Gregory K. Leonard & J. Douglas Zona, *Simulation in Competitive Analysis*, *in* 2 ISSUES IN COMPETITION LAW AND POLICY 1405 (ABA Section of Antitrust Law 2008)

Chapter 59

SIMULATION IN COMPETITIVE ANALYSIS

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Simulation is the use of a structural economic model to predict the effects of a change in economic conditions or policy. Simulation has wide application in competitive analysis, from merger review to the calculation of damages in antitrust and patent infringement litigation. It provides a scientific and quantitative approach that is a useful complement to making qualitative inferences based on a review of documents and other information. In this chapter, we describe simulation methods and techniques with a particular emphasis on merger review, where simulation has been most widely used.

1. Introduction

In some circumstances, history has provided a useful natural experiment. For example, consider the competitive analysis of a merger of two retailers. If retail stores in this industry compete in localized markets, it may be useful to examine historical data to determine the impact of store openings on the prices at nearby competing stores.[1] In the right circumstances, natural experiments may be very useful for measuring and characterizing what happened in the past and making inferences about what would likely happen in the future.[2] In many situations, however, history has not provided a natural experiment to guide answers to important policy and competitive questions. But economists still have a powerful tool available to help address these questions: structural modeling.[3]

A structural model is a representation of the underlying fundamental economic processes that determine observable outcomes. For example, a structural model of a merger might include components that describe firm behavior (i.e., firms' objectives such as short-run profit maximization); production technology (i.e., the nature of costs); and consumer response (i.e., the system of demand for the products at issue).[4]

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* NERA Economic Consulting and LECG, respectively. We thank Julia Lynch for excellent research assistance and Roy Epstein, Luke Froeb, Dan Rubinfeld, Greg Werden, and Lawrence Wu for valuable comments.

1. An example of this type of analysis was presented in *FTC v. Staples, Inc.*, 970 F. Supp. 1066 (D.D.C. 1997). For the views of the respective experts in that case, see Jerry A. Hausman & Gregory K. Leonard, Econometrics versus Documents in *Staples* (NERA, Working Paper, undated), *available at* http://www.nera.co.uk/image/PUB_Staples_963.doc; and Jonathan B. Baker, *Econometric Analysis in* FTC v. Staples, 18 J. PUB. POL'Y & MARKETING 11 (1999).
2. See Mary Coleman & James Langenfeld, *Natural Experiments*, which appears as Chapter 31 in this book.
3. Even where a natural experiment is available, structural modeling may be helpful for understanding the implications of the natural experiment.
4. Models are designed to capture the key economic elements of the real world, abstracting from those elements that are not crucial. It is precisely by boiling down to the key elements that a model becomes useful for economic analysis. The question then becomes which real world elements are

With a model in hand, the economist can predict the effects on economic outcomes of changes in the economic environment. For example, the competitive effects of a merger can be predicted if the economist is able to model the key economic processes of the postmerger world.

Structural modeling to assess the competitive effects (and potentially other effects) of a merger has come to be referred to as merger simulation over the last 15 years. Merger simulation has become popular in part because it has recently become easier as data have become more widely available and econometric techniques have advanced, and it is consistent with a particular general theory of competitive harm—unilateral effects.[5] There is widespread agreement among economists on the basic economic forces at work with unilateral effects. The question that arises for a particular merger is the quantification of the unilateral effects. Merger simulation techniques address precisely this question.

Given its popularity, it is perhaps not surprising that critics have emerged and merger simulation has come under fire. However, properly interpreted, the criticisms are not aimed at the technique of structural modeling in general. Indeed, structural modeling is well-established in economics—building models and using those models to predict the effects of changes in policy or other exogenous variables has long been a fundamental technique in the economist's toolkit. Instead, the criticisms of merger simulation concern the details of its implementation in practice. In this context, it is important to recognize that merger simulation, like all scientific methods, is continually subject to refinement. Merger simulation in its current state can provide (and indeed has provided) important insights that lead to better decisions by government agencies and others engaged in competitive analysis.[6] Simulation provides a substantially more scientific and quantitative approach to competitive analysis than the commonly used alternative of making qualitative inferences based on a review of documents, deposition transcripts, and other discovery materials. Thus, simulation is an important complement to the qualitative approach. This chapter summarizes the current state of structural modeling in the analysis of competitive issues, where it has wide application, from merger reviews to the calculation of damages in antitrust and patent infringement litigation. Sections 2 and 3 discuss merger simulation in both differentiated product industries and homogeneous product industries. Section 4 presents some of the particular methods for structural modeling of mergers. Section 5 discusses criticisms—both those with merit and those without—that have been leveled at merger simulation. Section 6 presents some nonmerger applications of simulation techniques.

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key and which are not. The choice as to which elements of the real world to incorporate into the model is part of model specification.

5. *See, e.g.*, Jonathan B. Baker, *Why Do the Antitrust Agencies Embrace Unilateral Effects?*, 12 GEO. MASON L. REV. 31 (2003).

6. Experience with merger simulation in court proceedings is limited. In *United States v. Oracle Corp.*, 331 F. Supp. 2d 1098 (N.D. Cal. 2004), an expert economist testified about the results of a merger simulation. However, there are other cases where experts have testified concerning own- and cross-price elasticities of demand—one of the important inputs of merger simulation. *See, e.g.*, FTC v. H.J. Heinz Co., 246 F.3d 708 (D.C. Cir. 2001).

2. Use of simulation in merger investigation

A merger combines the productive resources and decision-making responsibilities of two firms. As a result, the merged firm may have the incentive to implement different competitive strategies than those of the two merging firms prior to the merger. Thus, the merger may change market outcomes. Merger-induced changes in market outcomes are referred to as the competitive effects of the merger.[7] Competitive effects may result, for example, if the merger leads to marginal cost reductions or if the merger eliminates competition between the products of the merging firms. Antitrust has distinguished between unilateral effects and coordinated effects,[8] and simulation has been used only with the former.[9]

A merger could potentially affect any competitive behavior—price setting, advertising and promotional activities, choice of production capacity, product positioning, research and development activities, and the like. As a practical matter, merger simulation has generally been used to predict the price effects of mergers, thus implicitly holding constant the other types of competitive behavior.[10] In most industries, this approach is justified with respect to those aspects, such as capacity and product positioning, which are largely fixed in the short run, since the focus of a merger review is generally on the relatively short run.

The basic idea behind simulating a merger is to build a structural model of the postmerger world.[11] A structural model of an industry has three basic components: (1) the demand functions for the products, (2) the cost functions of the firms, and (3) the nature of strategic interaction between the firms. With these three components, the model can be solved to obtain predictions of the prices that would prevail in the postmerger world.

The basic procedure outlined above is used to simulate mergers in both homogeneous product industries and differentiated products industries. The details,

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7. This terminology arises from the fact that § 7 of the Clayton Act makes the legality of mergers turn on their effects on "competition," although this generally has been interpreted to implicate some notion of economic welfare.

8. *See, e.g.*, U.S. DEP'T OF JUSTICE & FEDERAL TRADE COMM'N, HORIZONTAL MERGER GUIDELINES (1992) (with Apr. 8, 1997 revisions to Section 4 on efficiencies) [hereinafter 1992 MERGER GUIDELINES], *reprinted in* 4 Trade Reg. Rep. (CCH) ¶ 13,104.

9. Unilateral effects typically start with the merged firm. However, the merged firm's rivals may also make unilateral changes in their actions postmerger, and these are also considered unilateral effects of the merger. For example, a rival may find it in its own best interest to increase its price if the merged firm has increased its price postmerger. It is a common misconception that unilateral effects involve only the merging firm.

10. Exceptions are Raymond Hartman, *Price-Performance Competition and the Merger Guidelines*, 18 REV. INDUS. ORG. 53 (2001), and Oliver Richard, *Flight Frequency and Mergers in Airline Markets*, 21 INT'L J. INDUS. ORG. 907 (2003). The implications of ignoring advertising competition are studied by Luke Froeb, Steven Tenn & Steven Tschantz, Mergers When Firms Compete by Choosing Both Price and Promotion (Vanderbilt Public Law Research Paper No. 07-09, 2007).

11. There are other types of empirically based analysis that one might perform when attempting to predict the competitive effects of a merger. *See, e.g.*, Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust*, 1 AM. L. & ECON. REV. 386 (1999); ABA SECTION OF ANTITRUST LAW, ECONOMETRICS ch. 12 (2005).

however, differ depending on the type of industry at issue. In the following sections, we address differentiated products mergers, and in the subsequent section we address homogenous product mergers.

3. Merger simulation in differentiated products industries

A differentiated products industry is one where the various rivals sell products that differ in their attributes and where consumers differ in their tastes for these attributes. The attributes can be physical attributes, such as flavor in soft drinks and softness in facial tissue, or intangible attributes, such as a brand name. Because of the differentiation, in general, each product appeals to a somewhat different set of customers.

We consider three ways in which a differentiated products merger might have an effect on prices.[12] First, the merger might change the incentives for the merged firm when setting the prices for its products. This change in incentives arises due to the relaxation of competitive constraints. When setting its price, a firm is constrained by the fact that an increase in its price would cause some consumers (marginal consumers) to switch to other brands (or leave the market altogether), in which case the firm would lose the profit from sales to these customers. Thus, each firm sets its price so that the gain from an increase in price to customers who would not switch (inframarginal consumers) would be exactly offset by the loss in profit from the marginal consumers. At this price level, the firm cannot increase its profits by raising its price any further (nor can it increase its profits by lowering its price, for that matter).

Now suppose that there is a merger of two firms, A and B. Before the merger, part of the pricing constraint faced by firm A might have involved some marginal consumers switching to firm B's product. After the merger, however, this part of the constraint faced by firm A is no longer present. The merged firm does not lose as many marginal consumers as firm A did by itself; some of the loss of marginal customers has been internalized by merger. The gain and loss that would result from a price increase on firm A's product are now no longer balanced: the gain exceeds the loss. Thus, the merged firm has the incentive to raise the price of firm A's product until the balance is restored. The strength of this change in incentives due to the merger depends largely on how many marginal consumers of firm A's product would switch to firm B's product. This, in turn, is a function of the own- and cross-price elasticities of demand for the products of the merged firm.

The second way that the merger might affect prices is if it generates marginal cost reductions for the merged firm. To the extent that a merger will lead to lower marginal costs for any of the merging parties' products, the merged firm will have the incentive to lower its prices on those products, all else equal. The reasoning is as follows. As described above, each firm sets its price at the level at which lowering or raising the price would not increase profits. A price reduction would not increase

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12. The first and third of these ways that a merger might lead to competitive effects were studied by Raymond Deneckere & Carl Davidson, *Incentives to Form Coalitions with Bertrand Competition*, 16 RAND J. ECON. 73 (1985).

profits because the gains associated with the profit from added marginal consumers is just offset by the losses from charging the lower price to inframarginal consumers. If the merger leads to lower marginal cost, however, this calculus is changed postmerger. In particular, the profit margin on each marginal consumer increases (because of the lower marginal cost). Thus, at premerger prices, the merged firm would gain more from lowering its price in terms of the profit from additional marginal consumers than it would lose from having a lower price on sales to inframarginal consumers. Thus, marginal cost reductions lead the merged firm to lower prices, all else equal.

The third way in which the merger can affect prices arises because a change in the merged firms' incentives in turn affects the incentives of other firms in the industry. Specifically, to the extent that the merged firm raises its prices postmerger, competition may be softened and the merged firm's rivals may then have the incentive to raise their prices as well. Conversely, if the merged firm decreases its prices postmerger, competition may be sharpened and the merged firm's rivals will have an incentive to decrease their prices.

3.1. Inputs to a merger simulation

To simulate a merger, one needs a model that explains the ongoing outcomes in the industry. This model requires as inputs a characterization of demand and marginal costs for the products and the nature of strategic interaction between the firms. The details of how exactly the structural model is specified are important. In general, competent model specification will reflect the economically important marketplace realities, which will be informed by historical data or other facts and circumstances of the context at hand. Most often, if not always, model specification involves trade-offs. Specification of a very general model may require extensive and detailed information. Specification of a very restrictive model, on the other hand, may require only very limited information, but the inferences from such a model are more likely to be driven by assumptions of the model rather than the facts and circumstances of the particular case at hand.

Choice of functional form for product demands. The demand functions can be econometrically estimated if data are available on consumer choices. Such data might come in the form of transaction data aggregated over space and time—such as so-called retail scanner data—or in the form of data on the decisions of individual consumers. In some cases, consumer surveys may provide data on respondent's decisions over "hypothetical" choices posed in the survey that are reliable enough to be used to estimate the demand functions, although such survey results must be used with care.[13]

Regardless of the type of data that is available, the economist faces the decision as to which functional form to use to represent the demand functions. There are many possible choices, with no one best solution for all circumstances. The

13. For an example of the use of a survey in a merger investigation, see Competitive Impact Statement, United States v. Vail Resorts, 62 Fed. Reg. 5,037 (Feb. 3, 1997).

available data limit the feasibility of certain approaches, so that the choice of functional form must be made in the context of those limitations. Also, certain features of the postmerger world, such as expected entry, exit, or new product positioning, may be more easily incorporated using one kind of demand function rather than another. Rather than catalog the variety of possible approaches for demand estimation, the focus here is on some of the approaches that have proved useful in merger simulation in the past. However, any particular case may require a modification of previous approaches to better reflect the relevant facts and circumstances.

In general, the economist chooses among various functional forms that specify the demand functions up to a set of parameters.[14] These parameters are then econometrically estimated by choosing those values for the parameters that provide the best fit of the demand functions to the data. However, this fit is "best" only given the choice of functional form. The best fit for one particular functional form may be inferior to the best fit of a second functional form. Moreover, some functional forms impose restrictions on the patterns of consumer substitution among products.

Hausman, Leonard, and Zona[15] advocated the consideration of second-order flexible functional forms generally and the almost ideal demand system (AIDS) specifically for situations where the available detailed data are aggregated across consumers.[16] A functional form is second-order flexible if it can approximate any pattern of own- and cross-price elasticities of demand at a given set of prices. Thus, a second-order flexible functional form has the advantage that it imposes no restrictions on the patterns of consumer substitution among products at the point of approximation. Hausman, Leonard, and Zona also proposed using AIDS within a multistage budgeting framework.[17] While imposing certain restrictions on the patterns of substitution among products, the multistage budgeting is helpful when there are many products because it reduces the number of demand function parameters that need to be estimated. In addition, the validity of the restrictions imposed by the multistage budgeting can be tested statistically.

When data on the individual choices of consumers is available, random utility models have been used to estimate consumer demand. These models have advanced

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14. In certain situations, such as studying the overall demand for a single product that constitutes a separate relevant market, it may be possible to use nonparametric techniques that do not require that any particular parametric functional form be chosen. For example, Jerry A. Hausman & Whitney Newey, *Nonparametric Estimation of Exact Consumers Surplus and Deadweight Loss*, 63 ECONOMETRICA 1445 (1995), estimate the demand for gasoline nonparametrically. However, in the situation where there are many differentiated products, parametric approaches would generally be used.

15. Jerry Hausman, Gregory Leonard & J. Douglas Zona, *Competitive Analysis with Differentiated Products*, 34 ANNALES D'ECONOMIE ET DE STATISTIQUE 159 (1994).

16. AIDS was introduced by Angus Deaton & John Muellbauer, *An Almost Ideal Demand System*, 70 AM. ECON. REV. 312 (1980).

17. Multistage budgeting was introduced by W.M. Gorman, *Two-Stage Budgeting*, *in* SEPARABILITY AND AGGREGATION: THE COLLECTED WORKS OF W.M. GORMAN 19 (C. Blackorby & A.F. Shorrocks eds., 1995).

substantially over the last 30 years.[18] The first random utility model widely used by economists was the multinomial logit model. However, this model exhibits the independence of irrelevant alternatives (IIA) property, which imposes significant restrictions on the substitution patterns between products. Specifically, IIA requires that the cross-price elasticities of demand with respect to a given product's price all be equal, an unrealistic restriction in many circumstances.[19] In an effort to escape the IIA property, economists developed more advanced random utility models such as the nested logit model, which allows the cross-elasticties of demand with respect to a given product's price differ across prespecified "nests" or groups of products.[20] Such models are still less than fully satisfactory because the nests must be specified a priori (although the validity of a choice of nests can be statistically tested) and because the models still exhibit the IIA property within a nest.

The next step in the development of random utility models was the random coefficients logit or mixed logit model.[21] In this model, tastes or preferences for product attributes are assumed to vary across consumers and, as a result, the model does not exhibit the IIA property at the aggregate level. Consider a consumer who has a strong taste for sports cars and at current prices would purchase a Ferrari. If the price of the Ferrari increased, and the consumer decided to switch to another product, the consumer would most likely turn to another sports car given the taste for sports cars. Similarly, a consumer with a strong taste for minivans would tend to switch from one minivan to another minivan if the price of the first minivan increased. The mixed logit model is able to capture these possibilities by allowing one consumer to have a strong taste for sports cars and another consumer to have a strong taste for minivans.

A variant of the mixed logit model has also been applied to the situation where aggregated data, rather than data on the choices of individual consumers, are available.[22] Other specific demand systems, such as proportionally calibrated almost ideal demand systems (PCAIDS), have been used in similar circumstances,[23] and other logit models also can be used.

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18. Daniel McFadden was instrumental in this development. For a review, see Daniel McFadden, *Economic Choices*, 91 AM. ECON. REV. 351 (2001).

19. Econometric tests have been developed to determine whether IIA holds in a given situation. *See* Jerry Hausman & Daniel McFadden, *Specification Tests for the Multinomial Logit Model*, 52 ECONOMETRICA 1219 (1984).

20. A related model is the "principles of differentiation" model. *See* Timothy Bresnahan, Scott Stern & Manuel Trajtenberg, *Market Segmentation and the Sources of Rents from Innovation: Personal Computers in the Late 1980s*, 28 RAND J. ECON. S17 (1997).

21. *See* Daniel McFadden & Kenneth Train, *Mixed MNL Models for Discrete Response*, 15 J. APPLIED ECONOMETRICS 447 (2000).

22. *See* Steven T. Berry, James Levinsohn & Ariel Pakes, *Automobile Prices in Market Equilibrium*, 63 ECONOMETRICA 841 (1995); Aviv Nevo, *Mergers with Differentiated Products: The Case of the Ready-to-Eat Cereal Industry*, 31 RAND J. ECON. 395 (2000) [hereinafter Nevo, *Mergers*]; Aviv Nevo, *Measuring Market Power in the Ready-to-Eat Cereal Industry*, 69 ECONOMETRICA 307 (2001).

23. *See, e.g.*, Roy L. Epstein & Daniel L. Rubinfeld, *Merger Simulation: A Simplified Approach with New Applications*, 69 ANTITRUST L.J. 883 (2001).

In addition, a variety of other alternatives may be available. For example, with two-part pricing (say a fixed monthly charge and a per use charge), an analysis of either price in isolation may not fully represent the potential competitive effects from the merger. However, it is difficult in the context of a demand system such as AIDS to incorporate separately the effects of the two-part prices on demand. Other econometric models that more fully reflect the facts and circumstances of the particular industry, such as two-part pricing, could be constructed to measure these effects more accurately. For example, the Department of Justice, in its evaluation of the proposed WorldCom-Sprint transaction, developed a model in which the choice of carrier depended on both a fixed monthly charge and a per-minute charge and then the level of usage, conditional on the carrier choice, depended only on the per-minute charge.[24] Dynamics, entry, and changing product characteristics might also be important features of a market, and economists are working toward models that incorporate these features.

As mentioned above, any second-order flexible form can provide a "perfect fit" to a given set of mean own- and cross-price demand elasticities. Thus, from that perspective, there is no basis to choose among the various second-order flexible forms in a given situation. However, the size of the predicted price changes produced by the merger simulation nevertheless varies in general depending upon which particular second-order flexible form is used. The reason is that different second-order flexible forms differ in their curvature and the degree of curvature affects the size of the predicted postmerger price changes produced by the simulation.[25]

For example, consider a market consisting of two firms each with unit sales of 100 and marginal cost of $1. The products' elasticity matrix is as follows:

$$\begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}$$

The diagonal elements (the –2s) are the respective own-price elasticities of demand for the two firms' products. The off-diagonal elements (the 0.5s) are the cross-price elasticities of demand between the two firms' products. The two firms engage in Nash-Bertrand competition. Under these assumptions, the price of each product is $2.[26] Of the three basic inputs required to perform the merger simulation, the

24. For a general description of the analysis, see ABA SECTION OF ANTITRUST LAW, ECONOMETRICS 422-23 (2005).

25. *See* Philip Crooke, Luke Froeb, Steven Tschantz & Gregory J Werden, *The Effects of Assumed Demand Form on Simulated Post-Merger Equilibria*, 15 REV. INDUS. ORG. 205 (1999); Jerry A. Hausman & Gregory K. Leonard, *Efficiencies From the Consumer Viewpoint*, 7 GEO. MASON L. REV. 707 (1999); Luke M. Froeb, Steven Tschantz & Gregory J. Werden, *Pass-Through Rates and the Price Effects of Mergers*, 23 INT'L J. INDUS. ORG. 793 (2005); Gregory J. Werden, Luke M. Froeb & Steven Tschantz, *The Effects of Merger Efficiencies on Consumers of Differentiated Products*, 1 EUR. COMPETITION J. 245 (2005).

26. As discussed further below, in a Nash-Bertrand model, the equilibrium prices are such that the price-to-marginal-cost ratio is equal to the inverse of the absolute value of the own-price elasticity of demand.

foregoing provides two—the firms' marginal cost and the nature of postmerger strategic interaction (since only one firm will remain postmerger)—leaving only the functional form for the product demand curves to be specified.

Given the elasticities and the other available information, it is possible to make a functional form assumption and then infer the associated demand curve parameters. For example, assuming a constant elasticity functional form, the demand curves would be

$$\begin{aligned} Q_1 &= 282.8 p_1^{-2} p_2^{0.5} \\ Q_2 &= 282.8 p_2^{-2} p_1^{0.5} \end{aligned} \tag{1}$$

Alternatively, assuming a linear functional form, the demand curves would be

$$\begin{aligned} Q_1 &= 250 - 100 p_1 + 25 p_2 \\ Q_2 &= 250 - 100 p_2 + 25 p_1 \end{aligned} \tag{2}$$

On the basis of the information available, there is no reasonable way to choose between these two alternative functional forms for demand. The parameters of both functional forms have been calibrated so that they yield the correct demand elasticities. No other information exists that would allow us to distinguish between them.

The choice has a significant impact on the results of the merger simulation in this case. With the constant elasticity functional form, the merger simulation would predict postmerger price increases of 50 percent for the two firms. With the linear functional form, the merger simulation would predict postmerger price increases of only about 8 percent.

The difference in the predictions based on the two alternative functional forms is due to the difference in the two forms' respective curvatures. The linear form is, as the name suggests, linear in prices. With this form, as the price of a product increases, its own elasticity of demand increases in magnitude. On the one hand, this tends to limit the degree to which the merged firm can increase its prices profitably postmerger. The constant elasticity form, on the other hand, exhibits substantial curvature. As its name suggests, with this functional form a product's own elasticity of demand does not change as its price increases. This gives the merged firm a more powerful incentive to increase prices postmerger. In this example—in which the only observable information is the set of elasticities and the premerger prices—there is no way to determine whether the true underlying demand curves exhibit curvature and thus no way to use data to inform the choice among the linear functional form, the constant elasticity functional form, or some other functional form.

While this may appear to be troubling, additional information may allow us to determine which functional form is more appropriate.[27] For example, in any

27. Gregory J. Werden, *A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Differentiated Products*, 44 J. INDUS. ECON. 409 (1996), suggests a method for identifying the marginal cost reduction necessary to offset a price increase that does not require a functional form assumption.

situation where price and quantity data are available for an extended period of time, econometric methods can be used to statistically test which of the two functional forms provides a better fit to the actual data. For instance, in the example above, if we had data over time, we could observe whether the actual consumer demand responses to prices over time were best modeled as linear or instead a functional form with curvature. The ability to test for the correct functional form may mitigate the concern that different functional forms lead to different predicted price effects in a merger simulation, although it is generally more difficult econometrically to determine precisely rates of curvature than it is to determine slopes.

Marginal costs and efficiencies. The second set of important inputs to a merger simulation consists of the products' marginal costs. Typically, merger simulations for differentiated products assume that marginal costs are constant over the relevant range of output. This assumption is likely to be reasonable in most common circumstances.

Given a constant marginal cost, and given an assumption regarding the form of strategic interaction engaged in by firms prior to the merger, the products' premerger marginal costs can be inferred. For example, if single brand firms engaged in Nash-Bertrand competition before the merger, the premerger margin of price over marginal cost for product *i* would be:

$$\frac{p_i - c_i}{p_i} = -\frac{1}{e_{ii}(\mathrm{p})} \tag{3}$$

Since the premerger price p_i is observed and $e_{ii}(\mathrm{p})$ is readily calculated by evaluating the demand function at premerger prices, this equation can be solved for the marginal cost c_i.

In simulating the merger, we are primarily interested in the postmerger marginal costs. If the merger is expected to have no effect on the merging firms' costs, the premerger marginal costs serve as a reasonable estimate of the postmerger marginal costs. However, if the merger is expected to generate marginal cost reductions, the postmerger marginal costs for the merging firms will be less than their premerger levels.[28] Typically, the merging parties are in the best position to estimate the size of any such efficiencies. Accounting for any marginal cost reductions is important because they can have a substantial impact on the assessment of the competitive effects of the merger.[29] An advantage of merger simulation is that it provides a straightforward way to account for marginal cost reductions.

The nature of strategic interaction between firms. The third required input for a merger simulation is the specification of the form of strategic interaction between firms after the merger. To date, merger simulations in differentiated products industries have typically assumed that firms engage in static Nash-Bertrand

28. The merger may also be expected to generate fixed cost savings, which may be important for explaining why the merging firms expect the merger to be profitable. However, because fixed cost savings normally have no effect on the firms' price setting, they would typically be ignored in a merger simulation that is designed to assess the effects of the merger on prices.
29. *See* Hausman, Leonard & Zona, *supra* note 15; Hausman & Leonard, *supra* note 25.

competition in which each firm sets price taking as given the prices of the other firms; each time period's prices are set without reference to past history. This assumption seems natural in many circumstances, but other assumptions are possible, e.g., joint profit maximization and leader-follower models. These are discussed further below.

It would be useful to be able to test the validity of the assumed model of strategic interaction between firms. Such testing should be based on the model's ability to predict outcomes. For example, to test whether the Nash-Bertrand model is a valid model of competition in the premerger world, one can compare the margins of price over marginal cost implied by the Nash-Bertrand model to the actual observed margins. If the Nash-Bertrand model is valid, the implied margins and the actual margins should be close.[30] Of course, in making this comparison, one must take care when calculating the actual margin based on accounting data; accounting cost definitions do not always coincide with economic cost definitions. As another example, the change in market outcomes from a structural change can be predicted using the model and then the predictions can be compared to the actual change in outcomes. Hausman and Leonard compared the actual price changes that resulted from the entry of a new product to the predicted price changes based on the Nash-Bertrand model.[31]

We note that one should resist the urge to "test" a model of strategic behavior (or, for that matter, any economic model) on the basis of the "realism" of its assumptions. The value of a model is in its ability to predict economic outcomes and, therefore, tests of the validity of models should be performed on the basis of outcomes. For example, it is not a valid criticism of the Nash-Bertrand model to assert that its assumption that competitors engage in a one-shot game is unrealistic; instead, a valid criticism would be based on the model's failure to predict outcomes, such as price-cost margins, very well.

3.2. Merger simulation literature

The 1992 *Merger Guidelines* recognize the potential effects of a merger in a differentiated products industry but do not offer a method for quantifying the size of any merger-induced effect on prices.[32] A number of studies have demonstrated how merger simulation can be used to assess the likely unilateral effects of a merger.

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30. *See* Jerry A. Hausman & Gregory K. Leonard, *Using Merger Simulation Models: Testing the Underlying Assumptions*, 23 INT'L J. INDUS. ORG. 693 (2005); Roy J. Epstein & Daniel L. Rubinfeld, *Merger Simulation with Brand-Level Margin Data: Extending PCAIDS with Nests*, 4 ADVANCES IN ECON. ANALYSIS & POL'Y (2004).

31. *See* Jerry A. Hausman & Gregory K. Leonard, *The Competitive Effects of a New Product Introduction: A Case Study*, 50 J. INDUS. ECON. 237 (2002).

32. Indeed, the 1992 *Merger Guidelines* focus on whether there is "a significant share of sales in the market accounted for by consumers who regard the products of the merging firms as their first and second choices." 1992 MERGER GUIDELINES, *supra* note 8, § 2.21. This is not particularly helpful since the term "significant" is not defined. Certainly, the condition that a plurality of the consumers of the products of one merging firm find the other merging firm's products to be their second choice is neither necessary nor sufficient for the merger to have a significant price effect.

Early studies. Hausman, Leonard, and Zona proposed a method for simulating the postmerger world, which allows for explicit quantification of the merger's effects on the prices of the products of the merged firm, incorporating any marginal cost reductions the merger was expected to generate.

Consider an industry that premerger has *n* firms, each of which produces a single product so that firm *i* produces product *i*.[33] Hausman, Leonard, and Zona assumed that each firm *i* had constant marginal cost c_i. Two firms, say firms 1 and 2, are going to merge. Hausman, Leonard, and Zona also assumed that the $n-1$ postmerger firms would engage in Nash-Bertrand competition.[34] Under this form of strategic interaction between firms, the merged firm would set the prices of its two products p_1 and p_2 so as to maximize its total profits, taking the prices of the other $n-2$ products as given. At the optimum, the merged firm's two prices would solve:

$$\begin{aligned} s_1 + s_1 e_{11}\frac{p_1 - c_1}{p_1} + s_2 e_{21}\frac{p_2 - c_2}{p_2} = 0 \\ s_2 + s_2 e_{22}\frac{p_2 - c_2}{p_2} + s_1 e_{12}\frac{p_1 - c_1}{p_1} = 0 \end{aligned} \tag{4}$$

where s_i is the postmerger (revenue) share of product *i* (*i* = 1, 2), and e_{ij} is the postmerger elasticity of demand of product *i* with respect to the price of product *j*. This set of equations implicitly defines the postmerger prices of the merged firm in terms of postmerger shares, postmerger demand elasticities, and marginal costs.

The important difference between the two first-order conditions for the merged firm and the corresponding first-order conditions for firms 1 and 2 in the premerger world is the presence of the terms with the cross-price elasticities. In the premerger world, firm 1 does not take into account its cross-price effect on firm 2 and vice versa. However, after the merger, given the merged firm's common ownership of products 1 and 2, it takes into account the cross-price effects. This reflects the merger's relaxation of the competitive constraint that firms 1 and 2 imposed on each other before the merger.

The two first-order equations can be rewritten as system of equations:

$$s + Ew = 0 \tag{5}$$

For example, the products could be each other's closest substitute, but there could be many other close substitutes; in that case, it is not necessarily the case that prices would increase significantly. Conversely, the products may not be each other's closest substitute (e.g., have the greatest cross-price elasticities of demand of any pair of competing products), but, nonetheless, their prices may increase significantly.

33. The analysis can be extended in a straightforward manner if one or more of the firms sell more than one product.

34. Under Nash-Bertrand competition, firms choose prices for each product they produce to maximize short-run profit for the firm. In equilibrium, each price is a best response, given the actions of the others, and no firm has any incentive to deviate from the chosen prices. There are other behavioral assumptions, but the Nash-Bertrand assumption is by far the most common in modeling differentiated products industries.

where s is a 2×1 vector consisting of the postmerger shares of products 1 and 2, E is a 2×2 matrix consisting of the postmerger own-elasticities of products 1 and 2 on the diagonal and the postmerger cross-elasticities in the off-diagonal elements, and w is a 2×1 vector where the *i*th element is $s_i\,(p_i - c_i)/p_i$. Thus, given s, E, and the c_i, it is possible to solve for w as

$$w = -E^{-1}s \tag{6}$$

and then solve $s_i\,(p_i - c_i)/p_i = w_i$ for p_i.

Of course at the time the analysis is performed (i.e., before the merger), s and E are unknown because these are the postmerger shares and elasticities. However, Hausman, Leonard, and Zona reasoned that to first order, the shares and elasticities would be approximately the same postmerger as premerger. Thus, the premerger values of s and E can be used in place of the postmerger values, and the procedure described above is used to solve for approximate values of the postmerger prices of the merged firm.

The Hausman, Leonard, and Zona method is relatively easy to implement computationally because the postmerger first-order conditions are essentially linearized in w. However, this method provides only an approximation to the solution of the postmerger model. The accuracy of the approximation would be expected to decrease with the size of the difference between premerger and postmerger prices.

The Hausman, Leonard, and Zona method addresses only the price effects for the products of the merging firms. In general, for the reasons discussed above, the prices of the nonmerging firms would be expected to change as well. Werden and Froeb addressed how to calculate the "exact" postmerger prices in the particular case where demand is of the logit form and firms engage in Nash-Bertrand competition, and Feldman did the same for the case of nested logit demand.[35] Hausman and Leonard demonstrated how to calculate the exact postmerger prices in the case of general demand and Nash-Bertrand competition.[36] The basic idea is to note that the n postmerger first-order conditions for the $n-1$ firms in the industry can be written as

$$\begin{aligned}
&s_1(\mathrm{p}) + s_1(\mathrm{p})e_{11}(\mathrm{p})\frac{p_1 - c_1}{p_1} + s_2(\mathrm{p})e_{21}(\mathrm{p})\frac{p_2 - c_2}{p_2} = 0 \\
&s_2(\mathrm{p}) + s_2(\mathrm{p})e_{22}(\mathrm{p})\frac{p_2 - c_2}{p_2} + s_1(\mathrm{p})e_{12}(\mathrm{p})\frac{p_1 - c_1}{p_1} = 0 \\
&s_j(\mathrm{p}) + s_j(\mathrm{p})e_{jj}(\mathrm{p})\frac{p_j - c_j}{p_j} = 0 \qquad j = 3,\dots,n
\end{aligned} \tag{7}$$

35. *See* Gregory J. Werden & Luke M. Froeb, *The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy*, 10 J. L., ECON. & ORG. 407 (1994); Roger Feldman, *The Welfare Economics of a Health Plan Merger*, 66 J. REG. ECON. 67 (1994).

36. *See* Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data*, 5 GEO. MASON L. REV. 321 (1997).

where p is the $n \times 1$ vector of prices for the *n* products and the other variables are as defined above. Note now that the shares and elasticities are written as functions of the price vector p. The share and elasticity functions are themselves derived from the underlying demand functions. Thus, as long as the demand functions (or at least how shares and elasticities change with price) and marginal costs for the *n* products are known, the foregoing is a system of *n* nonlinear equations in the *n* unknown prices p_i, $i = 1, \ldots, n$. The system can be solved numerically to obtain the exact postmerger prices.

Simulation in diverse merger situations. At this point, a large number of merger simulation studies—of actual mergers or hypothetical mergers—have been performed, both at the agencies, by economists consulting for the merging parties, and by interested and disinterested third parties. Published papers have addressed merger simulations in beer,[37] health plans,[38] bread,[39] bath tissue,[40] breakfast cereals,[41] spaghetti,[42] long-distance telephone service,[43] parking lots,[44] software,[45] electric utilities,[46] hotels,[47] railroads,[48] hospitals,[49] soft drinks,[50] airlines,[51] and commercial trucks.[52] Unpublished studies, typically done in the course of merger investigations, have addressed many other industries.

37. Hausman, Leonard & Zona, *supra* note 15; Joris Pinske & Margaret E. Slade, *Mergers, Brand Competition, and the Price of a Pint*, 48 EUR. ECON. REV. 617 (2004).
38. Feldman, *supra* note 35; Robert Town, *The Welfare Impact of HMO Mergers*, 20 J. HEALTH ECON. 967 (2001).
39. Gregory J. Werden, *Expert Report in United States v. Interstate Bakeries Corp. and Continental Baking Co.*, 7 INT'L J. ECON. BUS. 139 (2000); Atanu Saha & Peter Simon, *Predicting the Price Effects of Mergers with Polynomial Logit Demand*, 7 INT'L J. ECON. BUS. 149 (2000).
40. Hausman & Leonard, *supra* note 36.
41. Nevo, *Mergers*, *supra* note 22.
42. Oral Capps, Jr., Jeffrey Church & H. Alan Love, *Specification Issues and Confidence Intervals in Unilateral Price Effects Analysis*, 113 J. ECONOMETRICS 3 (2003).
43. Werden & Froeb, *supra* note 35; Froeb et al., *supra* note 25.
44. Luke Froeb, Steven Tschantz & Philip Crooke, *Bertrand Competition with Capacity Constraints: Mergers Among Parking Lots*, 113 J. ECONOMETRICS 49 (2003).
45. Jith Jayaratne & Carl Shapiro, *Simulating Partial Divestitures to "Fix" Mergers*, 7 INT'L J. ECON. BUS. 179 (2000). The industry studied in this paper may not have actually been software given that the authors indicated that they changed the industry to protect confidentiality.
46. John Morris, *Finding Market Power in Electric Power Markets*, 7 INT'L J. ECON. BUS. 167 (2000).
47. Hartman, *supra* note 10.
48. Joon Park, Michael Babcock, Kenneth Lemke & Dennis Weisman, *Simulating the Effects of Railroad Mergers*, 67 S. ECON. J. 938 (2001).
49. Martin Gaynor & William B. Vogt, *Competition Among Hospitals*, 34 RAND J. ECON. 764 (2003).
50. Andrew Abere, Oral Capps, Jr., Jeffery R. Church & H. Alan Love, *Mergers and Market Power: Estimating the Effect on Market Power of the Proposed Acquisition by the Coca-Cola Company of Cadbury Schweppes' Carbonated Soft Drinks in Canada*, *in* MEASURING MARKET POWER 233 (Daniel J. Slottje ed., 2002); Jean-Pierre Dubé, *Product Differentiation and Mergers in the Carbonated Soft Drink Industry*, 14 J. ECON. & MGMT. STRATEGY 879 (2005).
51. Richard, *supra* note 10.
52. Marc Ivaldi & Frank Verboven, *Quantifying the Effects from Horizontal Mergers in European Competition Policy*, 23 INT'L J. INDUS. ORG. 669 (2005).

Mergers with localized competition. In some circumstances, location is an important element in product differentiation. For example, competition among supermarkets, retail gasoline stations, movie theaters, and others have important geographic dimensions. Merger simulation can be used to address some of the competitive questions that arise in these contexts.

Specification of cost structure and the nature of strategic interaction with localized competition is analogous to that developed above; however, specification of demand needs to incorporate the effects of distance and location. A logical starting point for demand describes the relative preference for each store based on the characteristics, including distance, of each store for each consumer. A logit model, perhaps with nests, provides such a demand system. The basic idea is to specify a logit model for each consumer where the probability of demand from each consumer location depends on the prices at the alternative stores, the characteristics of the stores (such as size, number of screens, brand, etc.), distance from the consumer location to the stores, and the characteristics of the consumer and location. Aggregate demand for each store is computed by summing the demands at each customer location.[53] With this specification, differences in demand for different stores arise because of the geographic distribution of consumers, location of nearby alternative stores, the demographic characteristics of local consumers, and other characteristics of the local environment.

A number of recent papers have estimated such models for various retail products. This literature includes analysis of a number of industries, including video retailing,[54] fast food,[55] movie theaters,[56] eyeglasses,[57] and coffee houses.[58] The studies provide various estimates of the amount consumers are willing to trade off distance against price (generally about \$1 to \$4 per mile, but as a matter of economic theory the trade-off would depend on the cost of travel and the relative value of the purchase, among other things).

Recent studies have also begun to analyze competitive questions that arise in this context. For example, Froeb, Tschantz, and Crooke examine demand for parking spaces and consider the effects of capacity constraints on competition through simulations.[59] Thomasden considers the unilateral effects of certain mergers in the fast food industry using merger simulation.[60]

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53. This is a mixed logit model in the aggregate.
54. Katia Seim, Spatial Differentiation and Market Structure: The Video Retail Industry (2000) (unpublished Ph.D. dissertation, Yale University).
55. Raphael Thomasden, Price Competition in Industries with Geographic Differentiation: Measuring the Effect of Location on Price in the Fast Food Industry (Columbia Business School, Working Paper, 2001).
56. Peter J. Davis, *The Effect of Local Competition on Admission Prices in the United States Motion Picture Exhibition Market*, 48 J.L. & ECON. 677 (2005).
57. Randal Watson, Product Variety and Competition in the Retail Market for Eyeglasses (University of Texas at Austin, Working Paper, 2004).
58. Bryan McManus, Nonlinear Pricing in an Oligopoly Market: The Case of Specialty Coffee (Washington University, Olin School of Business, Working Paper, May 2006).
59. Froeb et al., *supra* note 44.
60. *See* Thomasden, *supra* note 55.

When multiple stores are controlled by the same firm (e.g., supermarket chains and movie theaters), proposed mergers create a number of potential competitive issues. In some cases, competitive problems can be addressed through divestiture of certain locations. Fung and Zona analyze appropriate store divestitures in proposed mergers with localized competition based on simulations.[61] They also propose other behavioral solutions to competitive problems (e.g., agreements by the acquiring firm to charge the same prices across its similar stores in a particular area).

In general, merger simulation in industries with localized competition is a tool that can be used to identify locations with competitive problems and prioritize them. Simulation in this context also provides a way of focusing negotiations between merging parties and antitrust regulators regarding consent decrees.

Details. Competent merger analysis requires the resolution of a number of issues (as occurs with any competent empirical analysis). Some assumptions of the simulation cannot be tested given available information. In that case, another method for assessing the impact of a particular modeling choice on the predictions of the model is to perform a sensitivity analysis.[62] A sensitivity analysis involves examining whether the predictions of the model change substantially when a particular modeling choice is replaced with various reasonable alternatives. It should be noted that for a critique of a modeling choice to be given any weight, it must at the very least be accompanied by a sensitivity analysis that demonstrates that the model predictions are sensitive to the particular modeling choice made. A critique based only on the fact that a modeling choice is "unrealistic" should be given no weight.

It is often the case that the parameters of the demand functions on which a merger simulation is based have been econometrically estimated. Because these parameter estimates are subject to sampling variation, the predictions of the simulation are subject to sampling variation as well. It is useful to have a measure of the statistical precision of the simulation predictions. This can be done in the usual way using standard errors. Well-developed econometric methods exist for calculating these standard errors. From the beginning, merger simulation studies have provided standard errors and conducted statistical hypothesis testing for the predicted price effects of mergers.[63]

Merger simulations are typically performed assuming that the products and the products' characteristics are fixed. Yet, repositioning of existing products or entry by new firms, in response to a postmerger price increase, may in part eliminate any

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61. Chienyo Fung & J. Douglas Zona, Competitive Analysis with Localized Competition (2005) (unpublished paper).

62. *See, e.g.*, Gregory J. Werden, Luke M. Froeb & David T. Scheffman, *A* Daubert *Discipline for Merger Simulation*, ANTITRUST, Summer 2004, at 89.

63. *See, e.g.*, Hausman et al., *supra* note 15, at 176; Hausman & Leonard, *supra* note 36, at 336. These papers use the delta method to calculate the standard errors, despite the statement of Capps et al., *supra* note 42, at 12, that use of the delta method in this context would be "highly complicated, or impossible." Capps and his colleagues suggest the use of another method, the bootstrap, for obtaining standard errors.

such increase at least in the long run.[64] In this sense, the typical merger simulation predicts the upper bound on long-run competitive effects. On the other hand, the merger may affect the entry and expansion strategies of competitors postmerger. In principle, repositioning or entry could be built into the merger simulation by specifying a more complex model, although this normally has not been done in practice to date. In one exception, the Department of Justice studied the impact of entry by regional Bell operating companies on long-distance markets in the context of the proposed MCI WorldCom-Sprint transaction.[65] In that case, the demand model used allowed the probability of switching long-distance provider to depend on current provider choice and a set of available alternatives and their characteristics. In that model, the effects of changes in the characteristics of the set of available alternatives allowed postmerger pricing to be evaluated in the context of new entry.

Many differentiated products are sold by the manufacturers to retailers who then sell to final consumers. Thus, the manufacturers set the wholesale prices at which they sell the products to retailers, but the manufacturers do not directly set the prices that the retailers charge to retail consumers. Moreover, the pricing relationship between the manufacturers and the retailers is sometimes complex, involving nonlinear elements such as slotting allowances. Merger simulation studies have tended to ignore the existence of the retailers and assumed, in essence, that the manufacturers set retail prices. This assumption is valid under certain types of retailer behavior, e.g., if retailers calculate retail prices by taking the wholesale price and adding on a fixed percentage or dollar markup.[66] In some circumstances, the nature of retailer behavior can be determined. For example, in his analysis of a merger of bread manufacturers, Werden found that supermarkets marked bread products up by a constant percentage and thus concluded that the role of the supermarkets could effectively be ignored in the merger simulation.[67] In general, however, examination of the wholesale-retail relationship and the necessity of including a model of this relationship as part of a merger simulation is worthy of future research.[68]

Aggregation issues also must be addressed in the context of merger simulation. For example, it may be reasonable to aggregate weekly data to monthly or quarterly in order to better account for consumer inventorying behavior. Similarly, aggregation of demand over products (such as different package sizes) may be reasonable if prices for different package sizes tend to change together. Competent

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64. Rivals may not have the incentive to reposition toward the products of the merging firms and some research has found evidence of this. *See* Amit Gandhi, Luke M. Froeb, Steven Tschantz & Gregory J. Werden, *Post-Merger Product Repositioning*, 56 J. INDUS. ECON. 49, 67 (2008).

65. For a general description of the analysis, see ABA SECTION OF ANTITRUST LAW, ECONOMETRICS 422-23 (2005).

66. *See* Hausman & Leonard, *supra* note 31.

67. *See* Werden, *supra* note 39.

68. Several papers study the impact of various relationships between the manufacturer and a monopoly retailer. *See* Luke Froeb, Steven Tschantz & Gregory J. Werden, *Vertical Restraints and the Effects of Upstream Horizontal Mergers*, *in* THE POLITICAL ECONOMY OF ANTITRUST 369 (Vivek Ghosal & Johann Stennek eds., 2007); Daniel P. O'Brien & Greg Shaffer, *Bargaining, Bundling, and Clout: The Portfolio Effects of Horizontal Mergers*, 36 RAND J. ECON. 573 (2005).

analysis will consider the effects of various aggregation choices on outcomes of the simulation.[69]

As discussed above, the static Nash-Bertrand model is the most commonly used to represent the nature of strategic interaction between firms in merger simulations for differentiated product industries. However, other models could be used. Other potential static models include joint profit maximization (i.e., "perfect tacit collusion") and some form of leader-follower behavior. In principle, supergame models or dynamic models could be used as well. A useful area for future research would be investigating the fit of such models as compared to Nash-Bertrand. As also was discussed above, nonprice aspects of competition such as advertising are generally ignored (i.e., presumed to be held constant postmerger) in a merger simulation. It would be useful to investigate the importance of incorporating these aspects into a merger simulation.

Summary. Merger simulation is a potentially powerful tool and with competent analysis can be used to address a number of competitive questions that arise in the context of mergers. Just as merger simulation can be used to predict the effects of a merger, it can also be used to predict the effects of a divestiture or the effects of a merger and subsequent divestiture. The antitrust agencies may require a divestiture of one or more brands (or locations) to remedy potential anticompetitive effects of a proposed merger. Merger simulation techniques can be used to identify the brands or locations whose divestiture would constitute an appropriate remedy.[70]

3.3. Other approaches in differentiated products industries

A number of short-cut approaches to merger simulation have been proposed that allow one to make inferences based on limited data and without econometric analysis. At the core, each approach in this section takes a restricted version of some more general (demand) model and calibrates the model to the particular competitive situation using limited data. The advantage of these approaches is that they can be implemented very quickly and inexpensively. Of course, the disadvantage is that the restrictions and limited connection to the actual market data may make the model a very poor tool for inferring changes in incentives. Often it is difficult to weigh the advantages and disadvantages of these short-cut approaches without performing additional tests and analysis requiring data that may not be available.

Basic logit model. Using very limited data and a series of assumptions, the basic logit model (BLM) is the starting place for short-cut methods in simulation.[71]

69. Nielsen and IRI only provide retail scanner data that is aggregated to a certain degree. For example, data are typically not provided at the level of the individual store. Thus, the economist does not have complete control over the level of aggregation in most cases. There are, however, exceptions. For example, Fei Deng, Measuring the Substitution Effects of Sales Promotions in Supermarkets: An Analysis Based on a Dynamic Model of Differentiated Products (NERA, Working Paper, 2006), estimates a demand model on an interesting dataset that combines purchasing data from individual households with price and promotions data from the stores at which these consumers purchased.

70. *See* Jayaratne & Shapiro, *supra* note 45.

71. *See* Werden & Froeb, *supra* note 35, at 409-12.

Application of the BLM does not require detailed data, or sophisticated econometric modeling. All that is required to compute unilateral price effects under the BLM assumptions are revenues, prices, and a cost-price margin for one product (or an industry own-price elasticity of demand). These data are often easily assembled. An estimate of price-cost margin often can be obtained from accounting data. If the information to derive a price-cost margin estimate is not available, an industry demand elasticity can be used instead of a price-cost margin. While obtaining an estimate of the demand elasticity may be problematic, the BLM can be recomputed with a range of reasonable values for industry elasticity if one cannot be obtained from marketing studies, academic literature, or direct estimation.

At the core of the BLM is the set of assumptions about demand. The underlying assumption is that demand is characterized by multinomial logit (MNL).[72] Under the MNL assumptions, (quantity) shares, denoted w_i,[73] and own- and cross-price elasticities for the n products are

$$\begin{aligned} w_i &= \frac{\exp(\alpha_i + \beta p_i)}{\sum_j \exp(\alpha_j + \beta p_j)} \\ e_{ii} &= \beta p_i (1 - w_i) \\ e_{ij} &= -\beta p_i w_j \end{aligned} \tag{8}$$

Given data on prices and shares, own- and cross-price elasticities are all determined, up to a single proportionality factor, β. As developed above, under the premerger Nash-Bertrand behavioral assumption, margin is related to own price elasticity[74] according to

$$\begin{aligned} m_i &= \frac{p_i - c_i}{p_i} = -\frac{1}{e_{ii}} \\ m_i &= \frac{-1}{\beta p_i (1 - w_i)} \end{aligned} \tag{9}$$

which can be used to solve for β using data on a single firm's margin, price, and share (or elasticity, price, and share).[75] Alternatively, an industry (or group) price elasticity can be used to solve for β.[76] In turn, all the α_i terms can be solved for using

72. The logit model, when properly estimated from actual data, may be a reasonable starting place, but as discussed above it imposes the IIA restrictions on the ways products can substitute for one another.

73. Here, we use different notation to differentiate quantity shares from revenue shares. For simplicity of exposition, we assume there is no "outside good." The model is easily extended to include an outside good, and the qualitative nature of the results we describe below are unchanged.

74. Assuming the firm produces a single good, although the results are easily generalized.

75. If more than one margin is available, a joint test of Nash-Bertrand and the logit demand can be constructed by comparing multiple estimates of the underlying parameter β. If the model were extended to allow for an outside good, a second margin would be needed to identify the total market size, which in turn is needed to calculate shares when there is an outside good.

76. Assuming an equal percentage change in all group prices, the elasticity of demand for the group is

these data. Based on the assumed functional form, very basic data can be used to compute all own- and cross-price elasticities at the premerger prices and shares, as well as all the underlying parameters of the demand system.

The postmerger optimality conditions for a merger of two firms, which premerger were each producing a single good, can be written generally as

$$\frac{(p_1 - c_1)}{p_1} = -\frac{1}{\varepsilon_{11}} - \frac{(p_2 - c_2)}{p_2}\frac{\varepsilon_{21}}{\varepsilon_{11}}\frac{p_2 q_2}{p_1 q_1} \tag{10}$$

and

$$\frac{(p_2 - c_2)}{p_2} = -\frac{1}{\varepsilon_{22}} - \frac{(p_1 - c_1)}{p_1}\frac{\varepsilon_{12}}{\varepsilon_{22}}\frac{p_1 q_1}{p_2 q_2} \tag{11}$$

For the BLM, manipulation of the optimality conditions implies that the postmerger prices satisfy

$$p_1 - c_1 = p_2 - c_2 = -\frac{1}{\beta(1 - w_1 - w_2)} \tag{12}$$

These imply a system of two equations in the two unknown postmerger prices, which can be easily solved iteratively. Pre- and postmerger prices are compared to compute the estimated unilateral effects.

This comparison implies that the unilateral effects are

$$\begin{aligned} \frac{p_1 - p_1^0}{p_1^0} &= m_1^0\left[\frac{w_1 - w_1^0 + w_2}{1 - w_1 - w_2}\right] \\ \frac{p_2 - p_2^0}{p_2^0} &= m_2^0\left[\frac{w_2 - w_2^0 + w_1}{1 - w_1 - w_2}\right] \end{aligned} \tag{13}$$

where the zero superscript indicates premerger values.

The unilateral effects can be approximated by setting the postmerger shares equal to their premerger values. This approximation, while easy to perform, will result in an overestimate of the true unilateral effects under the BLM as the numerator is overstated and the denominator is understated.[77] Given the approximation, a share-weighted average price increase (WAPI) for the two products of the merging parties is

$$WAPI^0 = \frac{w_1^0}{(w_1^0 + w_2^0)}\frac{p_1 - p_1^0}{p_1^0} + \frac{w_2^0}{(w_1^0 + w_2^0)}\frac{p_2 - p_2^0}{p_2^0} \tag{14}$$

which can be rewritten as

$$e_{GG} = \beta \bar{p}(1 - \sum_{j \in G} w_j)$$

where $\bar{p}$ is a share-weighted price index for the group.

77. The approximation may well be extremely poor if BLM is the wrong model or if the postmerger shares would be substantially different than the premerger shares. Thus, we do not endorse the use of this approximation in any real application. We use it here simply to demonstrate a connection between the BLM and the 1992 *Merger Guidelines*.

$$WAPI^0 = \left[\frac{\left(m_1^0 + m_2^0\right)/2}{(1 - w_1^0 - w_2^0)\left(w_1^0 + w_2^0\right)} \right] \Delta HHI^0 \tag{15}$$

so that the WAPI is proportional to the change in Herfindahl-Hirschman Index (HHI) as calculated using premerger shares as described in the 1992 *Merger Guidelines*. This demonstrates the connection between the BLM and the method based on change in HHI in the *Guidelines*. Under the assumptions of the BLM, all else equal: large premerger shares, a large change in the HHI, and large premerger margins imply large estimated price effects.

Antitrust logit model. Werden and Froeb also proposed the antitrust logit model (ALM) as a reparameterization of BLM.[78] Like BLM, application of the ALM does not require detailed price and quantity data or sophisticated econometric modeling. All that is required to compute unilateral price effects under the ALM assumptions are revenues, prices, a price-cost margin for one product (or an own-price demand elasticity for the product) and an industry own-price demand elasticity.

In the BLM, one product normally is designated as the "everything else" or "no buy" option (also known as the "outside good") and substitution away from one of the parties' products is to both the outside good and other products.[79] In ALM, the specification allows more direct analysis of products within a group ("inside goods") and less focus on analysis of the outside good.

In the ALM, the industry own-price elasticity of demand is used to compute β and the margin or the product own-price elasticity of demand is used to compute what is the equivalent of the share (or probability) for the outside good in the BLM. The rest of the calculations follow similarly as with the BLM.

Nested logit model. The nested logit model (NLM) typically incorporates an additional parameter relative to the BLM or ALM, which controls relative substitution within a group versus across groups. In a typical application, the goods of the merging parties are included in a group of inside goods while the outside goods are specified as a single "no buy" alternative. The additional parameter is identified with one more piece of data (a premerger margin, the share of consumers who elect the no buy alternative, the industry elasticity, or a product-level elasticity).

In this section, we refer to a very simple nested logit structure as the basis for a short-cut method. In principle, a nested or mixed logit based demand model can be quite flexible and can require a great deal of data to estimate. Again, as the demand model becomes more flexible it requires more data to match the model to the facts.

PCAIDS. Proportionally calibrated AIDS (PCAIDS) is an alternative merger simulation method first developed around a restricted version of the AIDS.[80] Like BLM, ALM, and NLM, it can be used to estimate the unilateral effects of proposed

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78. *See* Werden & Froeb, *supra* note 35.
79. Sometimes a model will not include an outside alternative, as when customers must pick one from the set of available alternatives (for example, primary interexchange carriers).
80. *See* Epstein & Rubinfeld, *supra* note 23.

mergers using very little data. At the core of PCAIDS is a demand model built on diversion ratios. Diversion ratios have been defined as the fraction of sales diverted from one product to another in response to an increase in the price of the former product or some other change in circumstances. This concept has been interpreted in a variety of ways—fraction of units diverted, fraction of dollars diverted, and in other ways.[81] For our present purposes, we will work with fraction of diverted dollars:

$$d_{ij} = \frac{\partial R_i}{\partial R_j} \tag{16}$$

Thus, the diversion measures the change in revenue from sales of one product relative to the change in revenue from sales of another product. Revenue can be diverted from one product for a variety of reasons: revenues can move from one product to another from changes in advertising expenditures, other marketing efforts, or changes in relative tastes for the products. Diversions caused by relative price movements are particularly important in competitive analysis, and that is the focus of our analysis here. Price diversions measure the relative changes in expenditures on various products caused by price changes. Formally, the price diversion to product *i* arising from a change in the price of product *j* is

$$d_{ij} \equiv -\frac{\partial R_i}{\partial p_j} \Bigg/ \frac{\partial R_j}{\partial p_j} \tag{17}$$

A matrix of price diversions, D, with elements d_{ij}, can be constructed. Note that this matrix has *n* rows and *n* columns (where *n* is the number of products) and the diagonal elements of the matrix will all be –1.

Standard consumer demand theory implies that price diversion ratios can be expressed in terms of demand elasticities and vice versa:

$$d_{ij} = -\frac{s_i \varepsilon_{ij}}{s_j (1 + \varepsilon_{jj})} \tag{18}$$

Given the relationships, assumptions about diversions imply assumptions about own- and cross-price elasticities.

In the basic PCAIDS model, without nests, diversion ratios are assumed to be proportional to revenue shares. This assumption implies a particular structure on own- and cross-price elasticities so that just a few pieces of additional data are required to determine the values of all the own- and cross-price terms. Like the BLM and others, data on a price-cost margin or a single own-price elasticity are sufficient to compute all price elasticities at the premerger prices. Other assumptions about diversion ratios, including nests, are also considered, but again, all elasticities are implied, given the diversion ratios.

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81. *See, e.g.*, Gregory J. Werden, *Demand Elasticities in Antitrust Analysis*, 66 ANTITRUST L.J. 363, 414 (1998).

The AIDS demand system, under the assumption of homothetic consumer preferences,[82] can be written as

$$\Delta s_i = \sum_j b_{ij} \Delta \ln p_j \tag{19}$$

where Δ indicates the change in the corresponding variable between two time periods. AIDS demand implies a simple relationship between a given set of price elasticities of demand, an industry elasticity, e, revenue shares, and the underlying parameters of the AIDS demand system. For example, the own-price terms (b_{ii}) are given by

$$b_{ii} = s_i((\varepsilon_{ii} + 1) - s_i(e+1)) \tag{20}$$

while the cross-price terms are given by

$$b_{ij} = s_i(\varepsilon_{ij} - s_j(e+1)) \tag{21}$$

This completely characterizes the PCAIDS demand system.

Thus, price diversions (and an additional piece of information, such as a price-cost margin) imply elasticities which in turn imply the underlying parameters of the demand model. The model can be used in the usual fashion to predict unilateral effects. The basic PCAIDS has an advantage over BLM or ALM in that it does not require detailed price data.

A more flexible version of PCAIDS is extended to include nests of products.[83] Like the nested logit model, PCAIDS with nests is not particularly restrictive. Again, however, this flexibility comes at a cost, as more detailed data are required to apply (and estimate) the model.

Summary. A number of short-cut methods that do not require sophisticated econometric models and detailed data have been developed to provide quick guesses at unilateral effects. These models can provide an indication of the nature of the changes in incentives associated with a particular transaction involving differentiated products. However, the ease of use comes at the cost of using highly restrictive assumptions about the nature of consumer demand. When these assumptions are incorrect, these methods can yield misleading assessments of unilateral effects.

4. Merger simulation in homogeneous product industries

The previous section discussed the use of merger simulation to predict the unilateral effects of mergers in differentiated products industries. This section turns to the use of merger simulation in homogenous products industries. Merger simulation is less commonly seen in the homogenous products context.[84] This is in

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82. An implication of homothetic consumer preferences is that the individual products' shares of industry revenue do not change as industry revenue increases, holding constant the products' relative prices. Obviously, this assumption may not be reasonable in any particular application.

83. *See* Epstein & Rubinfeld, *supra* note 30.

84. *See, e.g.*, Jan Brueckner & Pablo Spiller, *Economies of Traffic Density in the Deregulated Airline Industry*, 37 J.L. & ECON. 379 (1994) (assuming product homogeneity within an origin-destination

part due to the fact that the conditions necessary for a merger in a Cournot industry—that is, an industry where firms simultaneously choose their outputs in a one-shot static game—to be consumer welfare enhancing can be derived explicitly.[85] However, a simulation is still needed if one wishes to predict the exact size of the price increase. This might be important because it is arguably not good antitrust policy to spend significant resources to stop mergers for which the predicted price increases are small.

As with differentiated products industries, the simulation of the postmerger world in a homogeneous products industry requires as inputs (1) the (market) demand function for the (homogeneous) product, (2) the firms' cost functions, and (3) the nature of strategic interaction between the firms. However, while the focus in a differentiated products industry is primarily on the demand functions for the various products, the focus in a homogeneous products industry is primarily on the firms' cost functions. The differentiation among firms in a homogeneous products industry arises due to differences in the firms' cost functions, not their products.

4.1. Simulating the postmerger world

We again start with the first-order conditions for profit maximization. For concreteness, we assume that postmerger firms engage in Cournot competition, although the same basic approach outlined below can be applied with other assumptions regarding the nature of firms' strategic interaction. Under Cournot, each firm chooses its quantity. Assuming n firms compete after the merger, the first-order conditions that define the Cournot equilibrium are

$$P(Q) + P'(Q)q_i - c_i(q_i) = 0 \qquad i = 1,...,n \tag{22}$$

where q_i is the quantity of firm i, $c_i(\cdot)$ is the marginal cost function of firm i, $Q = \Sigma_i q_i$ is total market quantity, and $P(\cdot)$ is the market inverse demand curve. Given functional forms for $c_i(\cdot)$ and $P(\cdot)$, the n first-order conditions constitute a system of n equations which can be solved for the n unknown quantities. The postmerger world can be simulated by calculating the equilibrium quantities and the resulting market price given the inverse market demand function and marginal cost functions for the $n-1$ nonmerging firms and the merging firm.

The parameters of the market inverse demand function can be econometrically estimated. As in the differentiated products case, the particular functional form that is chosen can have an effect on the outcome of the simulation. Statistical testing can help determine the appropriate functional form.

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pair, although this assumption is debatable); Eirik S. Amundsen & Lars Bergman, *Will Cross-Ownership Re-Establish Market Power in the Nordic Power Market?*, ENERGY J., Apr. 2002, at 73; Alfredo Garcia & Luis E. Arbeláez, *Market Power Analysis for the Columbian Electricity Market*, 24 ENERGY ECON. 217 (2002).

85. *See* Joseph Farrell & Carl Shapiro, *Horizontal Mergers: An Equilibrium Analysis*, 80 AM. ECON. REV. 107 (1990); Luke M. Froeb & Gregory J. Werden, *A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of a Homogeneous Product*, 58 ECON. LETTERS 367 (1998).

4.2. Cost functions

The problem of determining the marginal cost functions of the *n* postmerger firms can be broken into two parts. The first part is determining the marginal cost functions of the $n + 1$ premerger firms. Typically, the merger would not be expected to change the marginal cost functions of the $n-1$ nonmerging firms, so their marginal cost functions will be the same postmerger as premerger. The second part is determining how the premerger marginal cost functions of the two merging firms change in the postmerger world.

To determine the marginal cost functions of the premerger firms, we start by noting that, given knowledge of premerger quantities for the $n + 1$ premerger firms and the market demand function, we could solve the system of $n + 1$ premerger first-order conditions for the value of marginal cost function $c_i(\cdot)$ evaluated at q_i, $i = 1,...,n+1$. However, this does not allow us to recover the entire cost function unless marginal costs are constant. As an alternative, the marginal cost functions for the premerger firms could be estimated using econometric methods or calibration based on premerger data. This approach may be feasible for the merging firms, although the necessary data would likely not be available for the nonmerging firms. The problem of determining the cost functions of the nonmerging firms is likely another reason that merger simulations are rarely carried out in the homogeneous product context.

Even assuming that the premerger marginal cost functions of the two merging firms could be identified, we would still face the problem of determining how the premerger marginal cost functions of the merging firms would translate into the marginal cost function of the merged firm. Several different possibilities exist.[86] First, each merging firm could have its own production facility and these facilities could continue to operate separately after the merger. After the merger, the merged firm could reallocate its production across the two production facilities so as to minimize its total production costs. In the extreme case of constant marginal costs (and no binding capacity constraints), the merged firm would shut down the higher cost of the two production facilities and produce all of its output in the lower cost facility.[87] Second, consider a situation where economies of scale exist and the merged firm could feasibly combine the production assets of the two merging firms. In that case, the merged firm could lower its marginal cost for a given level of total firm output by combining its productive assets. Finally, consider the situation where know-how, trade secrets, or other knowledge from one merging firm could be applied to improve the production assets of the other merging firm. In that case, the merged firm would experience lower marginal costs on those assets.

In the first two cases (reallocation of production and economies of scale), knowledge of the premerger cost functions of the two merging firms would likely be sufficient to determine the cost function of the merged firm. In the third case (knowledge transfer), one would need to have the premerger cost functions as well as

86. *See, e.g.*, Farrell & Shapiro, *supra* note 85.

87. *See, e.g.*, R. Preston McAfee & Michael A. Williams, *Horizontal Mergers and Antitrust Policy*, 40 J. INDUS. ECON. 181 (1992).

some estimate of the extent to which the knowledge transfer would affect the cost functions postmerger.[88]

4.3. Alternatives to Cournot

Little research has been done on the use of alternatives to the Cournot model in merger simulation for homogeneous product industries.[89] One possible alternative is a "dominant firm-competitive fringe" model (where the dominant firm perhaps acquires some portion of the fringe).

One particularly fruitful area for future research would be the further development of static Bertrand models for homogenous product industries. To be interesting, such models require some feature, such as capacity constraints, that allows for equilibrium prices to be above marginal cost. The one such existing model considers a duopoly, which limits its usefulness as the framework for merger simulation.[90] Moreover, the equilibria of this model are difficult to summarize easily because they can involve mixed strategies.[91] Generally, however, it can be said that the equilibria exhibit high prices when demand is large relative to industry capacity and marginal cost pricing otherwise. This suggests that the capacity-constrained Bertrand approach may prove useful for modeling the type of cyclical pricing often observed in homogeneous product industries where firms cannot change their capacities in the short run in response to unexpected market demand shifts. The (short-run) effects of mergers could be simulated in such industries by determining the equilibrium of the model when the production assets of the two merging firms have been combined under common ownership.

5. Criticisms

The use of merger simulation has come under criticism. We address several of those criticisms here. Contrary to the beliefs of some critics, proponents of merger simulation do not argue that merger simulation should replace the "traditional" modes of antitrust analysis, e.g., a review of documents, taking depositions, conducting interviews with customers, and investigating institutional details. Indeed, proponents of merger simulation view the traditional modes of analysis as important complements (and in certain instances inputs) to merger simulation.

Merger simulation in turn complements the traditional modes by taking a scientific approach to merger analysis.[92] Merger simulation is scientific in that it is

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88. Another possible approach is to determine the marginal cost reduction that would be necessary to offset the price increase. *See* Froeb & Werden, *supra* note 85.

89. An exception is Brueckner & Spiller, *supra* note 84.

90. *See* Martin J. Osborne & Carolyn Pitchik, *Price Competition in a Capacity-Constrained Duopoly*, 38 J. ECON. THEORY 238 (1986).

91. A firm employing a "mixed strategy" for each play of the game draws a price out of a set of potential prices according to a probability distribution. This contrasts to a "pure strategy," where the firm plays a single price with probability one.

92. There are other types of analyses that we would also characterize as scientific that can be helpful in assessing the likely competitive effects of a merger. An example would be reduced-form analyses of the effects of natural experiments, which we discussed at the outset.

objective, quantitative, reproducible, and a measure of its precision can be derived. The traditional modes do not typically share these attributes. For example, in a merger simulation, the assumptions are clearly specified and can be subjected to testing. The result of the simulation is a quantitative estimate of the competitive effects accompanied by a measure of precision (e.g., a standard error). The results of the simulation are reproducible by another analyst. In contrast, two reasonable people reviewing the same set of documents could easily reach different conclusions and not be able to put their finger on why because the process is inherently subjective and assumptions are not well specified. In addition, a document review is unlikely to provide a quantitative estimate of the competitive effects, let alone a measure of precision of that estimate.

To say that merger simulation is scientific is not to say that it is a "perfected" technique. However, this is hardly a shortcoming since the same is true for virtually all scientific methods and theories. No scientific inquiry is ever "complete" and no scientific theory is ever "final." Indeed, the strength of the scientific method is that it explicitly provides a process by which the state of science evolves and improves as older methods and theories are replaced by newer and better ones. Thus, we expect that the techniques of merger simulation will evolve in the future. However, merger simulation (or any scientific method or theory) need not reach a state of "perfection" in order to be useful in the present.[93]

Some critics find merger simulation unhelpful because it cannot always be relied upon to come up with an "answer." This is a consequence of merger simulation being a science. The available data may be inadequate to fit the model, or the assumptions of a tractable simulation model might be rejected by the data. The traditional modes of analysis, in contrast, are (almost) always able to come up with an answer precisely because they are not scientific. Indeed, the usual outcome under the traditional modes is that there are two "answers"—one from each side of the debate with no scientific way to choose between them. While disputes may also arise between opposing experts regarding a merger simulation, the difference is that the source of these disputes is easily identified and the disputes can often be settled via scientific testing.

Another strand of criticism is that merger simulation abstracts too much from the actual institutional details of the industry under study. This criticism was addressed above in the discussion of how modeling choices (including which institutional details are explicitly captured in the model) should properly be tested. A model is properly tested by examining the accuracy of its predictions of economic outcomes, rather than the "realism" of its assumptions.[94]

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93. An example from the physical sciences is illuminating. Newton's Laws of Motion served humanity quite well for many years (and indeed still do) even though they are incorrect and have been replaced by the Theory of Relativity.

94. Two papers compare actual postmerger price changes to the predictions of a merger simulation. Peters examines several airline mergers and concludes that the simulations can explain a large percentage, though not all, of the actual postmerger price changes. *See* Craig Peters, *Evaluating the Performance of Merger Simulation: Evidence from the U.S. Airline Industry*, 49 J.L. & ECON. 627 (2007). Weinberg analyzes two consumer products mergers and concludes that the simulation

Finally, some critics argue that from a practical point of view attorneys will be hesitant to put forward merger simulation analyses because (1) unlike document review, merger simulation requires a degree of expertise that prevents attorneys from fully understanding it; (2) given the level of required expertise, use of merger simulation ultimately results in a "battle of the experts" who cancel each other out; and (3) since merger simulation has not been widely used in court cases, courts might be hesitant to put much weight on it. It is true that the law typically evolves somewhat more slowly than science. But it still evolves. For example, the case law in the area of patent infringement damages has become increasingly sophisticated from an economics point of view with regard to what is meant by "acceptable noninfringing substitutes." As discussed below, simulation methods are the next evolutionary step in this area. With regard to whether simulation is too new to court cases to receive much weight from judges, it is important to recognize that simulation is hardly new to economics. Indeed, the use of structural modeling to assess the likely effects of changes in policy or other exogenous factors has long been a fundamental tool of economics.

6. Simulation in other competitive contexts

Structural modeling is a powerful tool that can be used to address a variety of competitive questions. We consider some of these questions below.

6.1. Analysis of auction markets

A recent development is the use of merger simulation techniques to predict unilateral competitive effects of mergers or collusion in auction markets.[95] The primary focus is auctions where bidders have private values of the item being auctioned.[96] Situations where the private value aspect of the auction would be most relevant would include most procurement auctions where each bidder has a different level of expected profit associated with winning the contract depending on its particular cost, technology, capital stock, and such.

An agreement between two bidders in a given auction would tend to have an effect on the outcome of the auction when the agreement involved the two bidders with the lowest costs. Consider a second-price procurement auction in which the winner (the bidder with the lowest bid) receives a payment equal to the second lowest bid. In this type of auction, the dominant strategy of each bidder is to bid its

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overpredicts the extent of price changes. *See* Matthew C. Weinberg, An Evaluation of Merger Simulations (Princeton University, Working Paper, 2005). While no broad conclusions can be drawn from these studies, more studies of this type would be useful.

95. *See, e.g.*, Steven Tschantz, Philip Crooke & Luke Froeb, *Mergers in Sealed Versus Oral Auctions*, 7 INT'L J. INDUS. ORG. 201 (2000); Serdar Dalkir, John Logan & Robert T. Masson, *Mergers in Symmetric and Asymmetric Noncooperative Auction Markets: The Effects on Prices and Efficiency*, 18 INT'L J. INDUS. ORG. 383 (2000).

96. In contrast, a common value auction is one in which the item being auctioned has a common, but uncertain, valuation to all bidders. An example where common value aspects would tend to dominate might be art auctions where the artwork will be resold in the future.

cost. The bidder with the lowest cost wins and earns a profit equal to its cost advantage over the bidder with the second-lowest cost. If the two bidders with the lowest cost were to merge prior to the auction, the combined firm would win the auction and earn a profit equal to its cost advantage over the third-lowest cost. Thus, the merger eliminates the closest competitor to the lowest cost bidder (i.e., the second-lowest cost bidder) and thereby allows the lowest cost bidder to achieve greater profit.

Collusion and mergers in auction markets can be simulated in much the same way as mergers in differentiated products industries. Distributions for the bidder costs are specified. The parameters of these distributions are fit to data on actual auction outcomes. The cost distribution under the agreement is determined (based on the cost distributions of the conspiring firms) and the expected postagreement winning bid can be calculated and compared to preagreement expected winning bids.

6.2. Estimating damages in patent infringement litigation

Another potential application for simulation methods is evaluating the lost profits damages to patentees in patent infringement litigation.[97] Under the patent law, a patentee is entitled to recover as damages the profits it lost (if any) as a result of having to face competition from the infringer. A patentee's lost profits can arise through, among other things, lost sales of its product as well as a diminution in the price that it is able to charge. From the point of view of an economist, a patentee's lost profits damages can be calculated as the difference between its profits "but for" the infringement and its actual profits. Determining the patentee's profits but for the infringement requires a "reconstruction" of the but-for world, which is the world as it would have been had the infringement not occurred.

The but-for world in a patent infringement analysis is similar to the postmerger world in a merger analysis. The but-for world is not observed but can be simulated to predict the economic outcomes that would have occurred. Specifically, a simulation can be used to determine what consumers would likely have done if the infringing product had not been available for purchase and how the infringer's rivals (including the patentee) would likely have priced their products had they not faced competition from the infringer. This information can then be used to calculate the patentee's profits in the but-for world.

In the but-for world, the infringer's product would not have been on the market.[98] A product not being on the market is conceptually equivalent to its price being set so high than no consumer would want to purchase it (the price at which no one would

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97. *See* Gregory J. Werden, Luke M. Froeb & James Langenfeld, *Lost Profits from Patent Infringement: The Simulation Approach*, 7 INT'L J. ECON. BUS. 213 (2000); Gregory K. Leonard, *Applying Merger Simulation Techniques to Estimate Lost Profits Damages in Intellectual Property Litigation*, *in* ECONOMIC APPROACHES TO INTELLECTUAL PROPERTY POLICY LITIGATION AND MANAGEMENT 111 (Gregory K. Leonard & Lauren J. Stiroh eds., 2005).

98. We will discuss the most straightforward case where the infringer had no noninfringing product that it could have offered in place of the infringing product in the but-for world. However, simulation can address more complex situations as well.

purchase the product is called its reservation price).[99] The high price, like a lack of availability, would force consumers to make other choices and would eliminate the price constraint the infringing product placed on other competitors in the marketplace. Thus, we can simulate the but-for world by raising the price of the infringing product to its reservation level and seeing what would happen to consumer demand and the pricing incentives of rivals. In so doing, we would be able to identify how many additional sales the patentee would have made in the but-for world (lost sales) and the extent to which the patentee would have been able to charge a higher price (price erosion).

The inputs required for this type of simulation are essentially the same as the inputs required for a merger simulation: demand conditions, cost conditions, and the nature of the strategic interaction between the firms in the industry. Information on these inputs can typically be obtained from the same sources as with merger simulation.

Simulation encompasses the approaches typically used to calculate lost profits damages in patent infringement cases while also providing a significant advance beyond them. For example, under the market share rule, the patentee's lost sales are calculated by multiplying the infringer's sales by the fraction $s/(1-w)$, where s is the patentee's market share and w is the infringer's market share.[100] The market share rule is itself just a simulation of the but-for world, albeit under a particular set of assumptions. Specifically, if (1) consumer demand is specified to be of the logit form, (2) the market level own-elasticity of demand is zero, and (3) the rivals of the infringer would not change their prices in the but-for world, a simulation of the but-for world, i.e., an increase in the price of the infringing product to infinity (its reservation price in the logit model), results in the same outcome as the market share rule.

Refinements of the market share rule have focused on whether the infringing product and the patentee's product are in the same segment of the market.[101] If they are in the same segment, the patentee gets a fraction of the infringer's sales equal to the patentee's share of the segment, while if they are in different segments, the patentee gets no lost sales. For example, suppose the patentee's product is in the premium market segment, while the infringer's product is in the economy market segment. In that case, the refinements of the market share rule would assign no lost sales to the patentee.

Both the market share rule and its refinements depend crucially on how the "market" or the "segment" is defined. This leads to an "all-or-nothing" outcome. A product is either "in" so that it receives some share of the infringing sales, or it is "out" in which case it receives none of the infringing sales.

Yet, the all-or-nothing approach in many cases will not correctly reflect the actual nature of competition. For instance, in general some competition will exist between

99. In this sense, it is the reverse of a new product introduction, which has been analyzed in a similar fashion. *See* Hausman & Leonard, *supra* note 31.

100. This approach was first used in *State Industries v. Mor-Flo Industries*, 883 F.2d 1573 (Fed. Cir. 1989).

101. BIC Leisure Prods. v. Windsurfing Int'l, 1 F.3d 1214 (Fed. Cir. 1995); Crystal Semiconductor Corp. v. Tritech Microelectronics Int'l, 246 F.3d 1336 (Fed. Cir. 2001).

segments. Thus, even if the infringer is in one segment and the patentee in another, one should expect some level of lost sales for the patentee, even though this level may be well below what would be indicated by the patentee share of the overall "market." Similarly, even if the infringer and the patent owner are in the same segment, we would expect as a general matter some of the infringing sales to go outside the segment as long as there existed some competition between the segments.

A second issue with the market share rule and its refinements is that they make strong assumptions on the own- and cross-price elasticities of demand. Specifically, they adopt some form of IIA (either within the market or within the segment) and thus they assume that market shares completely characterize the extent of competition between the products.

Simulation provides the opportunity to use a more nuanced and sophisticated approach. With a properly specified simulation, products need not be ruled entirely in or entirely out in order to assign infringing sales in the but-for world. Instead, the products are assigned more or fewer of the infringing sales based on the extent of their competition with the infringing product. However, the trade-off to introducing more sophistication into the analysis is that simulation has substantially higher data and information requirements than the market share rule and its refinements. Identifying the extent of competition—rather than just determining whether two products are in the same market or market segment—requires an understanding of the structure of consumer demand for the products. However, when data availability makes simulation feasible, it will provide a more accurate calculation of lost profits damages.

6.3. Other applications

Simulation techniques can be used to address a variety of other questions, provided the underlying structure of the market interaction can be specified. For example, in a price-fixing context, simulation provides a way to demonstrate the effect of a narrow conspiracy, say covering two specific products, on prices for all interrelated products. This calculation is done in much the same way as merger analysis where the price of the affected products are set by agreement while the other product's prices are set unilaterally using the respective first-order conditions. The first-order conditions are changed postagreement to the extent changes in prices of demand for the price-fixed products affects demand (elasticity) for the other products. Any of the merger simulation models discussed above could be used to assess the impact of these changes on equilibrium price and to compute overcharges arising from the conspiracy.

Structural modeling is a powerful and useful tool with application beyond merger simulation.

7. Conclusion

As we have attempted to demonstrate, simulation has wide application in competitive analysis, from merger reviews to the calculation of damages in antitrust and patent infringement litigation. Moreover, simulation is commonly used by economists working in these contexts. This should not be surprising given that simulation has long been a fundamental technique in the economist's toolkit.

As with any scientific method, simulation is subject to continual refinements as further research is conducted. In the case of merger simulation, for example, it will be useful to investigate the importance of modeling such things as the role of retailers and manufacturer competition through advertising. Research will also proceed in the areas of econometric demand estimation. Thus, simulation can be expected to become ever more sophisticated.

However, we also believe that simulation, in its present state, is well-established and has much to offer to courts, government agencies, and others engaged in competitive analysis. Simulation provides a substantially more scientific and quantitative approach to competitive analysis than the commonly used alternative of making qualitative inferences based on a review of documents, deposition transcripts, and other discovery materials. Moreover, simulation can focus the areas of debate by forcing analysts to identify their beliefs regarding the key economic factors and to specify clearly the assumptions they have made. Simulation should not be used in place of the qualitative approach; rather, the two approaches are complements. Competitive analysis, in its various forms, is substantially improved by making it a practice to use both approaches when it is feasible to do so.