8. Basic Competition Economics

For Class 12

Merger Antitrust Law

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Dale Collins

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Topics

- Introduction
- Consumer demand and the aggregate consumer demand curve
- Producer profit maximization and the aggregate supply curve
- Merger typology, substitutes and complements, and elasticities
- Perfect market equilibria
  - Perfectly competitive markets
  - Perfect monopoly
  - Incentives for coordination
  - Public policy re monopoly
- Imperfect market equilibria
  - Cournot equilibria
  - Bertrand equilibria
  - Dominant firm with a competitive fringe

Merger Typology, Substitutes and Complements, and Elasticities

Merger typology

- Horizontal mergers
  - Combination of two competitors
    - Two competing manufacturers
    - Two competing distributors
    - Two competing retail stores

Firm 1 + Firm 2

- Vertical mergers
  - Combination of two firms at adjacent levels in the chain of manufacture and distributions
    - Manufacturer + distributor
    - Wholesaler + retailer
  - May be extended to two firms that produce complementary products

Firm 1
+
Firm 2

- Conglomerate mergers
  - Mergers that are neither horizontal or vertical
  - Products, however, can be complements
    - E.g., a combination between a printer company and a printer cartridge company

Firm 1
+
Firm 2

Merger typology

- Multifacility multiproduct combinations
  - Can involve horizontal, vertical, and conglomerate aspects depending on locations of facilities and the products or services that each facility offers

Substitutes/Complements

- Substitutes
  - Two products or services are *substitutes* if, when consumer demand increases for one product, it will decrease for the other product
    - That is, consumers see the products as replacements for one another
    - If the consumer is only going to buy one unit, she will buy one or the other
      - E.g., a Ford Focus or a Honda Civic
    - If the consumer buys multiple units, buying more of one means buying less of the other
      - E.g., more Diet Coke and less Diet Pepsi
  - *Horizontal mergers* involve combinations of firms that offer substitute products
    - Firms compete with each other when they offer substitute products

Merger typology: Substitutes/Complements

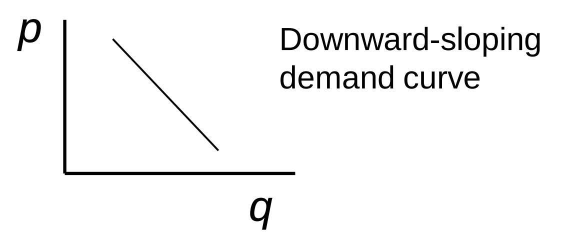
- Complements
  - Two products are *complements* if, when a consumer demand increases for one product, consumer demand also will increase for the other product
    - Examples
      - Razor and razor blades
      - Printers and printer ink cartridges
      - Product manufacturing and distribution
    - Complements do not have to be purchases in a one-to-one ratio (as the above examples show)
    - Complements may involve a product and a service
      - E.g., High-speed printers and high-speed printer repair services
  - *Vertical mergers* involve complement products and services that are in the same chain of manufacturing and distribution
  - *Conglomerate mergers* may or may not involve complement products or services
    - But if it does, they will not be in the same chain of manufacturing and distribution

Substitutes/Complements

- Mathematically (for those of you so inclined):
  - Notation
    - Let p_1 and p_2 be the prices of products 1 and 2
    - Let q_1 and q_2 be the respective quantities demand by consumers
  - Downward-sloping demand curve: Consumers demand less of a product the higher its price

Remember, read this as the change in q_i demanded with a (positive) increase in p_i

$$\frac{\partial q_i}{\partial p_i} < 0$$



  - *Substitutes*: Increased demand for product 1 means decreased demand for product 2:

$$\frac{\partial q_2}{\partial q_1} < 0 \quad \text{or equivalently} \quad \frac{\partial q_1}{\partial p_1}\frac{\partial q_2}{\partial q_1} < 0 \Rightarrow \frac{\partial q_2}{\partial p_1} > 0$$

As price of product 1 increases, demand for product 2 increases

  - *Complements*: Increased demand for product 1 means increased demand for product 2:

$$\frac{\partial q_2}{\partial q_1} > 0 \quad \text{or equivalently} \quad \frac{\partial q_1}{\partial p_1}\frac{\partial q_2}{\partial q_1} > 0 \Rightarrow \frac{\partial q_2}{\partial p_1} < 0$$

As price of product 1 increases, demand for product 2 decreases

Elasticities

- Elasticity of demand
  - *Problem*: Changes in the absolute quantities demanded can vary with changes in the unit of measure
    - *Example*: You get different numbers for the change in demand for razor blades with an increase in demand for razor if razor blades are measured in (a) units or (b) ounces
  - *Solution*: Find a measure of change that is dimensionless (free of units)
    - The percentage change in the quantity demanded for a given percentage change in price will do this. This is called an *elasticity of demand*.
    - The elasticity of demand will not change with a change in the unit of measure of either prices or quantities

Elasticities

- Elasticity of demand—Some definitions
  - *Own-elasticity of demand*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product.

When Δq_i *and* Δp_i are finite (not arbitrarily close to zero), these are called *arc elasticities*. They have the same type of error problems that we examined in the marginal revenue slides earlier.

$$\varepsilon = \frac{\dfrac{\Delta q_i}{q_i}}{\dfrac{\Delta p_i}{p_i}}$$

Percentage change q_i in the quantity of product *i* demanded

Percentage change p_i in the price of product *i*

Slope of the (residual) demand curve of firm i

- Using a little algebra, this is equivalent to $\dfrac{\Delta q_i}{\Delta p_i}\dfrac{p_i}{q_i}$ (or in calculus terms $\dfrac{\partial q_i}{\partial p_i}\dfrac{p_i}{q_i}$)
- Own-elasticities are negative, due to the downward-sloping nature of the demand curve

  - *Cross-elasticity of demand*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} = \frac{\dfrac{\Delta q_j}{q_j}}{\dfrac{\Delta p_i}{p_i}}$$

Percentage change q_j in the quantity of product *j* demanded

Percentage change p_i in the price of product *i*

- Cross-elasticities are positive for substitutes and negative for complements

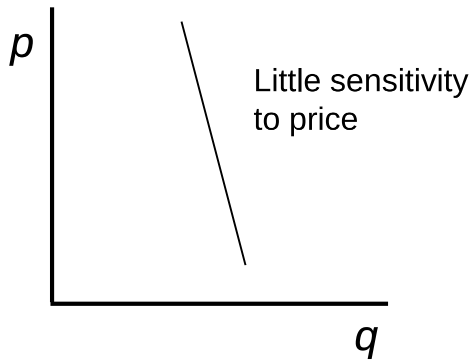
Terminology: Own-elasticity is often called just elasticity. Cross-elasticity is always called cross-elasticity.

Elasticities

These terms on this slide are part of the vocabulary of the antitrust bar. You need to be familiar with them.

- Elasticity of demand—More definitions
  - By convention, economists usually speak of elasticities in terms of their absolute values
    - Don't ask me why
  - Own-elasticities
    - *Inelastic demand*: Own demand where the quantity demanded does not change significantly with changes in the product's price. *Not price sensitive*. $\left(|\varepsilon| < 1\right)$
    - *Unit elasticity*: Where the percentage increase in the product's price results in the same percentage decrease in the quantity demanded $\left(|\varepsilon| = 1\right)$
    - *Elastic demand*: Own demand where the quantity demanded drops rapidly with small changes in price. *Very price sensitive*. $\left(|\varepsilon| > 1\right)$
    - Some (erroneous) graphics for the intuition:

Very inelastic demand



Very elastic demand

Elasticities

- Some conventions and definitions
  - By convention, economists speak of elasticities in terms of their absolute values
- Own-elasticities
  - *Inelastic demand*: Own demand where the quantity demanded does not change significantly with changes in the product's price. *Not price sensitive*. $\left(|\varepsilon| < 1\right)$

This means take the "absolute value (so, for example $|-0.5| = 0.5$), and so makes own-elasticities positive numbers.

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} < 1$$

Inelastic demand

  - *Unit elasticity*: Where a 1% change in the product's price results in a 1% decrease in the quantity demanded $\left(|\varepsilon| = 1\right)$

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} = 1$$

Unit elasticity

  - *Elastic demand*: Own demand where the quantity demanded drops rapidly with small changes in price. *Very price sensitive*. $\left(|\varepsilon| > 1\right)$

$$|\varepsilon| = \frac{\text{\%change in quantity}}{\text{\%change in price}} > 1$$

Elastic demand

Elasticities

Again, these terms on this slide are part of the vocabulary of the antitrust bar. You need to be familiar with them.

- Elasticity of demand—More definitions
  - Cross-elasticities
    - *High cross-elasticity of demand*: A small change in the price of product *i* will cause a large shift of demand to product *j*
      - As a result, product *j* brings a lot of competitive pressure on product *i*
    - *Low cross-elasticity of demand*: A large change in the price of product *i* will cause only a small shift of demand to product *j*
      - As a result, product *j* brings little competitive pressure on product *I*

Elasticities

- Elasticity of demand and the slope of the demand curve

$$\varepsilon = \frac{\text{\% change in quanity demanded}}{\text{\% change in price}} = \frac{\dfrac{\Delta q}{q}}{\dfrac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q}$$

This is the slope of the demand curve

  - *Note*: A linear demand curve has a constant slope $\Delta q/\Delta p$. But since p and q change going up or down the demand curve, the elasticity of demand is *not* constant
    - Assume that the slope of the demand curve is some constant k. Then:

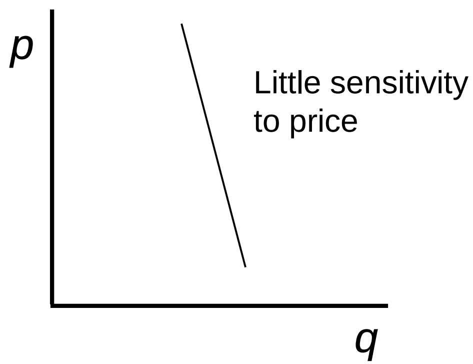
$$\varepsilon = k\frac{p}{q}$$

Given that the demand curve is downward sloping, as q increases in magnitude p decreases, so that p/q get smaller. This means that demand is inelastic $(|\varepsilon| > 1)$ at high prices and low quantities, and becomes more elastic (becomes a smaller number in absolute value) as q increases and p decreases.

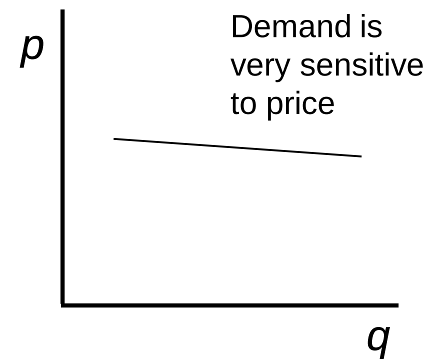
Elasticities

- Elasticity of demand and the slope of the demand curve
  - Some (erroneous) graphics for the intuition:

Very inelastic demand



Very elastic demand



  - Why are these diagrams erroneous?
    - Remember:

Slope of the (residual) demand curve

$$\varepsilon = \frac{\dfrac{\Delta q_i}{q_i}}{\dfrac{\Delta p_i}{p_i}} = \frac{\dfrac{p_i}{\Delta p_i}}{\dfrac{p_i}{\Delta p_i}} = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$

Elasticities are measured at a particular point (p,q) on the demand curve

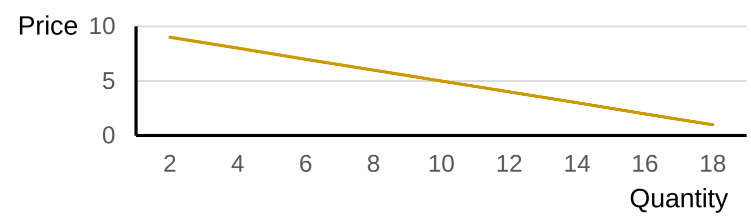
    - The slope of the demand curve is constant, but the ratio p_i/q_i changes along the curve. Therefore, the elasticity is *not* constant on a linear demand curve.

Elasticities

- Elasticity of demand and the slope of the demand curve
  - *Example:*
    - Demand curve: $q = 20 - 2p$ → Inverse demand curve: $p = 10 - \frac{1}{2}q$

Inelastic portion of the demand curve

Elastic portion of the demand curve



Elastic

Inelastic

p	q	Slope	p/q	ε	
1	18	-2	0.0556	-0.1111	Inelastic demand
2	16	-2	0.1250	-0.2500	
3	14	-2	0.2143	-0.4286	
4	12	-2	0.3333	-0.6667	
5	10	-2	0.5000	-1.0000	Unit elasticity
6	8	-2	0.7500	-1.5000	Elastic demand
7	6	-2	1.1667	-2.3333	
8	4	-2	2.0000	-4.0000	
9	2	-2	4.5000	-9.0000	

Rule for linear demand curves: Elasticity increases as price increases

Elasticities

- Elasticity of demand and the slope of the demand curve

Demand curve:
$p = 20 - 2q$

p	q	Slope	p/q	ε	Total revenue
1	18	-2	0.0556	-0.1111	18
2	16	-2	0.1250	-0.2500	32
3	14	-2	0.2143	-0.4286	42
4	12	-2	0.3333	-0.6667	48
5	10	-2	0.5000	-1.0000	50
6	8	-2	0.7500	-1.5000	48
7	6	-2	1.1667	-2.3333	42
8	4	-2	2.0000	-4.0000	32
9	2	-2	4.5000	-9.0000	18

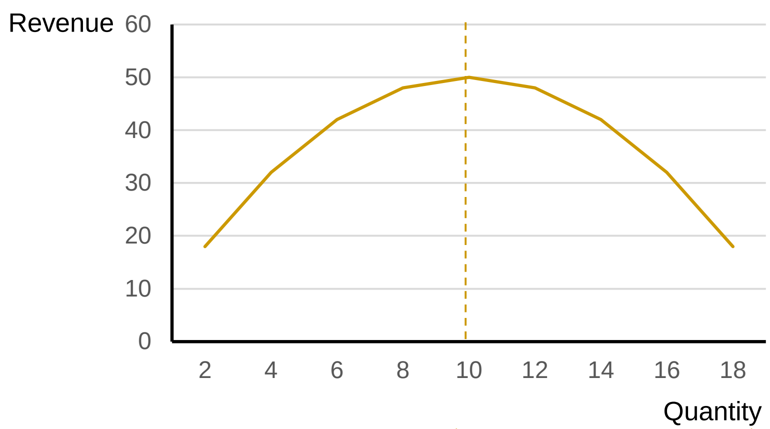
Inelastic demand

Unit elasticity

Elastic demand

Increasing revenue

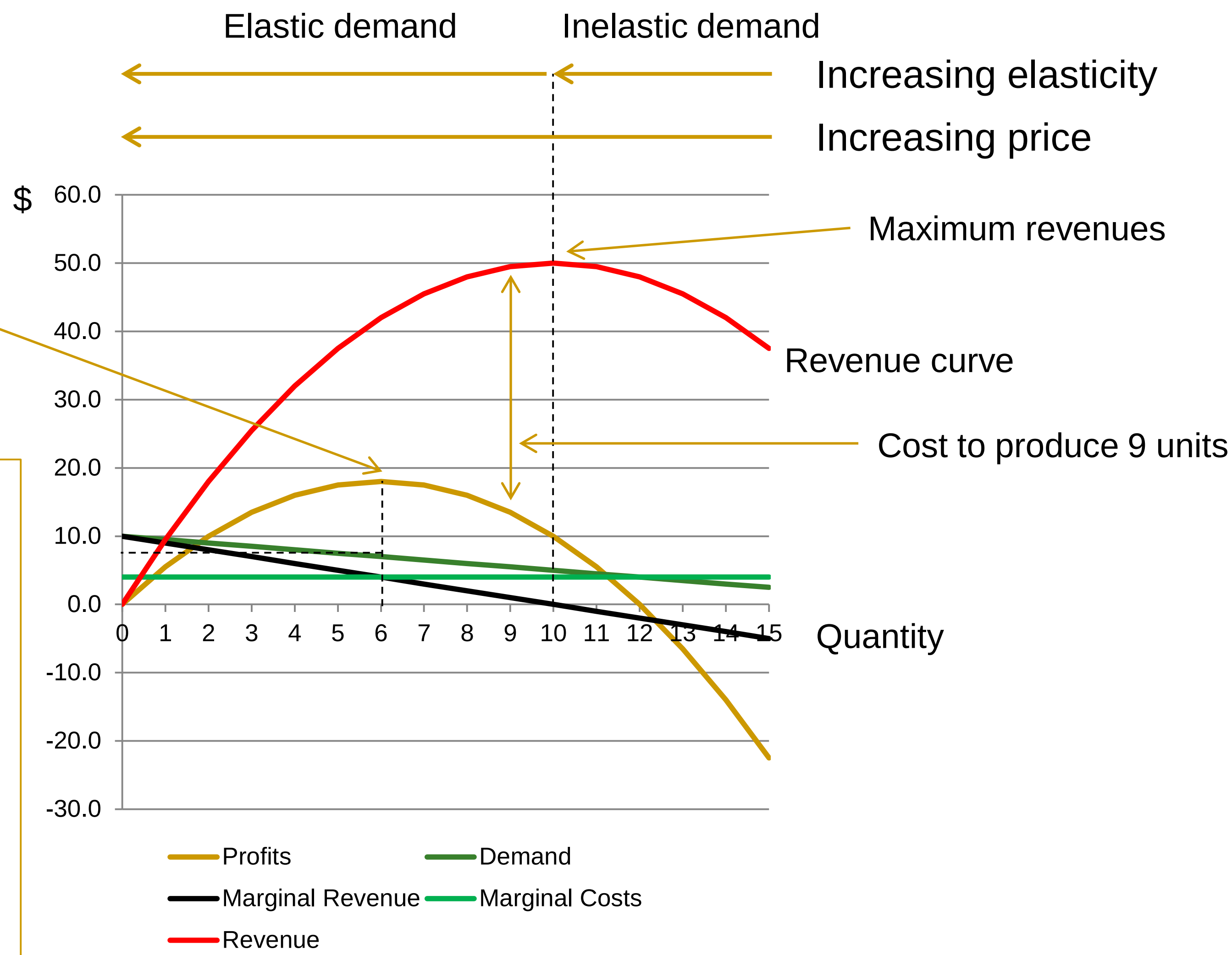
Decreasing revenue



Increasing revenue Decreasing revenue

This is why elasticities are meaningful

Elasticities



Note
1. The profit curve is below the revenue curve because profits equal revenues minus costs and costs are positive.
2. The gap between revenues and profits increases as quantity increases because total costs increase more quickly than revenues as quantity increases.

Elasticities

- Monopoly pricing and elasticities
  - The slide on previous slide suggests that a monopolist will not price in the inelastic portion of its demand curve (i.e., where $|\varepsilon| < 1$, or equivalently, $-1 < \varepsilon < 0$)
  - This is in fact a general rule

Proof:

First note that: $$\varepsilon = \frac{dq}{dp}\frac{p}{q} \Rightarrow \frac{1}{\varepsilon} = \frac{dp}{dq}\frac{q}{p} \Rightarrow \frac{dp}{dq} = \frac{p}{\varepsilon q}$$

Now look at:

$$\frac{d\pi}{dq} = p + q\frac{dp}{dq} - \frac{dc}{dq}$$

$$= p + q\left[\frac{p}{q\varepsilon}\right] - \frac{dc}{dq}$$

$$= p\left[1 + \frac{1}{\varepsilon}\right] - \frac{dc}{dq} < 0$$

Substituting terms

Remember, this is complicated-looking mathematical notation just says that marginal revenue is the difference between marginal revenue equals marginal cost

Remember, these are negative numbers

When demand is inelastic, the term in brackets is negative, which means that the entire express is negative

Proofs in this deck are always optional

Since marginal profits are negative, a monopolist could increase its profits by restricting output and increasing price. Q.E.D.

Elasticities

- Relation of the residual demand elasticity to the aggregate demand elasticity
  - *Rule*: In a market of *n* identical firms, a single firm's own-price elasticity is equal to *n* times the aggregate demand own-elasticity (i.e., $\varepsilon = n\varepsilon_i$, where ε is the market elasticity and ε_i is the firm elasticity)

Proofs in this deck are always optional

Proof: Assume that there are *n* identical firms in the market, each producing quantity *q* (for an aggregate market output of $nq = Q$) at price *p*. Recall the definitions of firm own-elasticity ε_i and aggregate demand own-elasticity ε:

$$\varepsilon_i \equiv \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \qquad \varepsilon \equiv \frac{\frac{\Delta Q}{Q}}{\frac{\Delta p}{p}}.$$

In this partial equilibrium analysis, assume that one firm changes its production by an amount Δq and that all other firms remain at their original production levels *q*, so that $\Delta Q = \Delta q$. Assume further that the one firm's quantity change is so small that we can ignore any effect on price (i.e., the changes are infinitesimally very small). Then:

$\Delta q = \Delta Q$

This is equal to 1

$$\varepsilon_i = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{\left(\frac{n}{n}\right)\frac{\Delta Q}{q}}{\frac{\Delta p}{p}} = n\left(\frac{\frac{\Delta Q}{Q}}{\frac{\Delta p}{p}}\right) = n\varepsilon.$$

Q.E.D.

Elasticities

- Relation of upstream and downstream elasticities
  - Say an upstream firm U produces an input that it sells to downstream firm D at price p_U.
  - If Firm D's production function requires k units of the inputs for each downstream unit (i.e., it is a *fixed proportions production function*), which it sells at price p_D (a multiple m of p_U), and has elasticity ε_D then:

$$\varepsilon_D = \varepsilon_U.$$

Key result

Proof:

$$x_D = kx_U \Rightarrow \frac{\partial x_D}{\partial x_U} = k \text{ and } p_U = \frac{p_D}{m} \Rightarrow \frac{\partial p_U}{\partial p_D} = \frac{1}{m},$$

so

$$\frac{\partial x_D}{\partial p_D} = \frac{\partial x_D}{\partial x_U}\frac{\partial x_U}{\partial p_U}\frac{\partial p_U}{\partial p_D} = k\frac{\partial x_U}{\partial p_U}\frac{1}{m}$$

$$= \frac{k}{m}\frac{\partial x_U}{\partial p_U}.$$

Substituting

$$\varepsilon_D = \frac{\partial x_D}{\partial p_D}\frac{p_D}{x_D} = \left[\frac{k}{m}\frac{\partial x_U}{\partial p_U}\right]\left[\frac{mp_U}{kx_U}\right] = \frac{\partial x_U}{\partial p_U}\frac{p_U}{x_U} = \varepsilon_U.$$

Q.E.D.

Proofs in this deck are always optional

Elasticities

- Relationship between own- and cross-elasticities
  - *Rule*: The own-elasticity of demand for product *i* is a function of the sum of the cross-elasticities of all of the other products weighted by their respective revenue expenditure shares or equivalently, their relative market shares[1]

Proof: We will prove this for product 1. Consider the consumer's budget constraint:

$$B = \sum_{i=1}^{n} p_i q_i(p),$$

where q_i is the amount of product *i* the consumer demand given the prevailing prices $p = (p_1, p_2, \ldots p_n)$ of all of the products available to the consumer to purchase (e.g., automobiles, houses, peanuts, jelly beans)

Proofs in this deck are always optional

Differentiating the budget constraint with respect to p_1 yields:

Since B is a constant, its derivative is zero

$$\frac{dB}{dp_1} = 0 = q_1 + p_1 \frac{\partial q_1}{\partial p_1} + \sum_{i=2}^{n} p_i \frac{\partial q_i}{\partial p_1}$$

Rearranging:

$$-p_1 \frac{\partial q_1}{\partial p_1} = q_1 + \sum_{i=2}^{n} p_i \frac{\partial q_i}{\partial p_1}$$

[1] The revenue expenditure share for product *i* is the share of the budget the consumer spends on product *i.*

Elasticities

- Relationship between own- and cross-elasticities (con't)

Dividing by q_1 and multiplying the last term by 1:

This term equals 1

$$-\frac{p_1}{q_1}\frac{\partial q_1}{\partial p_1} = \frac{q_1}{q_1} + \sum_{i=2}^{n} p_i \frac{\partial q_i}{\partial p_1}\frac{1}{q_1}\left(\frac{q_i}{q_i}\frac{p_1}{p_1}\right).$$

Rearragning a little more:

$$-\frac{\partial q_1}{\partial p_1}\frac{p_1}{q_1} = 1 + \sum_{i=2}^{n}\left(\frac{\partial q_i}{\partial p_1}\frac{p_1}{q_i}\right)\left(\frac{q_i}{q_1}\frac{p_i}{p_1}\right).$$

Recall that own elasticity ε is negative due to the downward-slope of the demand curve, so that $-\varepsilon$ is positive

Or:

$$-\varepsilon_{11} = 1 + \sum_{i=2}^{n}\varepsilon_{i1}\frac{r_i}{r_1},$$

where $r_i = p_i q_i$, that is, the consumer's expenditure on product *i*. Let $s_i = r_i / B$, that is, the revenue expenditure share for product *i*. Then:

$$-\varepsilon_{11} = 1 + \sum_{i=2}^{n}\varepsilon_{i1}\frac{s_i}{s_1}.$$

So the own-elasticity of demand for product 1 is a function of the sum of the cross-elasticities of all of the other products weighted by their respective revenue expenditure shares or, equivalently, their relative market shares (when share is measured in revenues). Q.E.D.

Proofs in this deck are always optional

Perfect Market Equilibria

Perfectly competitive markets

- *Definition*: A market in which no single firm can effect price, meaning:
  - The firm's residual demand curve is horizontal,
  - The firm can sell any amount of product without affecting the market price,
  - $\frac{dp}{dq} = 0$, or
  - $p = \frac{dc}{dq}$ (i.e., price = marginal cost)

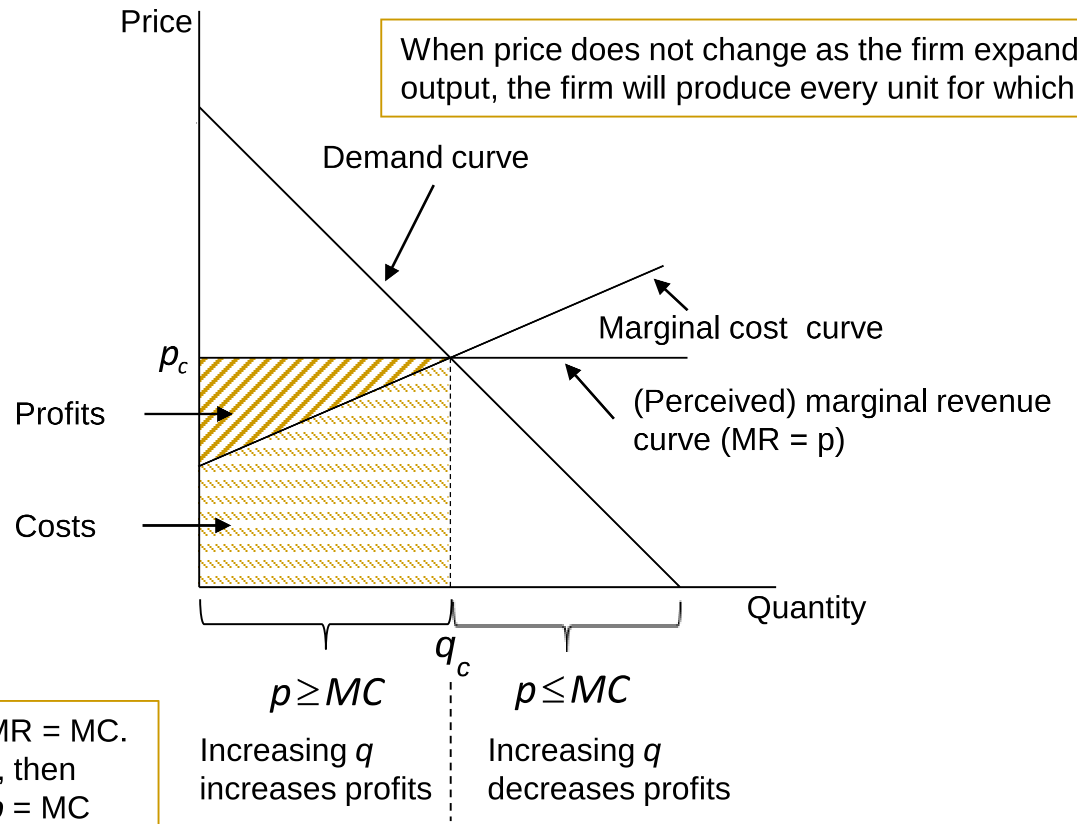
These four bullets are just different ways of saying exactly the same thing.

- What could cause a market to be perfectly competitive?
  - *Traditional theory*: Each individual firm's production is very small compared to aggregate demand at any price, so that individual production changes cannot move significantly along the aggregate demand curve
    - This implies that there are a very large number of firms in the market
  - *Modern theory*: Competitors in the market place react strategically but non-collusively to price or quantity changes by a firm in ways that maintain the competitive equilibrium

Competitive firms

- Competitive firms take prices as given
  - → Individual output decisions do not affect the market-clearing price

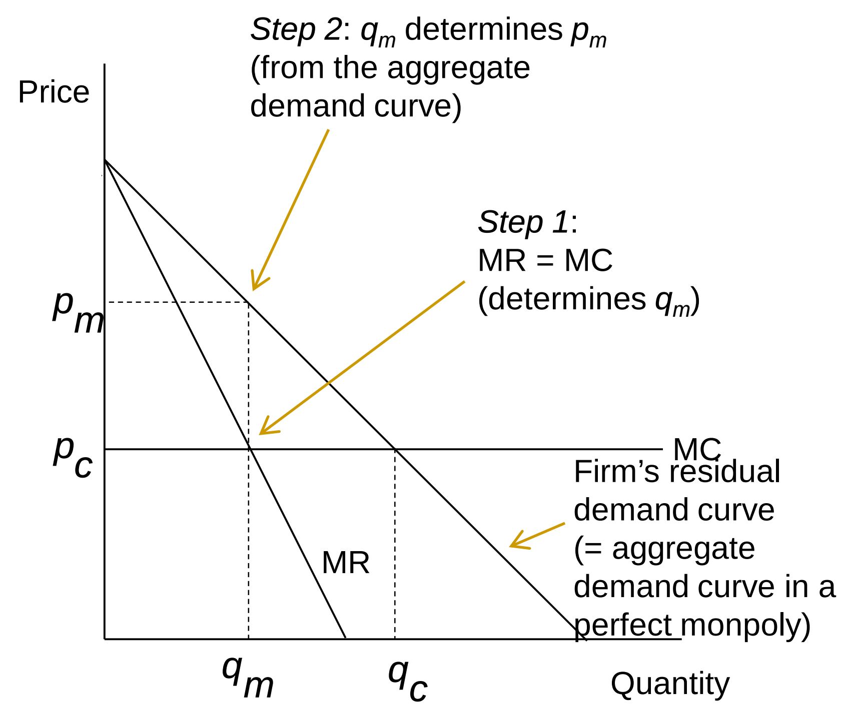
When price does not change as the firm expands output, the firm will produce every unit for which $p \geq MC$



As always, the FOC is MR = MC. If the firm is competitive, then MR = *p and so* FOC is *p* = MC

Perfect monopoly

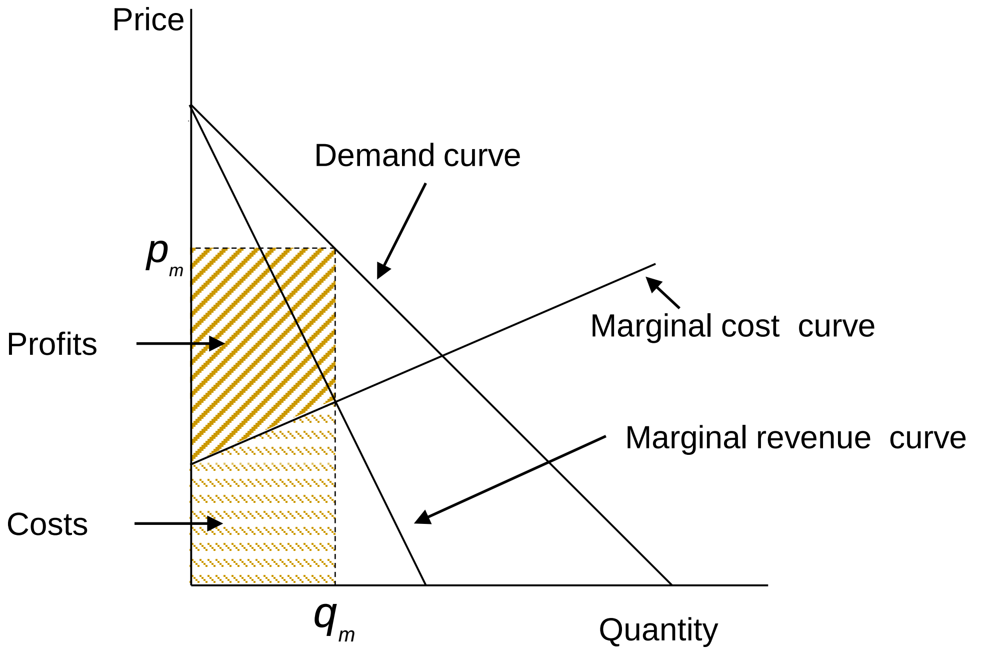
- *Economic definition*: A market in which only one firm operates
  - In this case, the firm's residual demand curve is the same as the aggregate demand curve
  - As always, the firm's chooses production so that its marginal cost is equal to marginal revenue



Keep in mind that this is the way economists define "perfect monopoly." As we shall see later, the legal definition of "monopoly" is quite different.

Monopolist firm

- A monopolist choice of output *q* affects the market-clearing price *p*
  - Contrast with a perfectly competitive firm, which cannot affect p through its choice of q (by definition)



Monopolists (like all profit-maximizing firms) price at MR = MC, where the monopolist's marginal revenue is determined by the aggregate demand curve

Perfect markets: Summary of key results so far

- Profit-maximizing firms choose production levels so that marginal revenue equals marginal cost (MR = MC) (in Cournot models)
  - Step 1: MR = MC determines the firm's profit-maximizing production level $q*$
  - Step 2: The firm's residual demand curve determines the firm's profit-maximizing price $p*$ given $q*$
- In a perfectly competitive market, a firm's choice of production level cannot effect market price, so:
  - Marginal revenue is equal to the market unit price (MR = p_{market}),
  - MR = MC implies that MC = p_{market} , so firm picks $q_{competitive}$ to satisfy this condition
    - We have not discussed how the market price is determined in a perfectly competitive market. For our purpose, just take market price as a given.
  - By definition, firm cannot affect market price, so $p_{competitive} = p_{market}$
- In a perfect monopoly market, consumers can only purchase from the monopolist, so:
  - The firm's residual demand curve is the same as the aggregate demand curve
  - MR = MC determines the monopolist's profit-maximizing quantity q_m
  - The aggregate demand curve determines p_m given q_m

Imperfectly Competitive Market Equilibria

Introduction

- The basic concept
  - An imperfectly competitive market exhibits—
    - A degree of competition, but not to the extent of a perfectly competitive market
    - A degree of market power, but not to the extent of perfect monopoly market
  - Role in merger antitrust analysis
    - Almost all mergers of interest occur in markets that are imperfectly competitive
  - Characteristics of imperfectly competitive markets
    - Some or all firms exercise some degree of market power
      - One way of things about this is that they each face a downward-sloping residual demand curve, so that changes in a firm's output level will have an effect on the firm's market-clearing price
    - Multiple firms, but few enough that each firm recognizes its optimal control variables (e.g., price, output, quality) depends on the choices
    - Firms are differentiated
      - Product differentiation (e.g., various makes and models of automobiles)
      - Spatial differentiation (e.g., gasoline stations located at different locations)

      NB: In both cases, firms are "close enough" to one another to exhibit significant cross-elasticities of demand

Market power

- Some definitions
  - Market power
    - "As an economic matter, market power exists whenever prices can be raised above the levels that would be charged in a competitive market."[1]
    - "Market power is usually stated to be the ability of a single seller to raise price and restrict output, for reduced output is the almost inevitable result of higher prices."[2]
    - "Market power generally is defined as the power of a firm to restrict output and thereby increase the selling price of its goods in the market."[3]
    - Market power means "by definition, means that the defendant can produce anticompetitive effects."[4]
    - "A merger enhances market power if it is likely to encourage one or more firms to raise price, reduce output, diminish innovation, or otherwise harm customers as a result of diminished competitive constraints or incentives."[5]

[1] Jefferson Parish Hosp. Dist. No. 2 v. Hyde, 466 U.S. 2, 27 n.46 (1984); *accord* NCAA v. Board of Regents, 468 U.S. 85, 109 n.38 (1984); Copperweld Corp. v. Independence Tube Corp., 467 U.S. 752, 789 n.19 (1984).
[2] Fortner Enters., Inc. v. United States Steel Corp., 394 U.S. 495, 503 (1969)
[3] Ryko Mfg. Co. v. Eden Servs., 823 F.2d 1215, 1232 (8th Cir. 1987).
[4] Agnew v. National Collegiate Athletic Ass'n 683 F.3d 328, 337 (7th Cir. 2012)
[5] U.S. Dept. of Justice & Fed. Trade Comm'n, Horizontal Merger Guidelines § 1 (rev. 2010).

Market power

- Some definitions
  - Monopoly power
    - "Monopoly power is the power to control prices or exclude competition."[1]
    - Practically, monopoly power is just an extreme form of market power that exists when the firm (or a combination of firms acting together) can behave in the market as if they were or were close to being the only firm in the market.

[1] United States v. E. I. du Pont de Nemours & Co., 351 U.S. 377, 391 (1956).

Market power

- Measuring market power
  - Recall that in a competitive market, firms set price equal to marginal cost
  - The traditional measure of market power is the *price-cost margin* or *Lerner index L*, which is a measure of how much price has been marked up:[1]

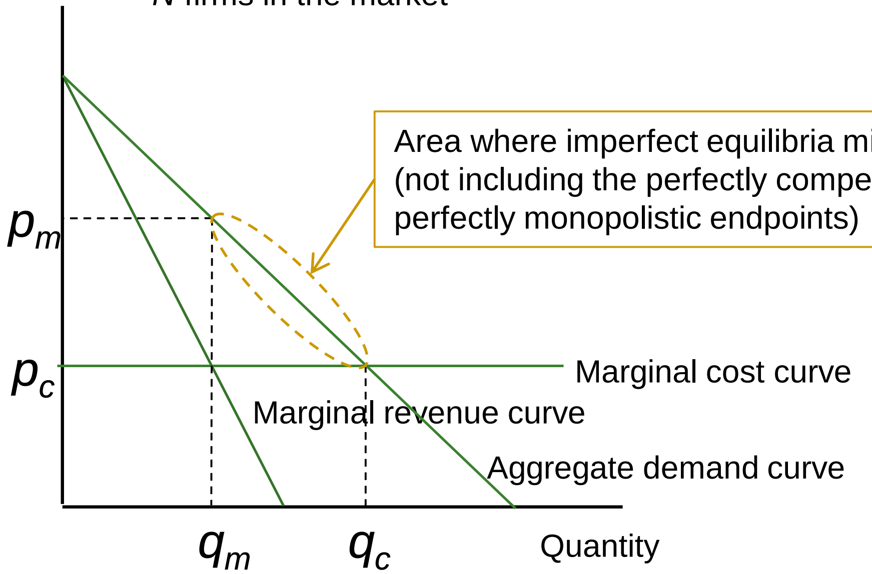
$$L = \frac{p - mc}{p}$$

  - Note that in a competitive market $L = 0$ and that L increases as price increases relative to marginal cost

[1] For more on the Lerner index, see Kenneth G. Elzinga & David E. Mills, *The Lerner Index of Monopoly Power: Origins and Uses*, 101 Am. Econ. Rev. (Papers & Proceedings) 558 (2011).

Homogeneous product models

- Range of imperfect equilibria in homogeneous product models
  - Assumes that products are undifferentiated (that is, *fungible* or *homogeneous*) in the eyes of the customer
    - *Common examples*: Ready-mix concrete, winter wheat, West Texas Intermediate (WTI) crude oil, wood pulp
    - Two properties of homogeneous products
      - Customers purchase from the lowest cost supplier → This forces all suppliers in the market to charge the same price
      - Since the goods are identical, their quantities can be added: Aggregate demand $Q = \sum_{i=1}^{N} q_i$ for the N firms in the market



Cournot oligopoly models

- The setup
  - The standard homogenous product model is the *Cournot model*
    - Recall that in a Cournot model the firm's control variable is quantity
    - The (download sloping) demand curve gives the relationship between the quantity produced and the market-clearing price
    - When their a multiple firms each producing some output, the market-clearing price is a function of all of their outputs
  - Assume that each firm produced an identical (homogeneous) product. Then the market-clearing price *p* is a function of the sum of the outputs of all of the firms in the market:

    $$p = p(Q), \text{ where } Q = \sum_{i=1}^{N} q_i,$$

    and where q_i is the output of the *i*th firm
  - So the profit equation for the *i*th firm is: $\pi_i = p(Q)q_i - c_i(q_i), \quad i = 1,2,\ldots N$

    and the first-order condition for a profit maximum for each firm *i* is:

    $$\frac{\partial \pi_i}{\partial q_i} = p(Q) + q_i \underbrace{\frac{\partial p(Q)}{\partial Q} \overbrace{\frac{\partial Q}{\partial q_i}}^{=1}}_{\text{Using the chain rule}} - \frac{\partial c_i}{\partial q_i} = 0.$$

Cournot oligopoly models

- The Cournot model
  - Rearranging the last equation yields: $p(Q) - \dfrac{\partial c_i}{\partial q_i} = q_i \dfrac{\partial p(Q)}{\partial q_i}$

    To simply the notation, define marginal cost as $c_i' \equiv \dfrac{\partial c_i}{\partial q_i}$ and the slope of the demand curve as $p' \equiv \dfrac{\partial p}{\partial Q}$

    Dividing both sides by *p(Q), multiplying by Q/Q, and* rearranging a bit yields:

$$\frac{p - c_i'}{p} = -\frac{q_i p'}{p} = \left[\frac{Q}{p}p'\right]\frac{q_i}{Q}$$

Now $\dfrac{Q}{p}p' = \dfrac{Q}{p}\dfrac{\partial Q}{\partial p} = \dfrac{1}{\varepsilon}$ and $\dfrac{q_i}{Q} = s_i$, where s_i is the market share of the *i*th firm, so:

Key result

$$\frac{p - c_i'}{p} = \frac{s_i}{\varepsilon}$$

Thus, in a Cournot oligopoly model, the *i*th firm's percentage gross margin is equal to the firm's market share divided by the industry own-elasticity. This implies that, holding marginal costs and industry elasticity constant, as the firm's market share increases, so does its percentage gross margin (and it market power).

Cournot oligopoly models

- The Cournot model and the HHI
  - Take the last equation and multiply both sides by s_i:

$$\frac{p - c_i'}{p} s_i = \frac{s_i^2}{\varepsilon}$$

Summing over all firms yields:

$$\sum_{i=1}^{N} \frac{p - c_i'}{p} s_i = \sum_{i=1}^{N} \frac{s_i^2}{\varepsilon} = \frac{HHI}{\varepsilon}$$

NB: The HHI here is in its decimal representation, which ranges from 0 to 1. This results when market shares are expressed as a fraction of 1 rather than a percentage share of 100. To convert the decimal form to the regular form, just multiply by 10,000.

The sum of the percentage gross margins of the firms in the market weighted by their respective market shares is a measure of the market power being exercised in the market. The above equation says the that this measure is equal to the HHI divided by the industry own-elasticity, so as the HHI increases, so does the market power being exercised. This is the primary theoretical justification for using the HHI in merger antitrust analysis.

  - From the above equation, we can see that Cournot equilibrium lies between the competitive equilibrium (where the HHI = 0) and the monopoly equilibrium where the HHI =1 (in decisional form)).
    - As the number of firms in the market becomes smaller and the HHI approaches 1, the Cournot equilibrium approaches the monopoly equilibrium.

Cournot oligopoly models

- Production levels in Cournot models
  - Equilibrium concept
    - When asking a question about market variable such as aggregate output in an imperfect equilibrium model, we need to specify the *equilibrium concept*, that is, the assumption of how firms will respond to the actions of their competitors
    - A fundamental characteristic of Cournot models that the firms act as if their individual decisions do not affect the decisions of their competitors (regardless of what the reality may be)
      - For example, firm 1 makes its decision on its production level as if all of other firms will not their production levels
    - A *Cournot equilibrium* exists at the point where each firm is maximizing its profit on the assumption that any changes it makes in its production level will not affect the production decisions of its competitors

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example
    - To illustrate the Cournot equilibrium concept, assume that there are two firms in the market with identical constant marginal costs. In a Cournot equilibrium, each firm will be maximizing its profits on the assumption that its competitors output is fixed.
    - Let q_1 and q_2 be the production choices of firms 1 and 2, respectively. Both firms have the same marginal cost of production c = 5. Say the aggregate demand curve has the form $Q = 100 - 2p$, where $Q = q_1 + q_2$ is aggregate demand and p is the market-clearing price. Then firm 1's profit-maximizing problem is:

$$\begin{aligned} \max_{q_1} \pi_1 &= pq_1 - cq_1 \\ &= \left[\frac{Q-100}{2}\right]q_1 - 5q_1 \\ &= \left[\frac{q_1+q_2-100}{2}\right]q_1 - 5q_1, \end{aligned}$$

assuming q_2 is constant.

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example (con't)

    To find the first order condition for a profit maximum, set the derivative of the right-hand side of this equation equal to zero assuming that q_2 is a constant:

$$\begin{aligned}\frac{d\pi_1}{dq_1} &= \frac{d}{dx}\left(\left[\frac{q_1+q_2-100}{2}\right]q_1 - 5q_1\right)\\ &= \frac{d}{dx}\left(\left[\frac{q_1^2+q_2q_1-100q_1}{2}\right]-5q_1\right)\\ &= \frac{2q_1+q_2-100}{2}-5q_1=0.\end{aligned}$$

    Solving for q_1:

$$q_1 = 45 - \frac{q_2}{2}.$$

    This is called firm 1's *reaction function* or the *best response function*, since it tells firm 1 what production level q_1 it should choose if firm 2's production level is q_2. Firm 2 has an analogous reaction function:

$$q_2 = 45 - \frac{q_1}{2}.$$

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example (con't)

    We now have two equations in two unknowns, which allows us to solve for q_1 and q_2:

$$q_1 = 45 - \frac{q_2}{2}$$
$$q_2 = 45 - \frac{q_1}{2}.$$

Substituting for q_2 in the first equation:

$$q_1 = 45 - \frac{1}{2}\left[45 - \frac{q_1}{2}\right].$$

Multiplying by 2:

$$2q_1 = 90 - \left[45 - \frac{q_1}{2}\right].$$

Simplifying:

$$3q_1 = 90,$$

so that

$$q_1 = 30.$$

Solving for q_2 also yields a production level of 30. So aggregate production Q is 60 and the market price is 20. Compare this to the competitive and monopoly levels:

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example
    - Compare the competitive, Cournot, and monopoly outcomes in this example

      Demand curve: $Q = 100 - 2p$

	Price	Quantity
Perfectly competitive	5 (= mc)	90
Cournot	20	60
Perfect monopoly	27.5	45

  - *General rule*: When demand is linear and there are *n* identical firms in a Cournot model, then:

$$Q_{Cournot} = \frac{n}{n+1} Q_{Competitive}.$$

  - Note when $n = 1$, $$Q_{Courant} = \frac{Q_{Competitive}}{2} = Q_{Monopoly}.$$
    and that as n gets larger, $Q_{Courant}$ approaches $Q_{Competititve}$. This simple model helps support the intuition that markets are competitive when n is large enough and that prices increase and market output decreases as n gets smaller.

Bertrand oligopoly models

- Homogeneous products case
  - Consider two firms producing homogeneous (identical) products at constant marginal cost *c* and use price as their control variable
  - Consumers also purchase from the lower priced firm; if both firms charge the same price, they split equally consumer demand
  - Consumer demand *Q* is a function of $\underline{p}$, the lowest price offered by a firm in the market
  - So if—
    - $p_1 < p_2$, , then $p_1 = \underline{p}$ and firm 1 sells all of consumer demand $Q(\underline{p})$ for profits $\pi_1 = \underline{p}Q - c(Q)$, and firm 2 sells nothing and earns zero profits
    - $p_1 = p_2$, then $p_1 = p_2 = \underline{p}$ firm 1 and firm 2 each sell one-half of consumer demand $Q(p)$ for profits

$$\pi_i = \frac{\underline{p}Q - c(Q)}{2}.$$

  - *Equilibrium*: As long as $\underline{p} > c'$, the higher priced firm can undercut the lower priced firm and take all of the market demand. This "race to the bottom" until $p_1 = p_2 = \underline{p} = c'$, so that both firms price at marginal cost and split equally market demand and total market profits
    - So in this model, a competitive equilibrium is achieved with only two firms

Bertrand oligopoly models

- Differentiated products case
  - When products are differentiated, a lower price charged by one firm will not necessarily move all of the market demand to that customer
    - Consider a market with only red cars and blue cars.
    - Some consumers like blue cars so much that even if the price of red cars is lower than the price of blue cars there will still be positive demand for blue cars
    - Moreover, if the price of blue cars increases, some (inframarginal) blue car customers will purchase blue cars at the higher price while some (marginal) customers will switch to red cars
    - This means that the demand for red cars (and separately for blue cars) is a function both of the price of red cars and the price of blue cars
    - It also means that the price of blue cars may not equal the price of red cars in equilibrium

Bertrand oligopoly models

- Differentiated products case
  - Simple linear model
    - Firms 1 and 2 produce differentiated products and face the following residual demand curves:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

NB: Each firm's demand decreases with increase in its own price and increases with increases in the price of the other firm

Assume that $b_1 > b_2$, so that each firm's residual demand is more sensitive to its own price than to the other firm's price

    - Assume each firm has a cost function with no fixed costs and constant marginal costs:

$$c_i(q_i) = cq_i$$

    - Firm 1 profit-maximization problem:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2)$$

    - First order condition for Firm 1:

$$\frac{\partial \pi_1}{\partial p_1} = 0 = a - 2b_1 p_1 + b_2 p_2 + cb_1$$

NB: This formulation does not take into account firm 2's reaction to a change in firm 1's price

Bertrand oligopoly models

- Differentiated products case
  - Simple linear model (con't)
    - Solving the last equation for p_1 gives firm 1 *best response function* $BR(p_2)$, that is, the price it should choose given firm 2's choice of p_2:

$$BR_1(p_2): p_1 = \frac{a + cb_1 + b_2 p_2}{2b_1}$$

    - Firm 2 has a similar best response function $BR(p_1)$:

$$BR_2(p_1): p_2 = \frac{a + cb_1 + b_2 p_1}{2b_1}$$

    - The price pair $\left(p_1^*, p_2^*\right)$ that simultaneously satisfies both best response functions is a *Bertrand equilibrium*
    - Solving these two simultaneous equations for p_1^* and p_2^* yields:

$$p_1^* = p_2^* = \frac{a + cb_1}{2b_1 - b_2}$$

Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with—
    - a dominant firm, which sees its output decisions as affecting price and so sets output so that *mr* =*mc*, and
    - a fringe of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., $p = mc$)
  - Choice question for the dominant firm: Pick the profit-maximizing level for its output given the competitive fringe

- The model
  - At market price p, let $q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe. Then the residual demand $q_d(p)$ for the dominant firm is $q(p) - q_f(p)$.

Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with
    - a dominant firm, which sees its output decisions as affecting price and so sets output so that *mr* =*mc*, and
    - a fringe of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., *p* = *mc*)
  - Choice question for the dominant firm: Pick the profit-maximizing level for its output given the competitive fringe
    - The model requires some constraint on the ability of the competitive fringe to expand it output. Otherwise, the competitive fringe will take over the market.
    - The constraint usually is either limited production capacity or increasing marginal costs
- The model
  - At market price p, let $q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe. Then the residual demand $q_d(p)$ for the dominant firm is $q(p) - q_f(p)$.

Dominant firm with a competitive fringe

- A simple example
  - Say aggregate demand is $Q = 100 – 5p$.
  - The competitive fringe has increasing marginal costs such that $c = 0.2q_f$, where q_f is the output of the competitive fringe
  - Since the competitive fringe supplies output so that p = c, then p $=0.2q_f$, which in turn yields a competitive fringe supply curve of $q_f = 5p$
  - Say the marginal cost of the dominant firm is constant at $c_d = 2$
  - The dominant firm's residual demand q_d at a price p is the market demand at price p minus the competitive supply at price p:

$$\begin{aligned} q_D &= Q(p) - q_F(q) \\ &= [100 - 5p] - 5p \\ &= 100 - 10p. \end{aligned}$$

  - The dominant firm's profit maximization problem is then:

$$\begin{aligned} \underset{p}{\text{Max}}\, \pi_D &= pd_D - c_D d_D \\ &= (p-2)[100 - 10p] \\ &= 100p - 10p^2 - 300 + 20p. \end{aligned}$$

Dominant firm with a competitive fringe

- A simple example (con't)
  - The first order condition for a profit maximum is:

$$\frac{d\pi_D}{dp} = \frac{d}{dp}\left[100p - 10p^2 - 300 + 20p\right]$$
$$= 100 - 20p + 20.$$

Solving for p:

$$20p = 120$$
$$p = 6.$$

We can calculate total demand Q from the demand curve:

$$Q(p) = 100 - 5p = 100 - 30 = 70,$$

and competitive supply from the competitive supply curve:

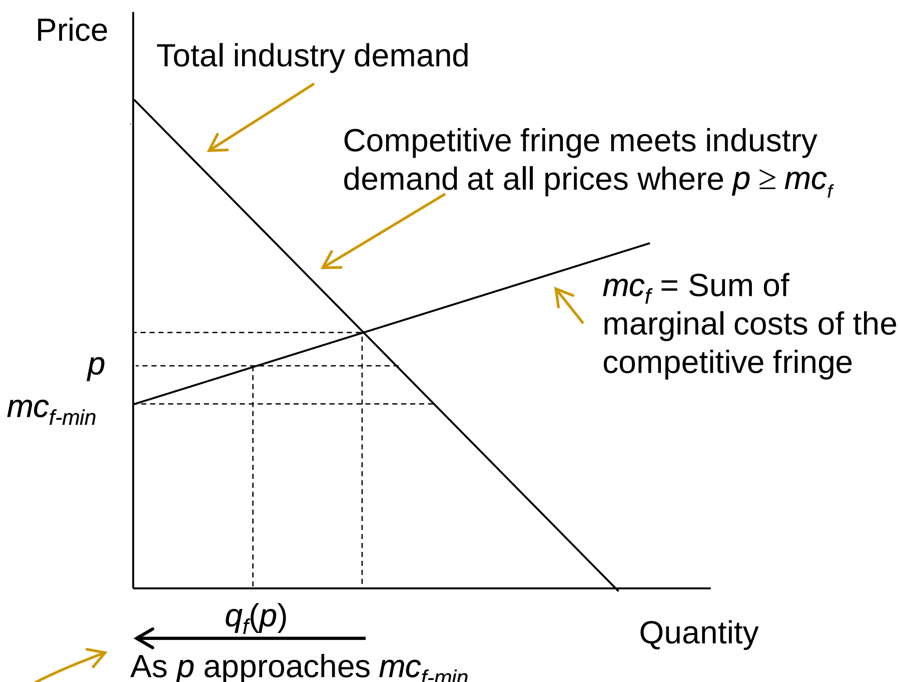
$$q_f = 5p = 30.$$

The dominant firm then supplies the residual demand d_D:

$$d_D = Q - d_F = 70 - 30 = 40.$$

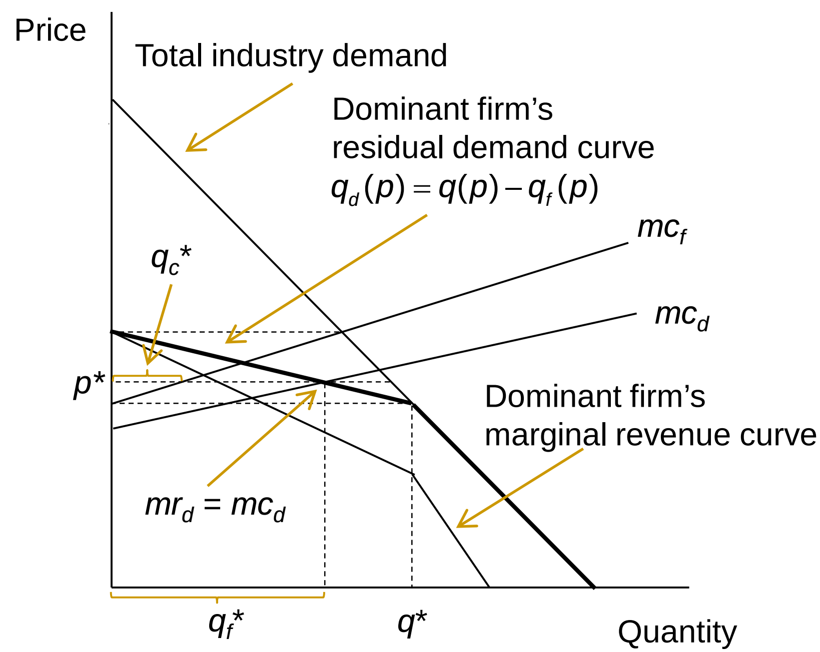
Dominant firm with a competitive fringe

Output of the Competitive Fringe



Competitive fringe reduces output until price equals mc_f-*min,* its minimum marginal cost. Below this price the competitive fringe does not produce.

Output of the Dominant Firm



Dominant firm maximizes profit at q_f*, where $mr_f = mc_f$. Total industry output $q^* = q_f^* + q_c^*$ at price p^*.

Dominant firm with a competitive fringe

- Dominant oligopolies
  - The model can be extended to the case where the dominant firm is replaced by a dominant oligopoly
    - The key is to specify the solution concept for the choice of output by the firms in the oligopoly (e.g., Cournot). You then create a residual demand curve for the oligopoly and apply the solution concept to that demand curve.
- Fringe firms
  - As we saw in Unit 2, the DOJ and the FTC typically ignore fringe firms. The dominant oligopoly model with a competitive fringe provides a theoretical justification.