Merger Antitrust Review: Formulas

Merger Antitrust Law

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Calculating HHIs

- Math notes
  - *Calculating the HHI*: Assume *n* firms in the market, with firm *i* having a market share of s_i:

$$HHI = \sum_{i=1}^{n} s_i^2$$

  - *Calculating the delta*: Let *a* and *b* be the market shares of the merging companies:

Premerger contribution to the HHI:	$a^2 + b^2$
Postmerger contribution to the HHI:	$(a+b)^2 = a^2 + 2ab + b^2$
Difference (= HHI delta):	$2ab$

  - *Calculating the HHI contribution for "other" firms*: Say an unknown number of "other" firms collectively have a market share of *x*. If we assume that the number of "other" firms is *k*, then each firm contributes $(x/k)^2$ to the HHI. The total contribution to the HHI is then:

$$k\left(\frac{x}{k}\right)^2 = \frac{x^2}{k}$$

Calculating HHIs

- *Application*: H&R Block/TaxACT

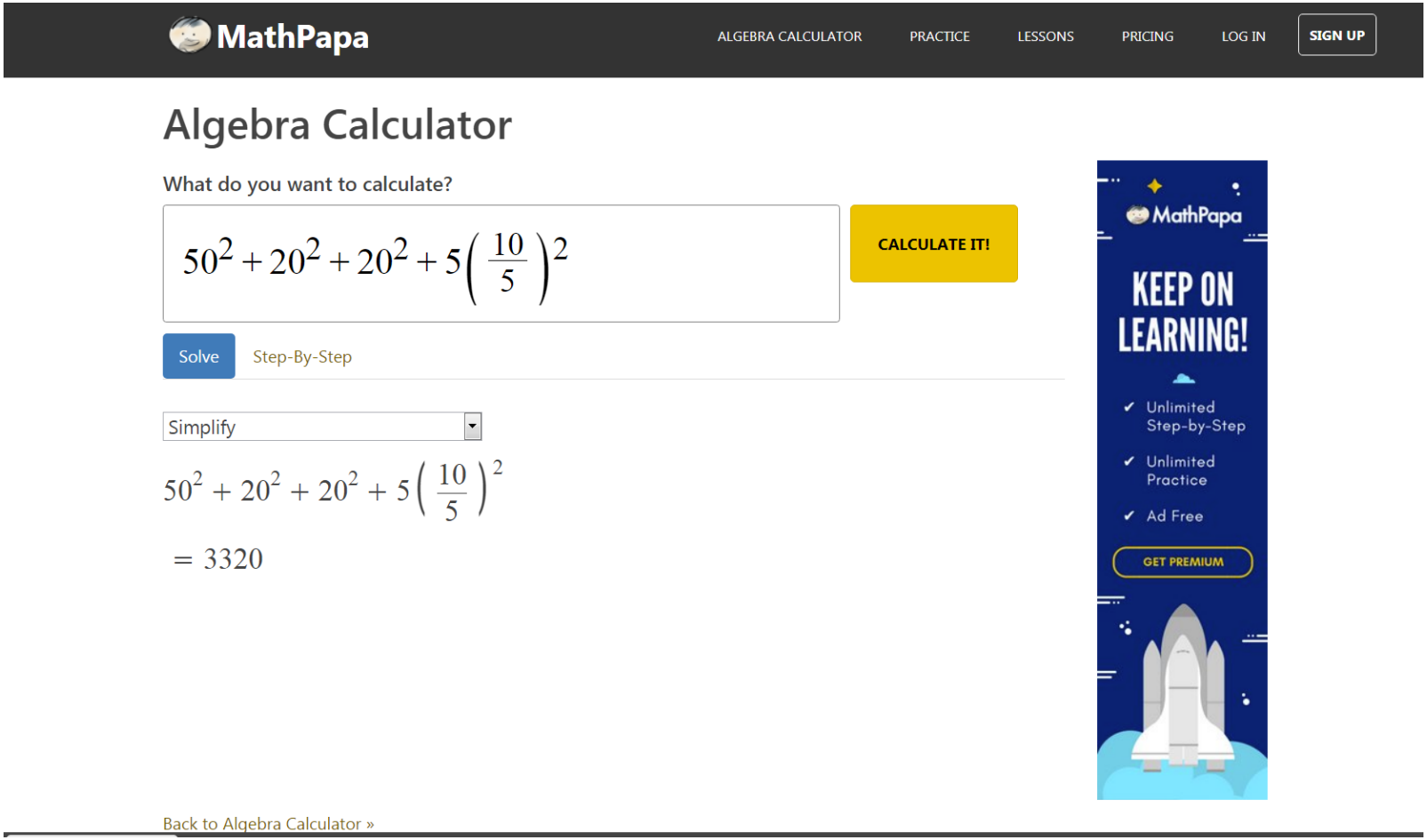
Almost everyone missed this on the graded homework assignment!!

	Premerger Shares	HHI Contribution	
Intuit	62.2%	3869	The square of the firm's market share
HRB	15.6%	243	
TaxACT	12.8%	164	
Others (6)	9.4%	15	Residual share (9.4%) divided by 6 firms and added six times
	100.0%	4291	The sum of the squared shares of all of the firms in the market
Combined share	28.4%		
Premerger HHI		4291	
Delta		400	2 × HRB share × Intuit share
Postmerger HHI		4691	

"Violates" the 2010 Guidelines:
Postmerger HHI exceeds 2500 and delta exceeds 200

Note: The court appears to have assumed that six equal-sized firms are in the "other" category

Math Papa



https://www.mathpapa.com/algebra-calculator.html

Elasticities

- Elasticity of demand—Some definitions
  - *Own-elasticity of demand*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product.

$$\varepsilon = \frac{\frac{\Delta q_i}{q_i}}{\frac{\Delta p_i}{p_i}}$$

Percentage change q_i in the quantity of product *i* demanded

Percentage change p_i in the price of product *i*

Slope of the (residual) demand curve

    - Using a little algebra, this is equivalent to $\frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$ (or in calculus terms $\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}$)
    - Own-elasticities are negative, due to the downward-sloping nature of the demand curve
  - *Cross-elasticity of demand*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} = \frac{\frac{\Delta q_j}{q_j}}{\frac{\Delta p_i}{p_i}}$$

Percentage change q_j in the quantity of product *j* demanded

Percentage change p_i in the price of product *i*

    - Cross-elasticities are positive for substitutes and negative for complements

Elasticities

- Some conventions and definitions
  - By convention, economists speak of elasticities in terms of their absolute values
  - Own-elasticities
    - *Inelastic demand*: Own demand where the quantity demanded does not change significantly with changes in the product's price. *Not price sensitive*. $\left(|\varepsilon| < 1\right)$

This means take the "absolute value ((so, for example $|-0.5| = 0.5$), and so makes own-elasticities positive numbers.

$$|\varepsilon| = \frac{\%\text{change in quantity}}{\%\text{change in price}} < 1$$

Inelastic demand

    - *Unit elasticity*: Where a 1% change in the product's price results in a 1% decrease in the quantity demanded $\left(|\varepsilon| = 1\right)$

$$|\varepsilon| = \frac{\%\text{change in quantity}}{\%\text{change in price}} = 1$$

Unit elasticity

    - *Elastic demand*: Own demand where the quantity demanded drops rapidly with small changes in price. *Very price sensitive* $\left(|\varepsilon| > 1\right)$

$$|\varepsilon| = \frac{\%\text{change in quantity}}{\%\text{change in price}} > 1$$

Elastic demand

Critical loss

- Four formulas for critical loss[1]
  1. Absolute terms (brute force):

Gain on retained sales

$$\Delta p\left(q-\Delta q^{*}\right)=\left(p-c\right)\Delta q^{*}$$

Loss of margin on lost sales

  2. Unit critical loss:

$$\Delta q^{*}=\frac{q\Delta p}{\left(p+\Delta p\right)-c}$$

Solving the above equality of Δq*

  3. Percentage terms ("percentage critical loss"):

$$\frac{\Delta q^{*}}{q}=\frac{\Delta p}{\left(p+\Delta p\right)-c}=\frac{\frac{\Delta p}{p}}{\frac{\Delta p}{p}+\frac{p-c}{p}}=\frac{\delta}{\delta+m}$$

Where δ is the percentage price increase and *m* is the percentage gross margin

  4. Critical elasticity:

$$\left|\varepsilon^{*}\right|=\frac{\frac{\Delta q^{*}}{q}}{\frac{\Delta p}{p}}=\frac{\frac{\Delta q^{*}}{q}}{\delta}=\frac{1}{\delta+m}$$

Dividing the above equality by Δp/p (= δ)

[1] This assumes constant marginal costs.

Critical loss

- Example 1
  - Products A and B are being tested as a candidate market. Each sells for $100, has an incremental cost of $60, and sells 1200 units. When the price for both products is increased by $5, each firm loses 100 units to outside the market. Do A and B constitute a relevant market under the 2010 Guidelines?

Parameters			**Brute force calculation**		**Unit critical loss**	
Price	p	100	Gain = (Q+ΔQ)Δp			
Cost	c	60	Q + ΔQ	2200	$\Delta q^* = \frac{q\Delta p}{(p+\Delta p)-c}$	
Gross margin	m	40	Δp	5		
Market output	Q	2400	Gain	11000		
SSNIP	Δp	5	Loss = mΔQ			
Customer loss	ΔQ	-200	ΔQ	-200	qΔp	12000
("actual loss")			m	40	(p+Δp)-c	45
			Loss	-8000	CL	266.6667
			Net	3000		
			Positive → RM		CL > AL → RM	

Critical loss

- Example 1A
  - We can also analyze Example 1 in terms of the percentage critical loss:

Summary:

P=$100

C=$60

Margin = $40

Total market $Q = q_1 + q_2 = 2400$

Percentage margin $m = \frac{p-c}{p} = \frac{100-60}{100} = 40.0\%$

SSNIP $\delta = 5\%$

Percentage critical loss $CL = \frac{\delta}{\delta + m} = \frac{5\%}{5\% + 40\%} = 11.1\%$

Percentage actual loss $L = \frac{100+100}{2400} = 8.33\%$

Conclusion: Since the percentage actual loss *L* does not exceed the percentage critical loss *CL*, a hypothetical monopolist of A and B could profitably raise price by 5% and so A and B are a relevant market

Critical loss

- Estimating actual loss
  - *Percentage actual* loss: First-order approximation:

$$\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \equiv \varepsilon \Rightarrow \frac{\Delta q}{q} \cong \frac{\Delta p}{p}\varepsilon = \delta\varepsilon,$$

that is, the percentage actual loss is approximately equal to the percentage price change times the own-elasticity of demand

  - *Unit actual* loss: First-order approximation:

$$\frac{\Delta q}{q} \cong \delta\varepsilon \Rightarrow \Delta q = q\delta\varepsilon.$$

Critical loss

- "Brute force" method for single product price increase—Example 1
  - We can apply the hypothetical monopolist test by looking at whether the gross profit gain to the hypothetical monopolist from a single product SSNIP would be greater than the gross profit loss from the loss of sales at the higher price to products outside the candidate market
  - Example 1
    - Assume that for a for a single product price increase of 5%, the hypothetical monopolist would retain 70 out of every 100 customers. Of the 30 lost customers, 24 would divert to another gourmet pizza and 6 would go to a standard pizza. Assume that the price of gourmet pizzas is $4.50 and that the dollar margin is $1.50 per pie.
    - *Query*: Under the single-product price increase test, are gourmet pizzas a relevant product market?

Out of every	100	Price	$3.00	
units sold:		Margin	$1.50	
		SSNIP (%)	5.00%	
		SSNIP ($)	$0.150	
Units retained	70	Gain on retained	$10.50	Units retained time dollar price incre
Total units lost	30	Loss	-$45.00	Total units lost times margin
Units recaptured	24	Gain on recapture	$36.00	Recaptured units times margin
Units lost to outside	6	Net gain	$1.50	

Since the net gain from a 5% price increase to one product is positive, under the one-product test gourmet pizza is a relevant market

Critical loss

- "Brute force" method for single product price increase—Example 2
  - We can use the brute force method for a single product price when margins differ among products within the candidate market.
  - Example 2

Gourmet pizza--Single product price increase

(brute force method--different margins for candidate market of three firms)

Out of every 100 units sold by G1 (the firm experiencing the price increase:

For G1		**For G2**		**For G3**	
Total units retained	70				
Total unit diverted	30	Total units recaptured	10	Total units recaptured	14
G1 price	$3.00				
G1 margin	$1.50	G2 margin	$1.75	G2 margin	$1.35
SSNIP (%)	5.00%				
SSNIP ($)	$0.15				
Gain on retained units	$10.50	Gain on recaptured units	$17.50	Gain on recaptured units	$18.90
Loss on diverted units	-$45.00				
Total gross gain to HM	$46.90	= $10.50 + $17.50 + $24.00			
Total gross loss to HM	-$45.00				
NET GAIN	**$1.90**				

Since the net gain to the hypothetical monopolist is positive, the candidate market is a relevant market

Aggregate diversion analysis

- Recapture rule
  - *Uniform price increase*: Let *R* be the recapture rate in the candidate market in the wake of a uniform SSNIP. Then if

$$R \geq \frac{\delta}{\delta + m} = \%CL$$

A uniform price increase will be profitable for the hypothetical monopolist

Aggregate diversion analysis

- Extension to single product recapture rates
  - Define the critical recapture rate R* as:

$$R^* = \frac{\delta}{\delta + m} = \%CL.$$

  - Rule:

$$\text{If } \min_i R_i \geq R^*, \text{ then } R \geq R^* \text{ and so } L < CL.$$

  - Example
    - Assume a three-product candidate market. Each product has a margin of 35%. Assume a uniform SSNIP of 5% across all products. Then R^* = 12.5%. Suppose that the SSNIP generates the following recapture rates:

Product	q	Δq	Recapture Units	Recapture Rate (R)
A	300	90	20	22.22%
B	400	125	40	32.00%
C	500	200	35	17.50%
Total	1200	415	95	22.89%

    - Applying extension, since the smallest R_i (17.5%) is greater than R* (12.5%), a hypothetical monopolist can profitably sustain a 5% uniform price and so the three products are a relevant market

Aggregate diversion analysis

- Warren-Bolton analysis in H&R Block/TaxACT
  - Question: Is DDIY a market?
  - Critical loss (*CL*): Use percentage critical loss formula
    - Starting point: Start with DDIY products (HRB, TaxACT, and TurboTax)
    - SSNIP (δ): 10%
    - Gross margin (*m*): 50% on each product

$$CL = \frac{\delta}{\delta + m} = \frac{10\%}{10\% + 50\%} = 16.7\%$$

  - Actual loss: Use Aggregate diversion ratio method (recapture rate *R*)
    - Test: If $R \geq CL$, then product grouping is a market
    - Using IRS switching data as a proxy for *R*, Warrant-Bolton found:
      - HRB: $R = 57\%$
      - TaxACT: $R = 53\%$
      - TurboTax: $R = 39\%$
    - Warren-Bolton concluded that, since each $R > CL$, a hypothetical monopolist of the DDIY product could profitably raise price by a SSNIP and therefore DDIY was a relevant product market

The 2010 Merger Guidelines

- "HHI thresholds"[1]
  - Not really *PNB* thresholds, but courts tend to use them that way[1]

Postmerger HHI	ΔHHI	Guidelines
--	< 100	"unlikely to have adverse competitive consequences and ordinarily require no further analysis"
< 1500	--	"unlikely to have adverse competitive consequences and ordinarily require no further analysis"
Between 1500 and 2500	≥ 100	"potentially raise significant competitive concerns and often warrant scrutiny"
> 2500	100-200	"potentially raise significant competitive concerns and often warrant scrutiny"
	≥ 200	"will be presumed to be likely to enhance market power. The presumption may be rebutted by persuasive evidence showing that the merger is unlikely to enhance market power."

[1] The "HHI" is a market concentration statistic. To calculate it, take the square of the market share of each firm in the relevant market and square it, and then add up all of the squared market shares. The "ΔHHI" is the difference between the HHI after the merger and the HHI before the merger.

[2] "The purpose of these thresholds is not to provide a rigid screen to separate competitively benign mergers from anticompetitive ones, although high levels of concentration do raise concerns. Rather, they provide one way to identify some mergers unlikely to raise competitive concerns and some others for which it is particularly important to examine whether other competitive factors confirm, reinforce, or counteract the potentially harmful effects of increased concentration." 2010 Merger Guidelines § 5.3.

HHIs in Successful DOJ/FTC Challenges

- The DOJ and FTC have not brought "close" cases in alleged markets

Agency	Complaint	Defendant	Combined share[1]	PreHHI	PostHHI	Delta	Deal Status
FTC	2018	Wilhelmsen	84.7	3651	7214	3563	Preclosing
DOJ	2017	Energy Solutions	100	6040	10000	3960	Preclosing
DOJ	2016	Anthem	47	2463	3000	537	Preclosing
DOJ	2016	Aetna			>5000[2]		Preclosing
FTC	2016	Penn State Hershey	64	3402	5984	2582	Preclosing
FTC	2015	Advocate Heath	55	2094	3517	1423	Preclosing
FTC	2015	Staples	75[3]	3036	5836	2800	Preclosing
FTC	2015	Sysco	71[4]	3153	5519	1966	Preclosing
DOJ	2015	Electrolux		3350[5]	5100	1750	Preclosing
DOJ	2013	Bazaarvoice	68	2674	3915	1241	Consummated
FTC	2013	Saint Alphonsus	57	4612	6129	1607	Consummated

[1] When the complaint alleged multiple markets, the market with the most problematic highest HHIs is reported.
[2] The DOJ challenged Aetna's proposed acquisition of Humana in 17 geographic markets. The complaint did not provide HHI statistics for each market, although it noted that in 75% of the markets, the post-HHI would be greater than 5000.
[3] The FTC also challenged the transaction in 32 alleged relevant local geographic markets, with the smallest combined share being 51% and the largest being 100%.
[4] The complaint alleged multiple markets in food distribution. The numbers given are for national broadline distribution.
[5] The complaint alleged three markets. The numbers given are for ranges. Cooktops and wall ovens were similar.

HHIs in Successful DOJ/FTC Challenges

- The DOJ and FTC have not brought "close" cases in alleged markets

Agency	Complaint	Defendant	Combined Share[1]	PreHHI	PostHHI	Delta	Deal Status
DOJ	2013	US Airways	100[2]	5258	10000	4752	Preclosing
DOJ	2013	ABInbev	100	5114	10000	4886	Preclosing
FTC	2011	OSF Healthcare	59	3422	5179	1767	Preclosing
FTC	2011	ProMedica	58	3313	4391	1078	Preclosing
DOJ	2011	H&R Block	28	4291	4691	400	Preclosing
FTC	2009	CCC	65	4900	5460	545	Preclosing
FTC	2008	Polypore	100	8367	10000	1633	Consummated
FTC	2007	Whole Foods	100[3]		10000		Preclosing
FTC	2004	Evanston	35	2355	2739	384	Consummated
DOJ	2003	UPM-Kemmene	20	2800	2990	190	Preclosing
FTC	2002	Libbey	79	5251	6241	990	Preclosing
FTC	2001	Chicago Bridge	73	3210	5845	2635	Consummated
FTC	2000	Heinz	33	4775	5285	510	Preclosing
FTC	2000	Swedish Match	60	3219	4733	1514	Preclosing
DOJ	2000	Franklin Electric	100	5200	10000	4800	Preclosing

[1] When the complaint alleged multiple markets, the market with the most problematic highest HHIs is reported.
[2] The complaint alleged 1043 markets.
[3] In some local geographic markets, this was a merger to monopoly in the FTC's alleged product market of premium, natural, and organic supermarkets.

Diversion ratios

- Diversion ratios
  - Unit diversion ratios (when firm A raises in price):

$$D_{A\to B} \equiv D_{AB} = \frac{\frac{\Delta q_B}{\Delta p_A}}{\frac{\Delta q_A}{\Delta p_A}} = \frac{\Delta q_B}{\Delta q_A},$$

This is the *diversion ratio* from A to B. It is the percentage of the total units that Firm A's loses when it raises price that go to Firm B.

where firm A loses total units of Δq_A, of which Δq_B go to Firm B

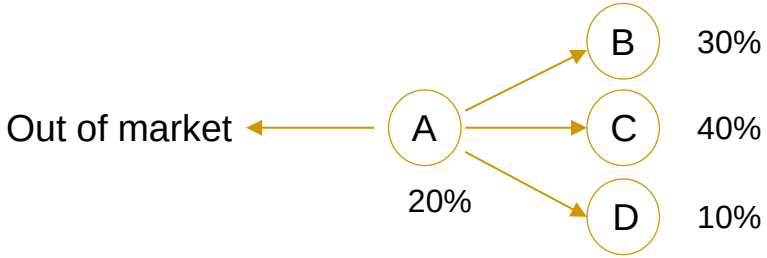
  - Market shares as proxies
    - Assumes that customers divert in proportion to the market shares of the competitor firms (after adjusting for any out-or-market diversion):

$$D_{A\to B} = \left(1 - \frac{\Delta q_{outside}}{\Delta q_A}\right)\frac{s_b}{1 - s_A},$$

where s_A and s_B are the market shares of firms A and B, respectively, in the market and $\frac{\Delta q_{outside}}{\Delta q_A}$ is the percentage of Frim A's lost sales that are diverted to firms outside of the market

Diversion ratios

- Diversion ratios
  - How are diversion ratios estimated?
    - Market shares as proxies--Example
      - *Example*: The market consists of four firms collectively producing 1200 units with market shares of 20%, 30%, 40%, and 10%. Firm A raises its prices by 5% and loses 100 units.



      - When Firm A raises its price, assume all of the diverted products stay within the market. Then estimating diversion ratio to Firm B based on market shares yields:

$$D_{A\to B} = \left(1 - \frac{\Delta q_{outside}}{\Delta q_A}\right)\frac{s_b}{1-s_A} = \frac{0.3}{1-0.2} = 37.5\%$$

      - Alternatively, assume that Firm A still losses 100 units but 10% diverts out of the market. Then:

$$D_{A\to B} = \left(1 - \frac{\Delta q_{outside}}{\Delta q_A}\right)\frac{s_b}{1-s_A} = (1-0.10)\times\frac{0.3}{1-0.2} = 33.75\%$$