
Market Definition

Class 14 class notes

Merger Antitrust Law

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Critical Loss Analysis

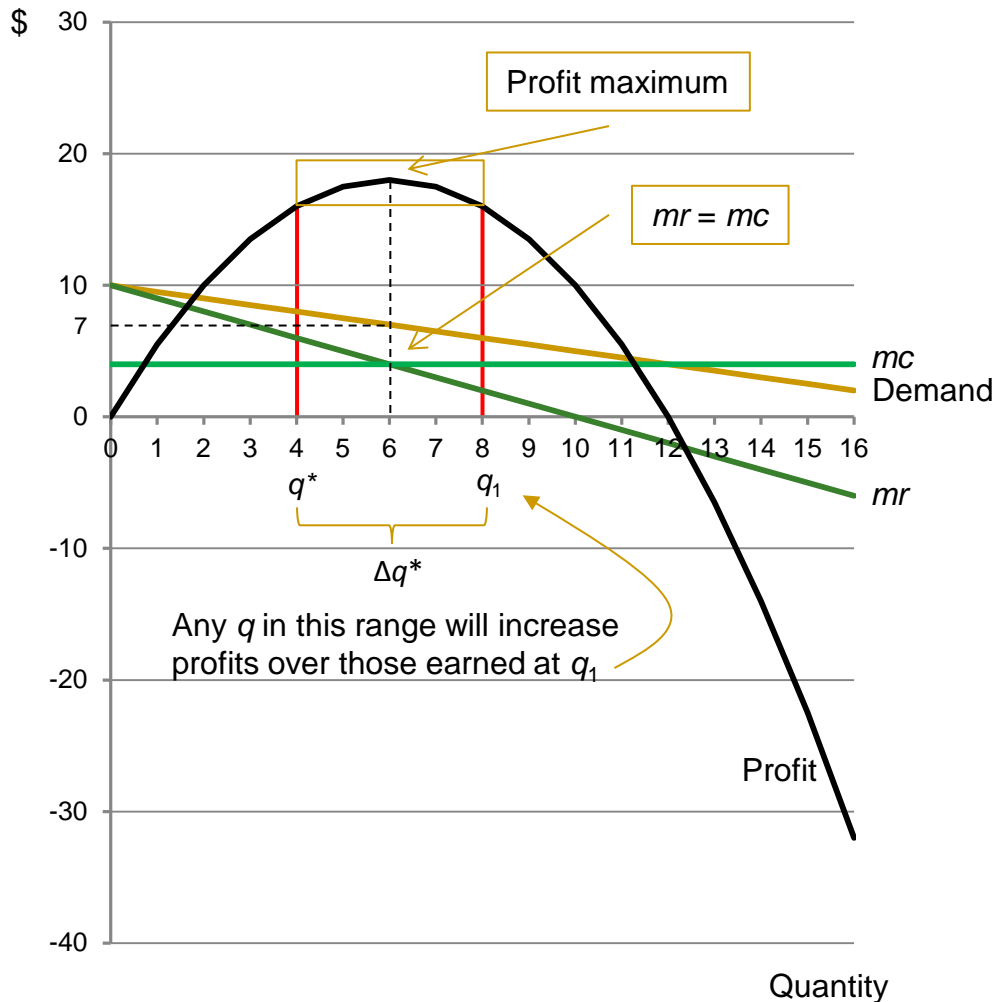
Critical loss

■ The basic idea

- Consider a price increase Δp in the product of a single-product hypothetical monopolist and an accompanying loss of sales Δq when the demand curve is downward sloping
 - When loss of sales is sufficiently small, then the gross gain in profits from higher prices on retained sales will be greater than gross loss in profits from lost sales and the price increase will be profitable
 - When the loss of sales is sufficiently large, the gross gain in profits from higher prices on retained sales will be smaller than gross loss in profits from lost sales and the price increase will be unprofitable
 - This is *precisely* the same question we asked in the competition economics discussion of monopoly pricing: When is it in the interest of the monopolist to raise price?
- *Definition:* The loss of sales Δq^* at the tipping point when the gross gain in profits just equals the gross loss is called the *critical loss*
 - Critical loss is often express in percentage terms as $\Delta q^*/q$, where q is the premerger level of sales
 - NB: A reduction in sales greater than Δq^* will mean a *loss* in profits compared to the starting point
- Dependencies
 - Critical loss (*CL*) is a function of the starting quantity q , the price p , and the price change Δp

Critical loss

■ The basic idea



Recall this diagram from Unit 8. The curves result from the inverse demand function $q = 20 - 2p$. While we originally saw this demand function in the context of a monopolist, we can reinterpret here as the aggregate demand function for the industry (where all firms produce identical products and have identical, constant marginal costs). The profit curve then shows aggregate profits for the firms in the market.

Suppose competition among the firms in the market yields an aggregate output q_1 , a quantity above the profit-maximizing level. The hypothetical monopolist tests asks whether a hypothetical monopolist can profitably raise profits by some SSNIP. An increase in price will decrease the quantity demanded, so q will move to the left. The critical loss is the Δq^* so that the profits at $q^* = q_1 - \Delta q^*$ are equal to the profits at q_1 . Note that the profits at q^* are *not* the profit maximum.

Critical loss

■ Formulas for critical loss

- We can express the critical loss Δq^* algebraically in two equivalent ways:¹
 - As an equality of total profits after and before the price increase:

$$(p + \Delta p - c)(q - \Delta q^*) = (p - c)q$$

Breakeven condition

- As an equality of the gross gain in profits on retained sales and the gross loss in profits from lost sales:

Gain on retained sales

$$\Delta p (q - \Delta q^*) = (p - c) \Delta q^*$$

Loss of margin on lost sales

- Note: Critical loss is a function of q , that is, the magnitude of q^* depends on the starting point q as well as on p and c

- Solving for Δq^* provides a formula for the critical loss in absolute units:

$$\Delta q^* = \frac{q \Delta p}{(p + \Delta p) - c}$$

or in percentage terms:

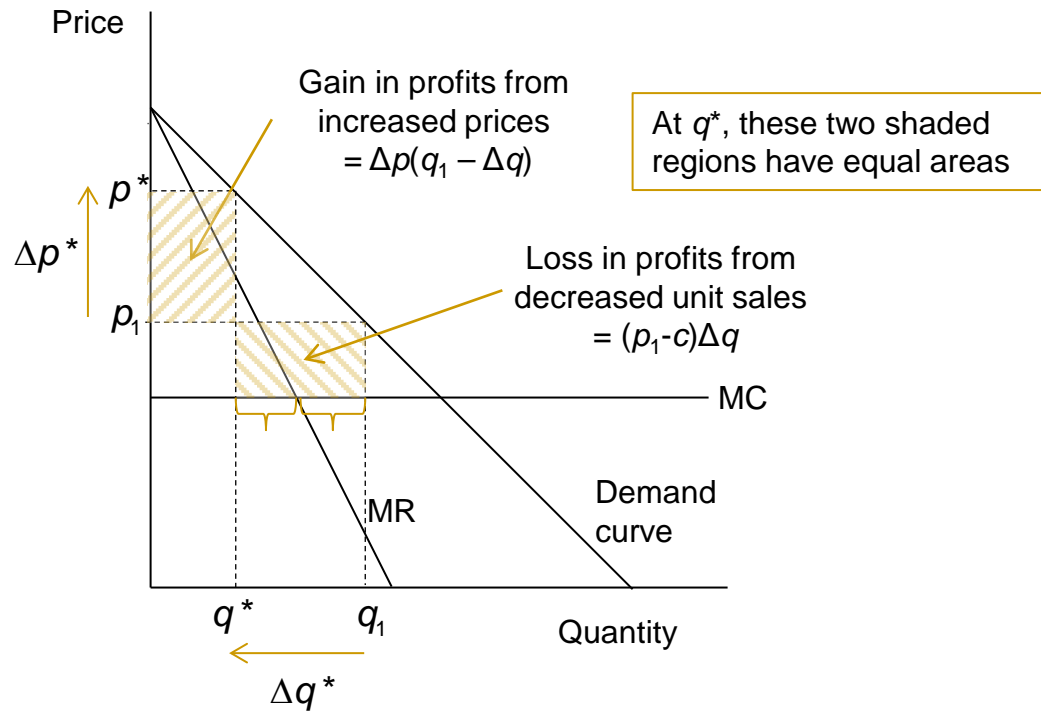
$$\frac{\Delta q^*}{q} = \frac{\Delta p}{(p + \Delta p) - c} = \frac{\frac{\Delta p}{p}}{\frac{\Delta p}{p} + \frac{p - c}{p}} = \frac{\delta}{\delta + m}$$

Where δ is the percentage price increase and m is the percentage gross margin

¹ This assumes zero fixed costs and constant marginal costs.

Critical loss

- Formulas for critical loss



NB: The profit-maximizing quantity lies equidistant between q^* and q_1

Critical loss and market definition

■ The basic idea

- Recall that under the hypothetical monopolist test, a candidate market is a relevant market if a hypothetical monopolist could profitably raise prices in the candidate market by a SSNIP.
 - So for any candidate market with prevailing aggregate output q and price p and a SSNIP Δp , then if the change in output Δq is less than the critical loss Δq^* a hypothetical monopolist could profitably raise price by the SSNIP and the candidate market is a relevant market
- Algorithm
 1. Start with a product of the merging firm
 - Or a product of the merging firm together with other closely related products (as in H&R Block/TaxACT)
 2. Assume a hypothetical monopolist over the group of products—the “candidate market”—and raise price by a SSNIP
 3. Compare actual loss Δq to critical loss Δq^* ,
 - If the actual loss $\Delta q < \Delta q^*$, then a hypothetical monopolist could profitably raise prices by the SSNIP and the product grouping is a relevant market
 - Whether the SSNIP is profitable will be determined by the candidate market’s *own-elasticity of demand*
 - If the actual loss $\Delta q \geq \Delta q^*$, then a hypothetical monopolist could not profitably raise prices the product grouping is not a relevant market → add to the product group another product with a high cross-elasticity of demand/diversion ratio and repeat Steps 2 and 3.
 - If the SSNIP is not profitable, the additional product to include the candidate market is determined by the *cross-elasticity of demand* between the products in the candidate market and the products outside the candidate market

Critical loss and market definition

■ Example 1

- Products A and B are being tested as a candidate market. Each sells for \$100, has an incremental cost of \$60, and sells 1200 units. When the price for both products is increased by \$5, each firm loses 100 units to outside the market. Do A and B constitute a relevant market under the 2010 Guidelines?

Parameters			Critical loss calculations		Critical loss	
Price	p	100	Gain = $(Q+\Delta Q)\Delta p$		$\Delta q^* = \frac{q\Delta p}{(p + \Delta p) - c}$	
Cost	c	60	$Q + \Delta Q$	2200		
Gross margin	m	40	Δp	<u>5</u>		
Market output	Q	2400	Gain	11000		
SSNIP	Δp	5	Loss = $m\Delta Q$			
Customer loss	ΔQ	-200	ΔQ	-200	$q\Delta p$	12000
			m	40	$(p+\Delta p)-c$	<u>45</u>
			Loss	<u><u>-8000</u></u>	CL	266.6667
			Net	3000		

From the breakeven condition (see earlier slide)

Conclusion: Since the gain exceeds the loss, a hypothetical monopolist of A and B could profitably raise price by 5% and so A and B are a relevant market. Note that the actual customer loss (200) is less than the critical loss (266.667).

Critical loss and market definition

■ Example 1A

- We can also analyze Example 1 in terms of the percentage critical loss:

Summary:

$$P = \$100$$

$$C = \$60$$

$$\text{Margin} = \$40$$

$$\text{Total market } Q = q_1 + q_2 = 2400$$

$$\text{Percentage margin } m = \frac{p - c}{p} = \frac{100 - 60}{100} = 40.0\%$$

$$\text{SSNIP } \delta = 5\%$$

$$\text{Percentage critical loss } CL = \frac{\delta}{\delta + m} = \frac{5\%}{5\% + 40\%} = 11.1\%$$

$$\text{Percentage actual loss } L = \frac{100 + 100}{2400} = 8.33\%$$

Conclusion: Since the percentage actual loss L does not exceed the percentage critical loss CL , a hypothetical monopolist of A and B could profitably raise price by 5% and so A and B are a relevant market

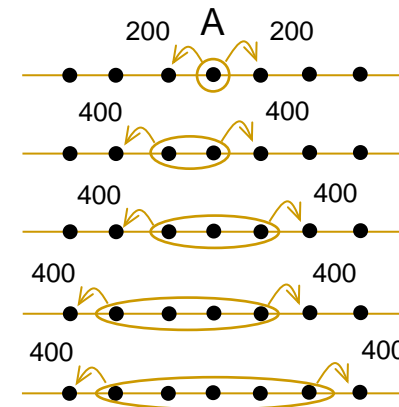
Critical loss and market definition

■ Example 2: Gas stations on a road

- Assume that there is an identical gas station every mile on a straight road. Each gas station charges \$3.25 per gallon, has an incremental costs of \$2.50, and sells 1000 gallons. When the price at any one station is increased by 5% (holding the price at all other gas stations constant), the station loses 400 customers. No customer will travel more than one mile, however, to avoid a 5% price increase. For a given station A, what is the relevant market?

Price	p	3.25
Cost	c	2.50
Gross margin	m	0.75
Percentage SSNIP		5.0%
Actual SSNIP		0.1625
Customers/station		1000
Customer loss		400

Stations in the market	Q	ΔQ	Gain	Loss	Net
1	1000	400	97.50	300.00	-202.50
2	2000	800	195.00	600.00	-405.00
3	3000	800	357.50	600.00	-242.50
4	4000	800	520.00	600.00	-80.00
5	5000	800	682.50	600.00	82.50



Critical loss and market definition

■ Estimating actual loss

- We can estimate the percentage critical loss if we know the aggregate own-elasticity of demand for the candidate market when:
 - Premerger pricing satisfies the Lerner Condition ($\varepsilon = 1/m$), and
 - All demand functions are linear in price in the vicinity of the premerger equilibrium point
- First-order approximation of the percentage actual loss:

$$\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \equiv \varepsilon \Rightarrow \frac{\Delta q}{q} \cong \frac{\Delta p}{p} \varepsilon = \delta \varepsilon,$$

where ε is the residual own-elasticity of demand for the candidate market (i.e., of the hypothetical monopolist)

that is, the percentage actual loss is approximately equal to the percentage price change times the own-elasticity of demand

- First-order approximation of the actual loss:

$$\frac{\Delta q}{q} \cong \delta \varepsilon \Rightarrow \Delta q = q \delta \varepsilon.$$

Remember, δ is the percentage price increase $\Delta p/p$

Critical loss and market definition

■ Critical elasticities

- We can also define the critical elasticity ε^* as the maximum elasticity that will profitably support a price increase of δ :

$$\underbrace{\frac{\Delta q^*}{q} = \frac{\delta}{\delta + m}}_{\text{}} \Rightarrow \delta |\varepsilon^*| \cong \frac{\delta}{\delta + m},$$

NB: This is the conventional way to write the critical loss equation, but it means that the Δq is a *positive* number. To make the signs work, we have to use the absolute value of the elasticity. *Always watch for the sign of Δq in any equation.*

or:

$$|\varepsilon^*| \cong \frac{1}{\delta + m}$$

- Accordingly, when the own-elasticity of demand ε is less than the critical elasticity ε^* (i.e., ε is more inelastic than ε^* or equivalently $|\varepsilon| < |\varepsilon^*|$), then for a small enough SSNIP the price increase will be profitable

- We can express this as:

$$|\varepsilon| < \frac{1}{\delta + m}.$$

Critical loss and market definition

OPTIONAL

■ Profit-maximization

- As noted earlier, the guidelines ask whether the hypothetical monopolist for the candidate market profit-maximizing price increase would be above a SSNIP
- The monopolist's profit-maximizing critical elasticity ε^{pm} —that is, the elasticity at which the hypothetical monopolist's profit-maximizing price increase will be at least as great as the SSNIP δ —is given by:

$$|\varepsilon^{pm}| = \frac{1}{2\delta + m}$$

- With a little algebra, we can rearrange the above equation to solve for δ :

$$\delta^{pm} = \frac{-m|\varepsilon| + 1}{2|\varepsilon|}$$

- This equation gives the profit-maximizing percentage price change δ^{pm} for a given group of product with an elasticity ε
- It is helpful to remember what is going on here. A profit-maximizing monopolist prices so that the Lerner equation is satisfied ($\varepsilon = 1/m$). Competition within the product grouping, however, may decrease the margin m , so that the Lerner equation is no longer satisfied. The profit-maximizing δ^{pm} gives the percentage price change that the monopolist would implement if it gained control of the product grouping. (Note that when $\varepsilon = 1/m$, $\delta^{pm} = 0$, as it should be.)

Aggregate diversion analysis

■ Basic idea

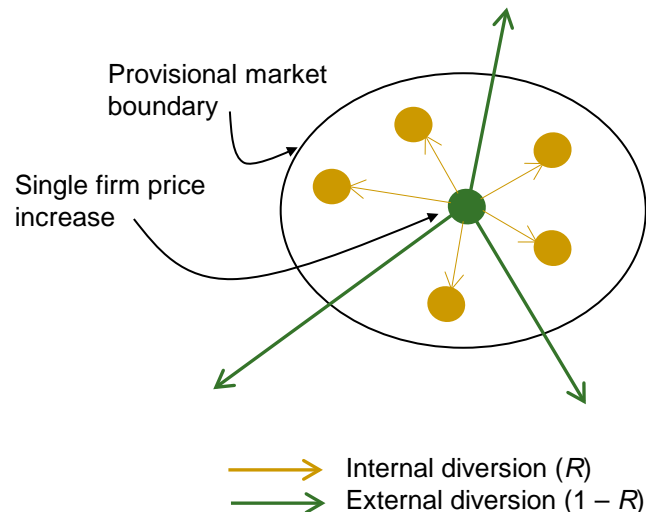
- 1982 Merger Guidelines
 - Required that all products in the provisional market be increased by the same percentage SSNIP
- 1992 Merger Guidelines
 - Allowed price discrimination in the SSNIP, at least where the premerger market exhibited some price discrimination (and sometimes when the postmerger market arguably would exhibit price discrimination even if the premerger market did not)
- 2010 Merger Guidelines
 - After the 2010 Merger Guidelines, some economists—including agency economists in court proceedings—used price discriminating SSNIPs in differentiated products markets
 - A *one-product SSNIP* creates the most narrow relevant markets, since internalizes the maximum amount of diversion
 - The “aggregate diversion ratio” method can whether a candidate market satisfies the hypothetical monopolist test under a one-product SSNIP
 - Some economists have used the aggregate diversion ratio method when imposing a uniform price increase across all products in the candidate market, but this requires some restrictive conditions
 - Examples: DOJ’s economist in H&R Block/TaxACT
FTC’s economist in Sysco/US Foods
DOJ’s economist in Aetna/Cigna
FTC’s economist in Wilhelmsen/Drew

Aggregate diversion/recapture analysis

■ Aggregate diversion or recapture ratio

□ Definition

- The percentage of total sales lost by a product in the wake of a uniform SSNIP that is captured by all of the other products inside the provisional market.



1. Raise the product of one merging firm by a SSNIP, leaving the prices of all other products in the provisional market at premerger levels
2. That product loses sales Δq , of which Δq_{inside} are diverted to products *inside* the provisional market (“recaptured”) and $\Delta q_{\text{outside}}$ are diverted to products *outside* the provisional market
3. $R \equiv \Delta q_{\text{inside}} / \Delta q$ is called the aggregate diversion ratio. It is much more descriptively called the “*recapture ratio*.”¹

- **Key result.** If the actual aggregate diversion ratio (recapture rate) R is greater than or equal to the critical loss, the provision market satisfies the hypothetical monopolist test:

$$R \equiv \frac{\Delta q_{\text{inside}}}{\Delta q} \geq \frac{\Delta q^*}{q} \Rightarrow \text{Hypothetical monopolist test is satisfied}$$

¹ The “aggregate diversion ratio” was the original term for the technique described in the next few slides. Some of the cases use this term. The 2010 Horizontal Merger Guidelines use “recapture percentage.” Other terms used in the economics literature include “retention ratio” and “group recapture ratio.”

Aggregate diversion/recapture analysis

- Proof (optional):

Assume that the candidate market has an own-elasticity of ε . Then:

$$\frac{\Delta q}{q} \cong \frac{\Delta p}{p} \varepsilon = \delta \varepsilon$$

Remember $\delta = \Delta p/p$ by our prior definition

Now assume that the hypothetical monopolist imposes a uniform SSNIP on all products in the candidate market. Some percentage R of the $\Delta q/q$ loss will be “recaptured” by other products in the candidate market and the remaining fraction $1 - R$ will exit the market. The actual percentage loss to the market is then $(1 - R)\varepsilon\delta$. The hypothetical monopolist will find the price increase profitable if and only if the actual percentage loss is less than the critical loss, that is:

$$(1 - R)\varepsilon\delta = (1 - R)\frac{\delta}{m} < \frac{\Delta q^*}{q} = \frac{\delta}{\delta + m}$$

Rearranging, this simplifies to:

$$R \geq \frac{\delta}{\delta + m}$$

That is, the price increase is profitable if the percentage recapture rate is greater than the percentage critical loss. Q.E.D.

Aggregate diversion/recapture analysis

■ Extension to single product recapture rates

- Some economists apply the aggregate diversion ratio test to individual products in the candidate market.
 - Say the hypothetical monopolist imposes a uniform SSNIP on all products in the candidate market and that some percentage R_i of the $\Delta q_i/q_i$ loss of each product i will be “recaptured” by other products in the candidate market and the remaining fraction $1 - R_i$ will exit the market.
 - Further assume that the percentage gross margin m is the same for each product.
 - Let R be the recapture rate for the candidate market as a whole
 - Define R^* to be the critical recapture rate for the candidate market as a whole, that is:

$$R^* = \frac{\delta}{\delta + m}.$$

- For example, in Sysco/U.S. Foods the FTC’s expert found a margin of 10% and used a SSNIP of 10%, so $R^* = 0.50$.¹
- Rule:

If $\min_i R_i \geq R^*$, then $R \geq R^*$ and so $L < CL$.

- Courts have accepted this test in H&R Block/TaxACT and Sysco/U.S. Foods
 - But subject to the limitations of the data (which reduced the probative value of the result)

¹ See *FTC v. Sysco Corp.*, 113 F.Supp.3d 1, 35 (D.D.C. 2015).

Aggregate diversion/recapture analysis

- Proof of rule (optional)

- Rule:

If $\min_i R_i \geq R^*$, then $R \geq R^*$ and so $L < CL$.

- Proof:

Recall that

$$R_i \equiv \frac{\Delta q_i^{inside}}{\Delta q_i}.$$

Without loss of generality, let R_1 be the smallest individual product recapture rate.

Now

$$R = \frac{\Delta Q^{inside}}{\Delta Q} = \frac{\sum_{i=1}^n \Delta q_i^{inside}}{\sum_{i=1}^n \Delta q_i} = \sum_{i=1}^n \frac{\Delta q_i^{inside}}{\Delta q_i} \frac{\Delta q_i}{\Delta Q}$$

If R_1 is the smallest recapture rate, then:

$$R = \sum_{i=1}^n \frac{\Delta q_i^{inside}}{\Delta q_i} \frac{\Delta q_i}{\Delta Q} \geq \sum_{i=1}^n \frac{\Delta q_1^{inside}}{\Delta q_1} \frac{\Delta q_i}{\Delta Q}$$

But
$$\sum_{i=1}^n \frac{\Delta q_1^{inside}}{\Delta q_1} \frac{\Delta q_i}{\Delta Q} = \frac{\Delta q_1^{inside}}{\Delta q_1} \sum_{i=1}^n \frac{\Delta q_i}{\Delta Q} = \frac{\Delta q_1^{inside}}{\Delta q_1} \frac{\Delta Q}{\Delta Q} = \frac{\Delta q_1^{inside}}{\Delta q_1} = R_1.$$

So

$$R \geq R_1 > R^*.$$

Q.E.D.

Aggregate diversion/recapture analysis

- Example: Extension to single product recapture rates

- Recall that the critical recapture rate R^* is:

$$R^* = \frac{\delta}{\delta + m}.$$

- Recall the rule:

If $\min_i R_i \geq R^*$, then $R \geq R^*$ and so $L < CL$.

- Example

- Assume a three-product candidate market. Each product has a margin of 35%. Assume a uniform SSNIP of 5% across all products. Then $R^* = 12.5\%$. Suppose that the SSNIP generates the following recapture rates:

Product	q	Δq	Recapture	
			Units	Rate (R)
A	300	90	20	22.22%
B	400	125	40	32.00%
C	500	200	35	17.50%
Total	1200	415	95	22.89%

- Applying extension, since the smallest R_i (17.5%) is greater than R^* (12.5%), a hypothetical monopolist can profitably sustain a 5% uniform price and so the three products are a relevant market

Aggregate diversion/recapture analysis

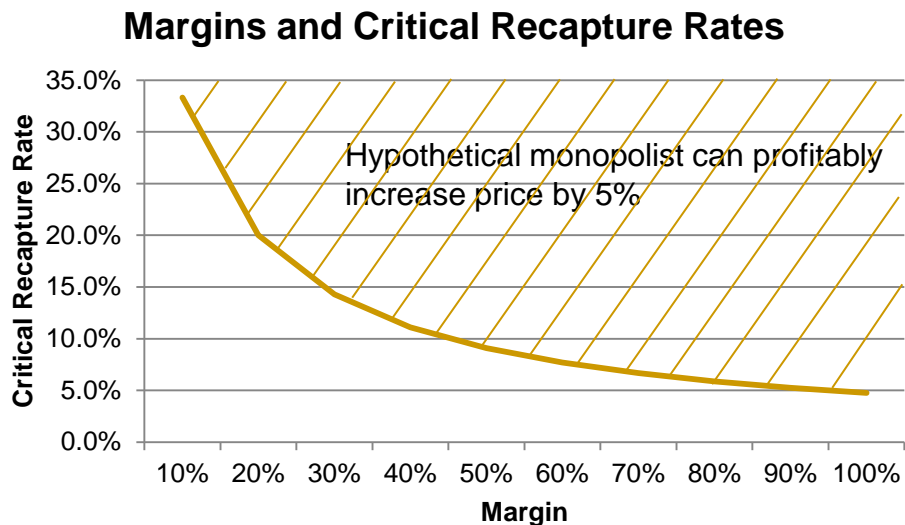
■ Critical recapture rates and margins

- The critical recapture rate R^* is:

$$R^* = \frac{\delta}{\delta + m}.$$

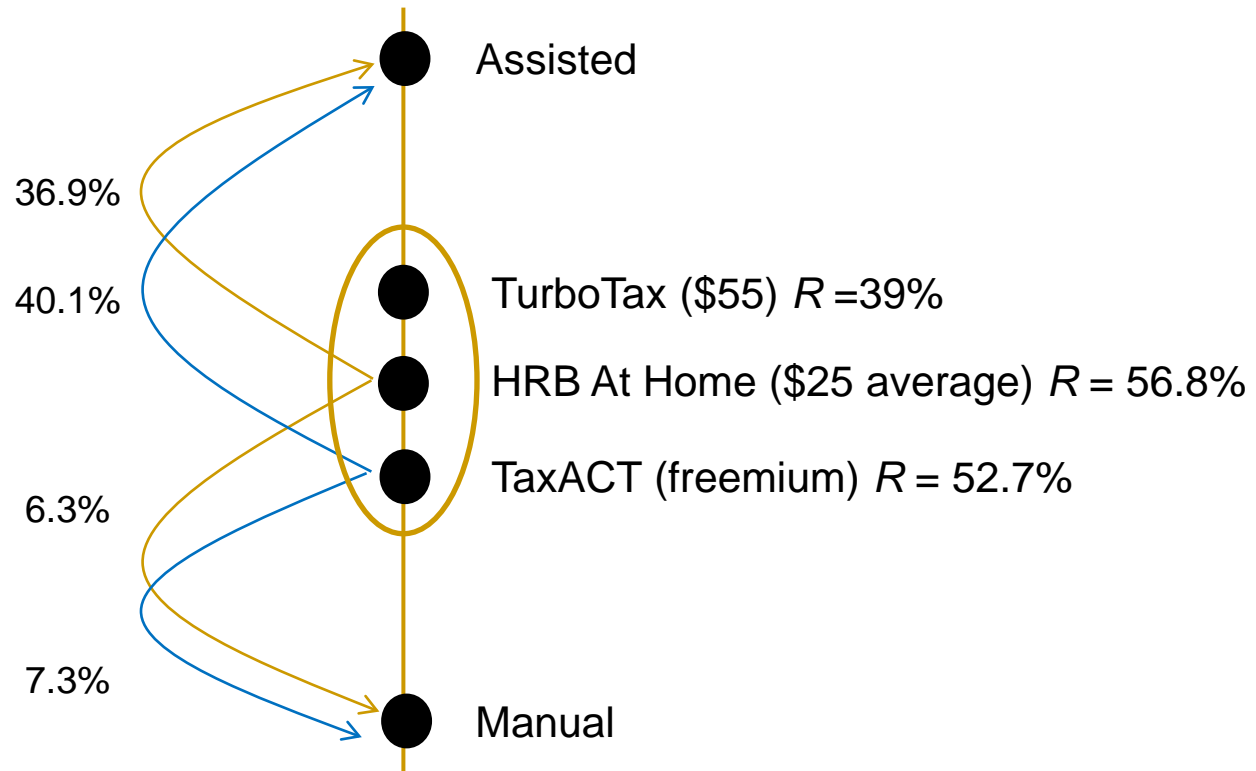
- For a fixed SSNIP of 5% (0.05), we can graph the relationship between the critical recapture rate and the margin:

Margin	R^*
10%	33.3%
20%	20.0%
30%	14.3%
40%	11.1%
50%	9.1%
60%	7.7%
70%	6.7%
80%	5.9%
90%	5.3%
100%	4.8%



Aggregate diversion/recapture analysis

- Warren-Bolton analysis in H&R Block/TaxACT



Aggregate diversion/recapture analysis

- Warren-Bolton analysis in H&R Block/TaxACT
 - Question: Is DDIY a market?
 - Critical loss (CL): Use percentage critical loss formula
 - Starting point: Start with DDIY products (HRB, TaxACT, and TurboTax)
 - SSNIP (δ): 10%
 - Gross margin (m): 50% on each product

$$CL = \frac{\delta}{\delta + m} = \frac{10\%}{10\% + 50\%} = 16.7\%$$

- Actual loss: Use aggregate diversion ratio method (recapture rate R)
 - Test: If $R \geq CL$, then product grouping is a market
 - Using IRS switching data as a proxy for R , Warrant-Bolton found:
 - HRB: $R = 57\%$
 - TaxACT: $R = 53\%$
 - TurboTax: $R = 39\%$
 - Warren-Bolton concluded that, since each $R > CL$, a hypothetical monopolist of the DDIY product could profitably raise price by a SSNIP and therefore DDIY was a relevant product market

Aggregate diversion/recapture analysis

- “Brute force” method for single product price increase—Example 1
 - We can apply the hypothetical monopolist test by looking at whether the gross profit gain to the hypothetical monopolist from a single product SSNIP would be greater than the gross profit loss from the loss of sales at the higher price to products outside the candidate market
 - Example 1
 - Assume that for a for a single product price increase of 5%, the hypothetical monopolist would retain 70 out of every 100 customers. Of the 30 lost customers, 24 would divert to another gourmet pizza and 6 would go to a standard pizza. Assume that the price of gourmet pizzas is \$4.50 and that the dollar margin is \$1.50 per pie.
 - *Query:* Under the single-product price increase test, are gourmet pizzas a relevant product market?

Out of every	100	Price	\$3.00	
units sold:		Margin	\$1.50	
		SSNIP (%)	5.00%	
		SSNIP (\$)	\$0.150	
Units retained	70	Gain on retained	\$10.50	Units retained time dollar price incre
Total units lost	30	Loss	-\$45.00	Total units lost times margin
Units recaptured	24	Gain on recapture	<u>\$36.00</u>	Recaptured units times margin
Units lost to outside	6	Net gain	\$1.50	

- *Note:* If the hypothetical monopolist would lose 20 out of 100 customers if it uniformly raised the price of *all* gourmet pizzas, the price increase would not be profitable

Aggregate diversion/recapture analysis

- “Brute force” method for single product price increase—Example 2
 - We can use the brute force method for a single product price when margins differ among products within the candidate market.
 - Example 2

Gourmet pizza--Single product price increase

(brute force method--different margins for candidate market of three firms)

Out of every 100 units sold by G1 (the firm experiencing the price increase):

For G1		For G2		For G3	
Total units retained	70				
Total unit diverted	30	Total units recaptured	10	Total units recaptured	14
G1 price	\$3.00				
G1 margin	\$1.50	G2 margin	\$1.75	G2 margin	\$1.35
SSNIP (%)	5.00%				
SSNIP (\$)	\$0.15				
Gain on retained units	\$10.50	Gain on recaptured units	\$17.50	Gain on recaptured units	\$18.90
Loss on diverted units	-\$45.00				
Total gross gain to HM	\$46.90	= \$10.50 + \$17.50 + \$24.00			
Total gross loss to HM	-\$45.00				
NET GAIN	\$1.90				

- Since the net gain to the hypothetical monopolist is positive, the candidate market is a relevant market