)
[5] U.S. Dept. of Justice & Fed. Trade Comm'n, Horizontal Merger Guidelines § 1 (rev. 2010).

# Market power

- Some definitions
  - Monopoly power
    - "Monopoly power is the power to control prices or exclude competition."[1]
    - Practically, monopoly power is just an extreme form of market power that exists when the firm (or a combination of firms acting together) can behave in the market as if they were or were close to being the only firm in the market.

[1] United States v. E. I. du Pont de Nemours & Co., 351 U.S. 377, 391 (1956).

# Market power

- Measuring market power
  - Recall that in a competitive market, firms set price equal to marginal cost
  - The traditional measure of market power is the *price-cost margin* or *Lerner index L*, which is a measure of how much price has been marked up:[1]

$$L = \frac{p - mc}{p}$$

  - Note that in a competitive market $L = 0$ and that $L$ increases as price increases relative to marginal cost

[1] For more on the Lerner index, see Kenneth G. Elzinga & David E. Mills, *The Lerner Index of Monopoly Power: Origins and Uses*, 101 Am. Econ. Rev. (Papers & Proceedings) 558 (2011).

# Cournot oligopoly models

- The setup
  - Recall that in a Cournot model, the firm's control variable (that is, the variable the firm controls) is quantity
  - The (download sloping) demand curve gives the relationship between the quantity produced and the market-clearing price
  - When their a multiple firms each producing some output, the market-clearing price is a function of all of their outputs
  - Assume that each firm produced an identical (homogeneous) product. Then the market-clearing price *p* is a function of the sum of the outputs of all of the firms in the market:

$$p = p(Q), \text{ where } Q=\sum_{i=1}^{N} q_i,$$

  and where $q_i$ is the output of the *i*th firm
  - So the profit equation for the *i*th firm is: $\pi_i = p(Q)q_i - c_i(q_i), \quad i = 1, 2, \ldots N$

  and the first-order condition for a profit maximum for each firm *i* is:

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + q_i \frac{\partial p(Q)}{\partial Q}\frac{\partial q_i}{\partial q_i} - \frac{\partial c_i}{\partial q_i} = 0.$$

= 1

Using the chain rule

We saw a version of this equation on slide 23

# Cournot oligopoly models

- The model
  - Rearranging the last equation yields: $p(Q) - \frac{\partial c_i}{\partial q_i} = q_i \frac{\partial p(Q)}{\partial q_i}$

  To simply the notation, define marginal cost as $c_i' \equiv \frac{\partial c_i}{\partial q_i}$ and the slope of the demand curve as $p' \equiv \frac{\partial p}{\partial Q}$

  Dividing both sides by *p(Q),* multiplying by *Q/Q, and* rearranging a bit yields:

$$\frac{p - c_i'}{p} = -\frac{q_i p'}{p} = \left[\frac{Q}{p} p'\right] \frac{q_i}{Q}$$

  Now $\frac{Q}{p} p' = \frac{Q}{p} \frac{\partial Q}{\partial p} = \frac{1}{\varepsilon}$ and $\frac{q_i}{Q} = s_i$, where $s_i$ is the market share of the *i*th firm, so:

$$\frac{p - c_i'}{p} = \frac{s_i}{\varepsilon}$$

  Thus, in a Cournot oligopoly model, the *i*th firm's percentage gross margin is equal to the firm's market share divided by the industry own-elasticity. This implies that, holding marginal costs and industry elasticity constant, as the firm's market share increases, so does its percentage gross margin (and it market power)

# Cournot oligopoly models

- The Cournot model and the HHI
  - Take the last equation and multiply both sides by $s_i$:

$$\frac{p - c_i'}{p} s_i = \frac{s_i^2}{\varepsilon}$$

Summing over all firms yields:

$$\sum_{i=1}^{N} \frac{p - c_i'}{p} s_i = \sum_{i=1}^{N} \frac{s_i^2}{\varepsilon} = \frac{HHI}{\varepsilon}$$

NB: The HHI here is in its decimal representation, which ranges from 0 to 1. This results when market shares are expressed as a fraction of 1 rather than a percentage share of 100. To convert the decimal form to the regular form, just multiply by 10,000.

The sum of the percentage gross margins of the firms in the market weighted by their respective market shares is a measure of the market power being exercised in the market. The above equation says the that this measure is equal to the HHI divided by the industry own-elasticity, so as the HHI increases, so does the market power being exercised. This is the primary theoretical justification for using the HHI in merger antitrust analysis.

  - From the above equation, we can see that Cournot equilibrium lies between the competitive equilibrium (where the HHI = 0) and the monopoly equilibrium where the HHI =1 (in decisional form)).
    - As the number of firms in the market becomes smaller and the HHI approaches 1, the Cournot equilibrium approaches the monopoly equilibrium.

# Cournot oligopoly models

- The Cournot model and the HHI
  - One last point. If all firms are symmetrical (identical in their cost functions), then the profit-maximizing quantity for each firm will be identical. This implies that $s_i = 1/N$, so that the HHI in a Cournot oligopoly equilibrium of N firms is:

$$HHI = \sum_{i=1}^{N} \left( \frac{1}{N} \right)^2 = N \left( \frac{1}{N^2} \right) = \frac{1}{N}$$

    So in a market where N = 5, the HHI is .2 (or 2000 in regular form)
  - In the symmetrical case, we can also calculate the Lerner index for the market at a Cournot equilibrium:

$$\sum_{i=1}^{N} \frac{p - c_i'}{p} s_i = \frac{p - c'}{p} \sum_{i=1}^{N} s_i = \frac{p - c'}{p} = \lambda$$

    since $c_i' = c'$ for all firms by symmetry and $\sum_{i=1}^{N} s_i = 1$ by the definition of market share

# Bertrand oligopoly models

- Homogeneous products case
  - Consider two firms producing homogeneous (identical) products at constant marginal cost *c′* and use price as their control variable.
  - Consumers also purchase from the lower priced firm; if both firms charge the same price, they split equally consumer demand.
  - Consumer demand *Q* is a function of $\underline{p}$, the lowest price offered by a firm in the market
  - So if—
    - $p_1 < p_2$, , then $p_1 = \underline{p}$ and firm 1 sells all of consumer demand $Q(\underline{p})$ for profits $\pi_1 = \underline{p}Q - c(Q)$, and firm 2 sells nothing and earns zero profits
    - $p_1 = p_2$, then $p_1 = p_2 = \underline{p}$ firm 1 and firm 2 each sell one-half of consumer demand $Q(p)$ for profits

$$\pi_i = \frac{\underline{p}Q - c(Q)}{2}.$$

  - *Equilibrium*: As long as $\underline{p} > c'$, the higher priced firm can undercut the lower priced firm and take all of the market demand. This "race to the bottom" until $p_1 = p_2 = \underline{p} = c'$, so that both firms price at marginal cost and split equally market demand and total market profits
    - So in this model, a competitive equilibrium is achieved with only two firms

# Bertrand oligopoly models

- Differentiated products case
  - When products are differentiated, a lower price charged by one firm will not necessarily move all of the market demand to that customer
    - Consider a market with only red cars and blue cars.
    - Some consumers like blue cars so much that even if the price of red cars is lower than the price of blue cars there will still be positive demand for blue cars
    - Moreover, if the price of blue cars increases, some (inframarginal) blue car customers will purchase blue cars at the higher price while some (marginal) customers will switch to red cars
    - This means that the demand for red cars (and separately for blue cars) is a function both of the price of red cars and the price of blue cars

# Bertrand oligopoly models

- Differentiated products case
  - Simple linear model
    - Firms 1 and 2 produce differentiated products and face the following residual demand curves:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

NB: Each firm's decreases with increase in its own price and increase with increases in the price of the other firm

Assume that $b_1 > b_2$, so that each firm's residual demand is more sensitive to its own price than to the other firm's price

    - Assume each firm has a cost function with no fixed costs and constant marginal costs:

$$c_i\left(q_i\right) = cq_i$$

    - Firm 1 profit-maximization problem:

$$\max_{p_1} \pi_1 = \left(p_1 - c\right)\left(a - b_1 p_1 + b_2 p_2\right)$$

    - First order condition for Firm 1:

$$\frac{\partial \pi_1}{\partial p_1} = 0 = a - 2b_1 p_1 + b_2 p_2 + cb_1$$

# Bertrand oligopoly models

- Differentiated products case
  - Simple linear model (con't)
    - Solving the last equation for $p_1$ gives firm 1 *best response function* $BR(p_2)$, that is, the price it should choose given firm 2's choice of $p_2$:

$$BR_1(p_2): p_1 = \frac{a + cb_1 + b_2 p_2}{2b_1}$$

    - Firm 2 has a similar best response function $BR(p_1)$:

$$BR_2(p_1): p_2 = \frac{a + cb_1 + b_2 p_1}{2b_1}$$

    - The price pair $(p_1^*, p_2^*)$ that simultaneously satisfies both best response functions is a *Bertrand equilibrium*
    - Solving these two simultaneous equations for $p_1^*$ and $p_2^*$ yields:

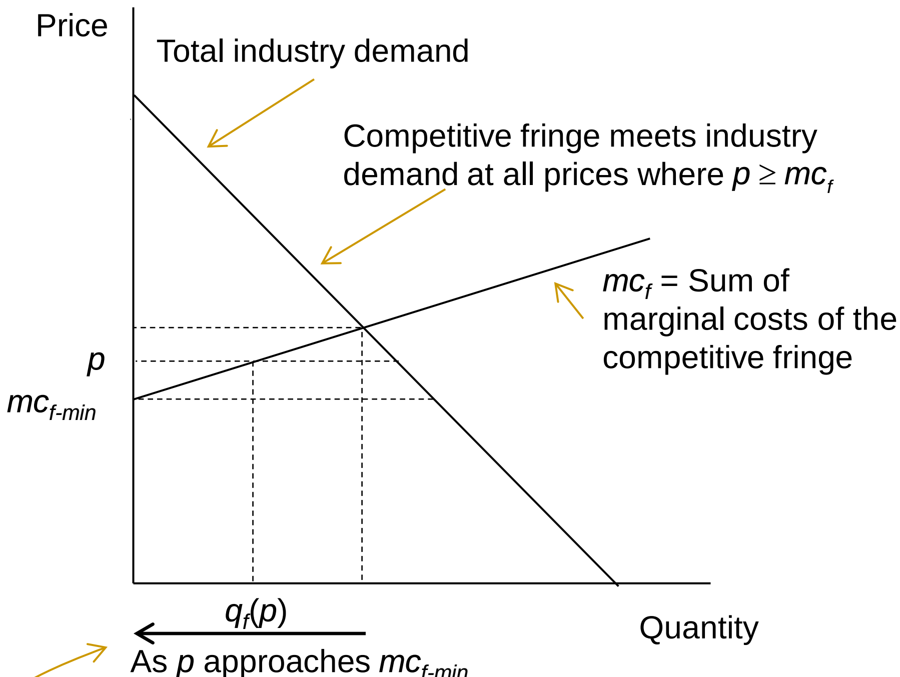
$$p_1^* = p_2^* = \frac{a + cb_1}{2b_1 - b_2}$$

# Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with
    - a dominant firm, which sees its output decisions as affecting price and so sets output so that $mr = mc$, and
    - a fringe of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., $p = mc$)
  - Choice question for the dominant firm: Pick the profit-maximizing level for its output given the competitive fringe
- The model
  - At market price $p$, let $q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe. Then the residual demand $q_d(p)$ for the dominant firm is $q(p) - q_f(p)$.

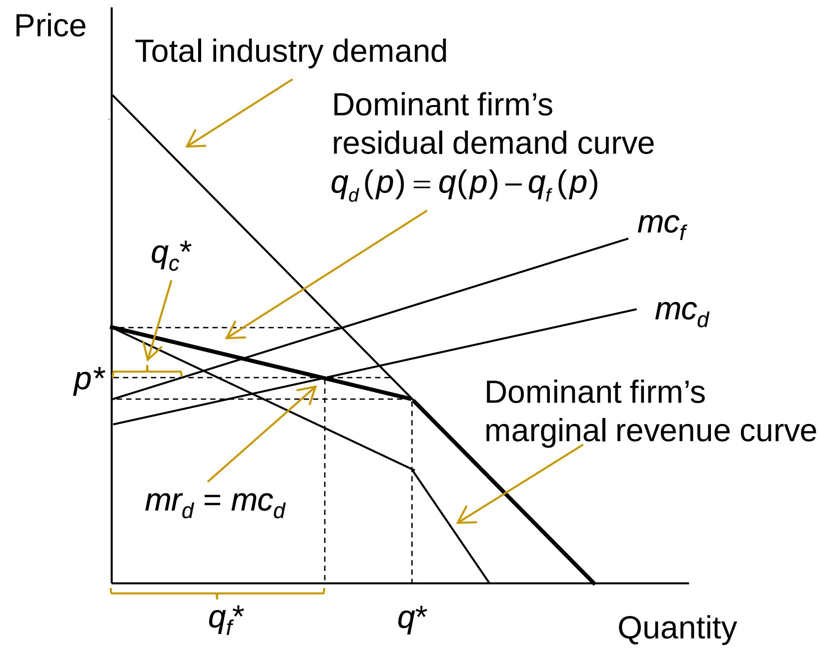
# Dominant firm with a competitive fringe

## Output of the Competitive Fringe




Competitive fringe reduces output until price equals $mc_f$*-min*, its minimum marginal cost. Below this price the competitive fringe does not produce.

## Output of the Dominant Firm




Dominant firm maximizes profit at $q_f{}^*$, where $mr_f = mc_f$. Total industry output $q^* = q_f{}^* + q_c{}^*$ at price $p^*$.

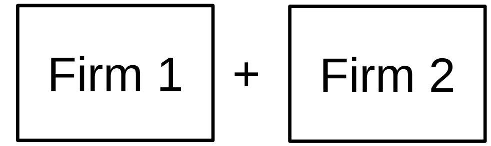
# Dominant firm with a competitive fringe

- Dominant oligopolies
  - The model can be extended to the case where the dominant firm is replaced by a dominant oligopoly
    - The key is to specify the solution concept for the choice of output by the firms in the oligopoly (e.g., Cournot). You then create a residual demand curve for the oligopoly and apply the solution concept to that demand curve.
- Fringe firms
  - As we saw in Unit 1, the DOJ and the FTC typically ignore fringe firms. The dominant oligopoly model with a competitive fringe provides a theoretical justification.

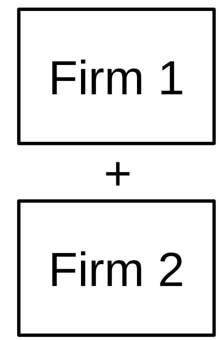
# Merger Typology, Substitutes and Complements, and Elasticities

# Merger typology

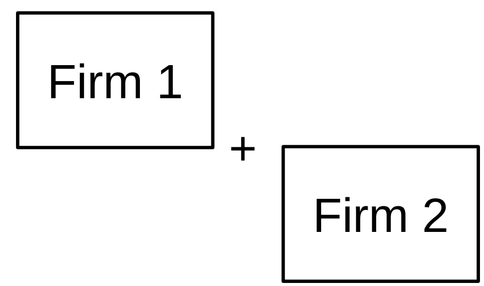
- Horizontal mergers
  - Combination of two competitors
    - Two competing manufacturers
    - Two competing distributors
    - Two competing retail stores




- Vertical mergers
  - Combination of two firms at adjacent levels in the chain of manufacture and distributions
    - Manufacturer + distributor
    - Wholesaler + retailer
  - May be extended to two firms that produce complementary products




- Conglomerate mergers
  - Mergers that are neither horizontal or vertical
  - Products, however, can be complements
    - E.g., a combination between a printer company and a printer cartridge company



# Merger typology

- Multifacility multiproduct combinations
  - Can involve horizontal, vertical, and conglomerate aspects depending on locations of facilities and the products or services that each facility offers

# Substitutes/Complements

- Substitutes
  - Two products or services are *substitutes* if, when consumer demand increases for one product, it will decrease for the other product
    - That is, consumers see the products as replacements for one another
    - If the consumer is only going to buy one unit, she will buy one or the other
      - E.g., a Ford Focus or a Honda Civic
    - If the consumer buys multiple units, buying more of one means buying less of the other
      - E.g., more Diet Coke and less Diet Pepsi
  - *Horizontal mergers* involve combinations of firms that offer substitute products
    - Firms compete with each other when they offer substitute products

# Merger Typology: Substitutes/Complements

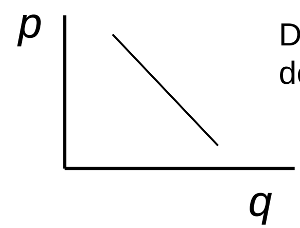
- Complements
  - Two products are *complements* if, when a consumer demand increases for one product, consumer demand also will increase for the other product
    - Examples
      - Razor and razor blades
      - Printers and printer ink cartridges
      - Product manufacturing and distribution
    - Complements do not have to be purchases in a one-to-one ratio (as the above examples show)
    - Complements may involve a product and a service
      - E.g., High-speed printers and high-speed printer repair services
  - *Vertical mergers* involve complement products and services that are in the same chain of manufacturing and distribution
  - *Conglomerate mergers* may or may not involve complement products or services
    - But if it does, they will not be in the same chain of manufacturing and distribution

# Substitutes/Complements

- Mathematically (for those of you so inclined):
  - Notation
    - Let $p_1$ and $p_2$ be the prices of products 1 and 2
    - Let $q_1$ and $q_2$ be the respective quantities demand by consumers
  - Downward-sloping demand curve: Consumers demand less of a product the higher its price

Remember, read this as the change in $q_i$ demanded with a (positive) increase in $p_i$

$$\frac{\partial q_i}{\partial p_i} < 0$$




Downward-sloping demand curve

  - *Substitutes*: Increased demand for product 1 means decreased demand for product 2:

$$\frac{\partial q_2}{\partial q_1} < 0 \quad \text{or equivalently} \quad \frac{\partial q_1}{\partial p_1}\frac{\partial q_2}{\partial q_1} < 0 \Rightarrow \frac{\partial q_2}{\partial p_1} > 0$$

As price of product 1 increases, demand for product 2 increases

  - *Complements*: Increased demand for product 1 means increased demand for product 2:

$$\frac{\partial q_2}{\partial q_1} > 0 \quad \text{or equivalently} \quad \frac{\partial q_1}{\partial p_1}\frac{\partial q_2}{\partial q_1} > 0 \Rightarrow \frac{\partial q_2}{\partial p_1} < 0$$

As price of product 1 increases, demand for product 2 decreases

# Elasticities

- Elasticity of demand
  - *Problem*: Changes in the absolute quantities demanded can vary with changes in the unit of measure
    - Example: You get different numbers for the change in demand for razor blades with an increase in demand for razor if razor blades are measured in (a) units or (b) ounces
  - *Solution*: Find a measure of change that is dimensionless (free of units)
    - The percentage change in the quantity demanded for a given percentage change in price will do this. This is called an *elasticity of demand*.
    - The elasticity of demand will not change with a change in the unit of measure of either prices or quantities

# Elasticities

- Elasticity of demand—Some definitions
  - *Own-elasticity of demand*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product.

$$\varepsilon = \frac{\dfrac{\Delta q_i}{q_i}}{\dfrac{\Delta p_i}{p_i}}$$

Percentage change $q_i$ in the quantity of product *i* demanded

Percentage change $p_i$ in the price of product *i*

Slope of the (residual) demand curve

    - Using a little algebra, this is equivalent to $\frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$ (or in calculus terms $\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$)
    - Own-elasticities are negative, due to the downward-sloping nature of the demand curve
  - *Cross-elasticity of demand*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} = \frac{\dfrac{\Delta q_j}{q_j}}{\dfrac{\Delta p_i}{p_i}}$$

Percentage change $q_j$ in the quantity of product *j* demanded

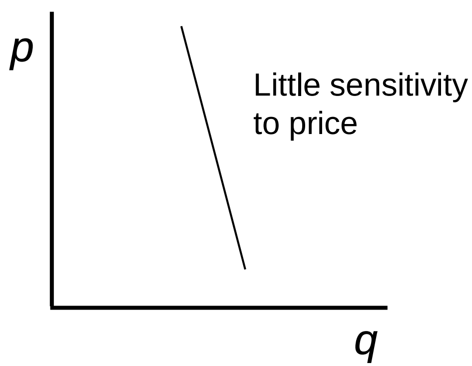
Percentage change $p_i$ in the price of product *i*

    - Cross-elasticities are positive for substitutes and negative for complements

# Elasticities

- Elasticity of demand—More definitions
    - By convention, economists speak of elasticities in terms of their absolute values
        - Don't ask me why
  - Own-elasticities
    - *Inelastic demand*: Own demand where the quantity demanded does not change significantly with changes in the product's price. *Not price sensitive*. $(|\varepsilon| < 1)$
    - *Unit elasticity*: Where a 1% change in the product's price results in a 1% decrease in the quantity demanded $(|\varepsilon| = 1)$
    - *Elastic demand*: Own demand where the quantity demanded drops rapidly with small changes in price. *Very price sensitive*. $(|\varepsilon| > 1)$
    - Some (erroneous) graphics for the intuition:

Very inelastic demand




Very elastic demand

# Elasticities

- Elasticity of demand and the slope of the demand curve

$$\varepsilon = \frac{\text{\% change in quanity demanded}}{\text{\% change in price}} = \frac{\dfrac{\Delta q}{q}}{\dfrac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q}$$

This is the slope of the demand curve

  - *Note*: A linear demand curve has a constant slope $\Delta q/\Delta p$. But since $p$ and $q$ change going up or down the demand curve, the elasticity of demand is not constant

# Elasticities

- Some conventions and definitions
  - By convention, economists speak of elasticities in terms of their absolute values
- Own-elasticities
  - *Inelastic demand*: Own demand where the quantity demanded does not change significantly with changes in the product's price. *Not price sensitive*. $(|\varepsilon| < 1)$

This means take the "absolute value ((so, for example $|-0.5| = 0.5$), and so makes own-elasticities positive numbers.

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} < 1$$

Inelastic demand

  - *Unit elasticity*: Where a 1% change in the product's price results in a 1% decrease in the quantity demanded $(|\varepsilon| = 1)$

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} = 1$$

Unit elasticity

  - *Elastic demand*: Own demand where the quantity demanded drops rapidly with small changes in price. *Very price sensitive*. $(|\varepsilon| > 1)$

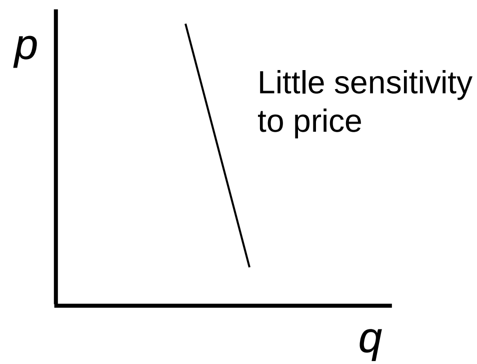
$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} > 1$$

Elastic demand

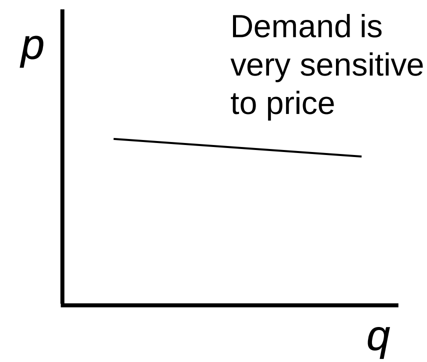
# Elasticities

- Own-elasticities and linear demand curves
  - Some (erroneous) graphics for the intuition:

Very inelastic demand




Very elastic demand




  - Why are these diagrams erroneous?
    - Remember:

$$\varepsilon = \frac{\frac{\Delta q_i}{q_i}}{\frac{\Delta p_i}{p_i}} = \frac{\frac{p_i}{\Delta p_i}}{\frac{p_i}{\Delta p_i}} = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$

Slope of the (residual) demand curve

Elasticities are measured at a particular point ($p$,$q$) on the demand curve

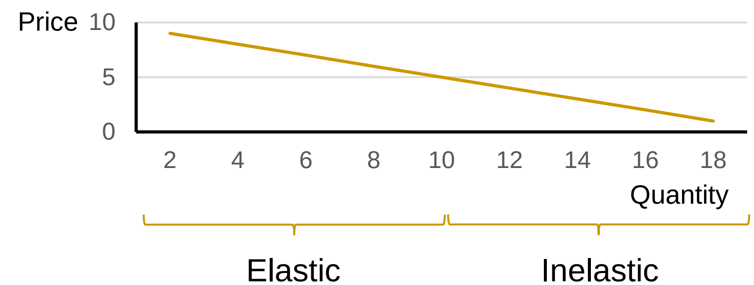
    - The slope of the demand curve is constant, but the ratio $p_i/q_i$ changes along the curve. Therefore, the elasticity is not constant on a linear demand curve.

# Elasticities

- Elasticity of demand and the slope of the demand curve
  - *Example:*
    - Demand curve: $q = 20 - 2p$ → Inverse demand curve: $p = 10 - \frac{1}{2}q$


Inelastic portion of the demand curve

Elastic portion of the demand curve




| $p$ | $q$ | Slope | $p/q$ | $\varepsilon$ | |
|---|---|---|---|---|---|
| 1 | 18 | -2 | 0.0556 | -0.1111 | Inelastic demand |
| 2 | 16 | -2 | 0.1250 | -0.2500 | |
| 3 | 14 | -2 | 0.2143 | -0.4286 | |
| 4 | 12 | -2 | 0.3333 | -0.6667 | |
| 5 | 10 | -2 | 0.5000 | -1.0000 | Unit elasticity |
| 6 | 8 | -2 | 0.7500 | -1.5000 | Elastic demand |
| 7 | 6 | -2 | 1.1667 | -2.3333 | |
| 8 | 4 | -2 | 2.0000 | -4.0000 | |
| 9 | 2 | -2 | 4.5000 | -9.0000 | |

*Rule for linear demand curves*: Elasticity increases as price increases

# Elasticities

- Elasticity of demand and the slope of the demand curve

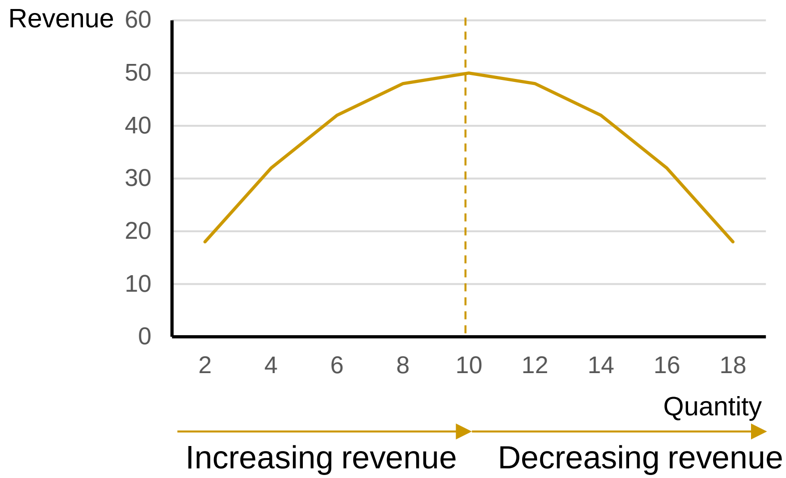
Demand curve:
$p = 20 - 2q$

| $p$ | $q$ | Slope | $p/q$ | $\varepsilon$ | Total revenue |
|---|---|---|---|---|---|
| 1 | 18 | -2 | 0.0556 | -0.1111 | 18 |
| 2 | 16 | -2 | 0.1250 | -0.2500 | 32 |
| 3 | 14 | -2 | 0.2143 | -0.4286 | 42 |
| 4 | 12 | -2 | 0.3333 | -0.6667 | 48 |
| 5 | 10 | -2 | 0.5000 | -1.0000 | 50 |
| 6 | 8 | -2 | 0.7500 | -1.5000 | 48 |
| 7 | 6 | -2 | 1.1667 | -2.3333 | 42 |
| 8 | 4 | -2 | 2.0000 | -4.0000 | 32 |
| 9 | 2 | -2 | 4.5000 | -9.0000 | 18 |

Inelastic demand (p = 1–4) — Increasing revenue

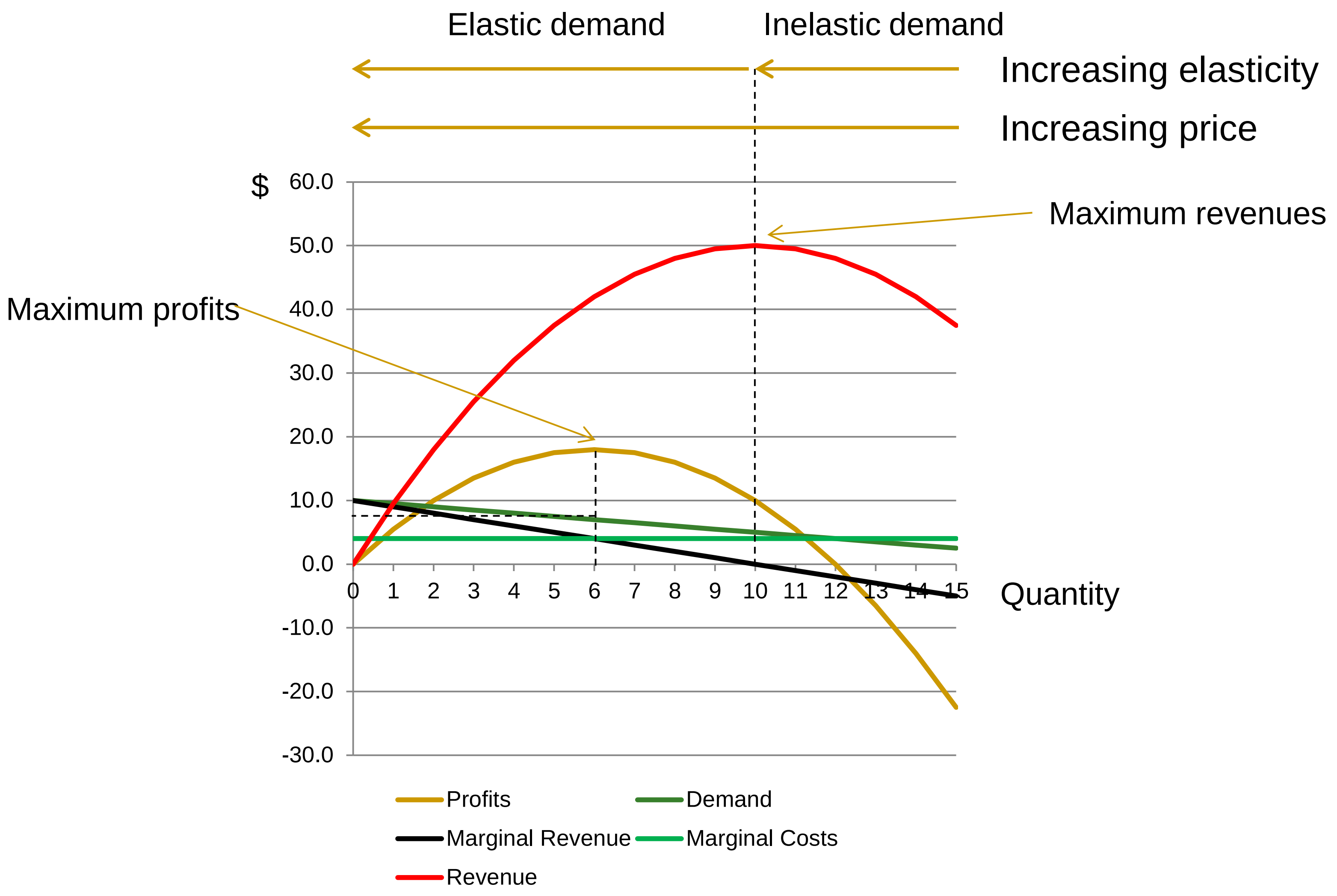
Unit elasticity (p = 5)

Elastic demand (p = 6–9) — Decreasing revenue




This is why elasticities are meaningful

# Elasticities



# Elasticities

- Monopoly pricing and elasticities
  - Slide on previous slide suggests that a monopolist will not price in the inelastic portion of its demand curve
  - This is in fact a general rule
    - Proof

First note that: $\varepsilon = \frac{dq}{dp}\frac{p}{q} \Rightarrow \frac{1}{\varepsilon} = \frac{dp}{dq}\frac{q}{p} \Rightarrow \frac{dp}{dq} = \frac{p}{\varepsilon q}$

Now look at:

$$\frac{d\pi}{dq} = p + q\frac{dp}{dq} - \frac{dc}{dq}$$

$$= p + q\left[\frac{p}{q\varepsilon}\right] - \frac{dc}{dq}$$

$$= p\left[1 + \frac{1}{\varepsilon}\right] - \frac{dc}{dq}$$

Substituting

# Elasticities

- Monopoly pricing and elasticities
  - This is in fact a general rule
    - Proof (con't)

If demand is inelastic, then $|\varepsilon| < -1$ by definition. This means that $-1 < \varepsilon < 1$. Moreover, $\varepsilon < 0$ since demand is downward sloping. This means that $-1 < \varepsilon < 0$, which makes the fraction above in the brackets less than -1, which in turn makes the expression in brackets negative.

$$\frac{d\pi}{dq} = p\left[1 + \frac{1}{\varepsilon}\right] - \frac{dc}{dq} < 0$$

Since costs are also subtracted, the entire expression is negative. So profits are decreasing in output, which in turn means that the output is above the profit-maximizing level. The firm needs to raise prices to reduce demand: the monopolist is pricing too low, even if marginal costs are zero.

# Elasticities

- Elasticity of demand—More definitions
  - Cross-elasticities
    - *High cross-elasticity of demand*: A small change in the price of product *i* will cause a large shift of demand to product *j*
      - As a result, product *j* brings a lot of competitive pressure on product *i*
    - *Low cross-elasticity of demand*: A large change in the price of product *i* will cause only a small shift of demand to product *j*
      - As a result, product *j* brings little competitive pressure on product *I*