Class 9-10 slides

Unit 8. Competition Economics

Part 1. Demand, Costs, and Profits

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0. Opening Thoughts

Economics is common sense made difficult

To hide the fact that their discipline is no more than common sense, economists have created a thicket of esoteric mumbo-jumbo.
—Mail & Guardian (Mar. 13, 1998)

Economic science is but the working of common sense aided by appliances of organized analysis and general reasoning, which facilitate the task of collecting, arranging, and drawing inferences from particular facts.
—Alfred Marshall, Principles of Economics (1890)

Antitrust and economics

- The role of economics in antitrust
  - In per se violations, no need to prove actual or likely anticompetitive effect
    - So only the role for economics is proof of damages
  - In rule of reason violations, need to prove actual or likely anticompetitive effect
    - Economics is critical to predicting competitive effects
    - But very few rule of reason cases are investigated or litigated
    - Challenges are to practices that are already in place and can observe competitive effects
      - But still need economics for assessing the “but for” world
  - In monopolization or attempted monopolization cases, need to prove anticompetitive exclusionary conduct
    - Some role for economics in identifying anticompetitive exclusionary conduct
    - But relatively few Section 2 cases are investigated or litigated
    - Challenges are to practices that are already in place and can observe competitive effects
      - But still need economics for assessing the “but for” world
  - In merger cases, need to prove actual or likely anticompetitive effect
    - Economics is essential (under current law)
    - Many mergers are investigated and challenged
    - With the HSR Act, almost all are investigated prior to closing when likely effects must be predicted and not observed
    - Economics provides the principal tool for predicting likely future competitive effects both with and without the merger

More on motivation

- The purpose of merger antitrust law
  - Section 7 of the Clayton Act prohibits mergers and acquisitions that "may be substantially to lessen competition, or to tend to create a monopoly"[1]
  - In modern terms, a transaction may substantially lessen competition when it threatens, with a reasonable probability, to create or facilitate the exercise of market power to the harm of consumers
  - Operationally, a transaction harms consumer when it result in—
    - Higher prices
    - Reduced market output
    - Reduced product or service quality in the market as a whole, *or*
    - Reduced rate of technological innovation or product improvement in the market

    compared to what would have been the case in the absence of the transaction (the "but for" world) and without any offsetting consumer benefits

Merger antitrust analysis typically focuses on price effects (see Unit 2)

Consequently, a central focus in merger antitrust law is the effect a merger is likely to have on the profit-maximizing incentives and ability of the merged firm to raise price in the wake of the transaction. In the first instance, this requires us to know how a profit-maximizing firm operates. The basic tools to enable us to do this analysis is the subject of this unit. These same tools are also fundamental to an understanding of merger antitrust law defenses.

[1] 15 U.S.C. § 18.

Antitrust economics

- Three starting points
  1. The *law of demand*: Demand curves are downward sloping
  2. *Profit maximization*: Firms act to maximize their profits
  3. *Single price markets*: Identical products sell at the same price in the market
     - That is, firms cannot price discriminate among customers
- With these starting points, economics enables us to—
  1. Analyze the incentives and abilities of a profit-maximizing firm given the demand curve facing the firm (the *residual demand curve*)
  2. Analyze how the firm's residual demand curve might change with a merger
  3. Predict how the merged firm might act differently postmerger from the two merging firms premerger
  4. Predict the consumer welfare consequences of this change in behavior

Profit maximization

To begin the analysis, we must understand how a firm makes its choices of price, production level, and other operating variables to maximize its profits

Profit maximization

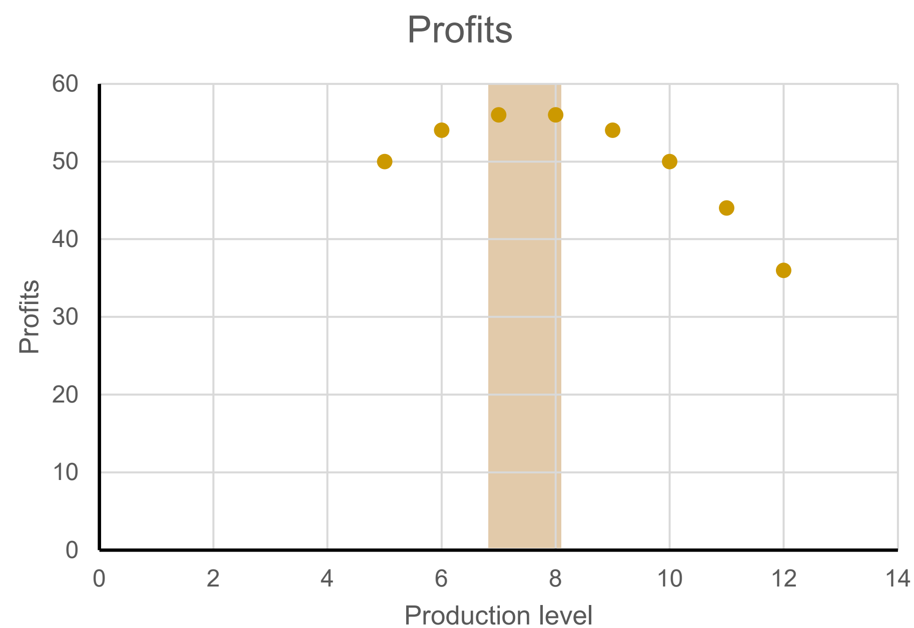
- Consider a very simple problem:
  - Avco makes widgets at a cost of $5 each
    - When Avco makes 5 widgets, it can sell out at a price of $15 per widget. Since Avco makes $10 on each widget, Avco makes profits of $50
  - Avco is thinking of increasing its production. It will do so only if this will increase its profits.
    - If Avco makes 6 widgets, it must drop its price to $14 to sell out. Since Avco makes $9 on each widget, Avco would now make profits of $54. Avco should increase its production.
  - Should Avco increase its production even more?
    - If Avco makes 7 widgets, it must drop its price to $13 to sell out. Since Avco makes $8 on each widget, Avco would now make profits of $56.
    - If Avco makes 8 widgets, it must drop its price to $12 to sell out. Since Avco makes $7 on each widget, Avco would now make profits of $56.
    - If Avco makes 9 widgets, it must drop its price to $11 to sell out. Since Avco makes $6 on each widget, Avco would now make profits of $54.
    - If Avco makes 10 widgets, it must drop its price to $10 to sell out. Since Avco makes $5 on each widget, Avco would now make profits of $50.
    - If Avco makes 11 widgets, it must drop its price to $9 to sell out. Since Avco makes $4 on each widget, Avco would now make profits of $44.

So Avco should increase its production to 7 (or 8) widgets in order to maximize its profits

Profit maximization

- We can see this on a graph:

Quantity	Price	Revenues	Cost	Profits
5	15	75	25	50
6	14	84	30	54
7	13	91	35	56
8	12	96	40	56
9	11	99	45	54
10	10	100	50	50
11	9	99	55	44
12	8	96	60	36



Profit maximization

- Let's look at this in another way that better illustrates the underlying economics
  - If Avco were to increase its production from 5 units to 6 units and drop its price from $15 to $14, two things would happen:
    1. Avco would gain an additional sale, *and*
    2. Avco would have to lower its price on all the units it would sell to clear the market
  - These two effects would have two consequences for Avco's profits:
    1. On the customer Avco gained, Avco would make an additional profit of $9
       - Additional sale of 1 unit times the profit margin of $9 (at a sales price of $14 and a unit cost of $5) equals $9 profit gain
    2. On its original sale, it would have to lower the price by $1 and so reduce its profits on those sales by $5
       - Original sale price of $15 minus the new sales price of $14 equals a $1 loss on each original sale
       - Five original sales times $1 loss on each sale equals a $5 profit loss
  - The change in Avco's profits is then:
    - The gain in profits from the additional sales at the new price ($9)
    - *Minus* the loss in profits from lowering the price on the original sales ($5)
    - For a net profit gain of $4

Rule: Avco should increase its production whenever the incremental profit gain is positive

Profit maximization

- Let's look at this in another way that better illustrates the underlying economics
  - Now if Avco were to increase its production from 10 units to 11 units and drop its price from $10 to $9, the same two things would happen:
    1. Avco would gain an additional sale
    2. Avco would have to lower its price on all the units it would sell
  - These two effects would have two consequences for Avco's profits:
    1. On the customer Avco gained, Avco would make an additional profit of $4
       - Additional sale of 1 unit times the profit margin of $4 (at a sales price of $9 and a unit cost of $5) equals $4 profit gain
    2. On its original sale, it would have to lower the price by $1 and so reduce profits on those sales by $10
       - Original sale price of $10 minus the new sales price of $9 equals $1 loss on each original sale
       - Ten original sales times $1 loss on each sale equals a $10 profit loss
  - The change in Avco's profits is then:
    - The gain in profits from the additional sales at the new price ($4)
    - Minus the loss in profits from lowering the price on the original sales ($10)
    - For a net profit loss of $6
  - Indeed, running the same analysis on a decrease in production from 10 units to 9 units would show that Avco would increase its profits

Rule: Avco should decrease its production whenever the incremental profit gain is negative

Profit maximization

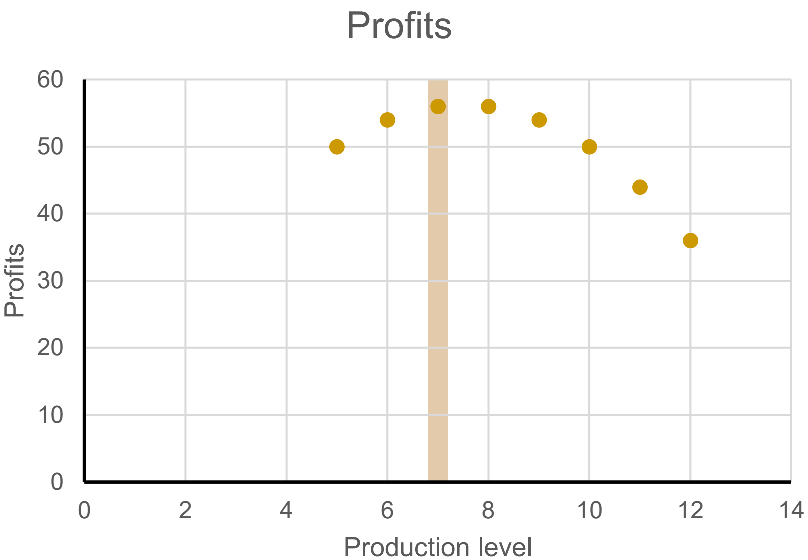
- Bottom line:

Avco maximizes its profit when its incremental profit is zero

  - *Incremental profit* is the profit earned on selling the *next* unit

- We can see this on the chart:

Quantity	Price	Revenues	Cost	Profits	Incremental Gain
5	15	75	25	50	4
6	14	84	30	54	2
7	13	91	35	56	0
8	12	96	40	56	-2
9	11	99	45	54	-4
10	10	100	50	50	-6
11	9	99	55	44	-8
12	8	96	60	36	



Profit maximization

- Some definitions
  - *Marginal sales*: Sales that are lost with an increase of one unit in price
    - *Marginal customers* are the customers connected with marginal sales
  - *Inframarginal sales*: Original sales that are retained when price increases
    - *Inframarginal customers* are the customers connected with inframarginal sales
  - *Marginal profit*: The net profits a firm would make by increasing its production by one unit
    - May be positive or negative
    - *Incremental profits* are the net profits a firm would make increasing its production by some specified amount (which may be more than one unit)
  - *Marginal revenue*: The net revenue a firm would earn by increasing its production by one unit
    - May be positive or negative
    - *Incremental revenue* are the net revenues a firm would earn increasing its production by some specified amount (which may be more than one unit)
  - *Marginal cost*: The net cost to the firm of increasing its production by one unit
    - Always positive
    - *Incremental costs* are the costs a firm would incur by increasing its production by some specified amount (which may be more than one unit)

Profit maximization

- Some important relationships
  1. At a profit maximum, marginal profits are zero
  2. Marginal profit is equal to marginal cost minus marginal revenue
  3. Therefore, to maximize profits, a firm operates
     so as to set its marginal revenue equal to its marginal cost
  4. For a linear inverse demand curve of the form $p = a + bq$,
     the marginal revenue curve is $mr = a + 2bq$
     - The parameter b will always be negative (since the demand curve is downward sloping)
  5. Marginal revenue can be decomposed into two parts:
     - The gross gain in profits from the sale of an additional unit at the new price, *and*
     - The gross loss in the profit margin from the sale of the inframarginal units at the new lower price

What you should be able to do after Part 1

For a firm—

- Facing a downward sloping residual (inverse) demand curve $p = a + bq$
- With fixed costs f and constant marginal costs c

1. Determine and graph the profit-maximizing levels of—
   - Output q^*
   - Price p^*
   - Profits π^*

   "*" indicates that the variable is at its profit-maximizing level

2. Determine and graph the net incremental revenue for a firm increasing output by some amount Δq, including—
   - The gross gain in revenues from the increase in output, and
   - The gross loss in revenues from the reduction of price for sales at the original price

   "Δ" indicates the change in the variable

3. Derive and graph an inverse demand curve given a demand curve

1. Profit Maximization

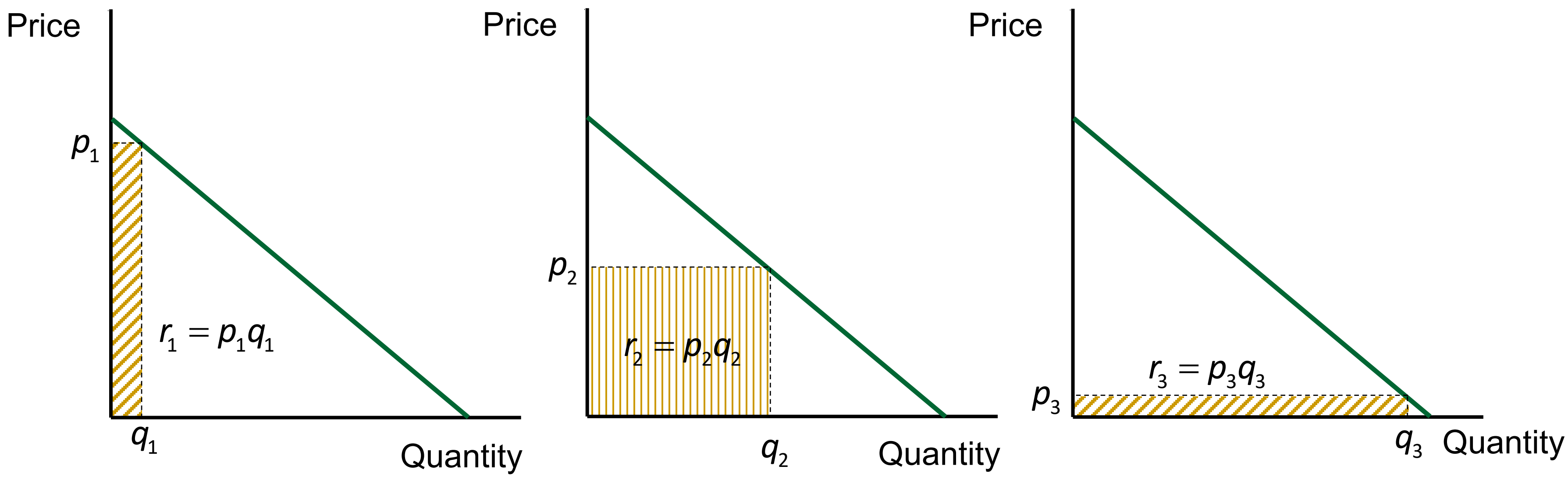
An observation by Dave Berry

Later on, Newton also invented calculus, which is defined as “the branch of mathematics that is so scary it causes everybody to stop studying mathematics.” That's the whole point of calculus. At colleges and universities, on the first day of calculus, professors go to the board and write huge, incomprehensible “equations” that they make up right on the spot, knowing that this will cause all the students to drop the course and never return to the mathematics building. This frees the professors to spend the rest of the semester playing cards and regaling one another with stories about the “mathematical symbols” they've invented over the years. (“Remember the time Professor Hinkwattle drew a ‘cosine derivative’ that was actually a picture of a squid?” “Yes! Students were diving out the windows! From the fourth floor!”)[1]

[1] Dave Berry, *Up in the Air on the Question of Gravity,* Baltimore Sun, Mar. 16, 1997, at 3J.

Profits

1. When the firm produces output *q*, its profits $\pi(q)$ are equal to its revenues *r*(*q*) minus its total costs *t*(*q*):
$$\pi(q) = r(q) - t(q)$$
2. Revenues *r*(*q*) are equal to price *p* times output *q*:
$$r(q) = pq$$
3. Revenues can be shown as a rectangle in a price-quantity chart:

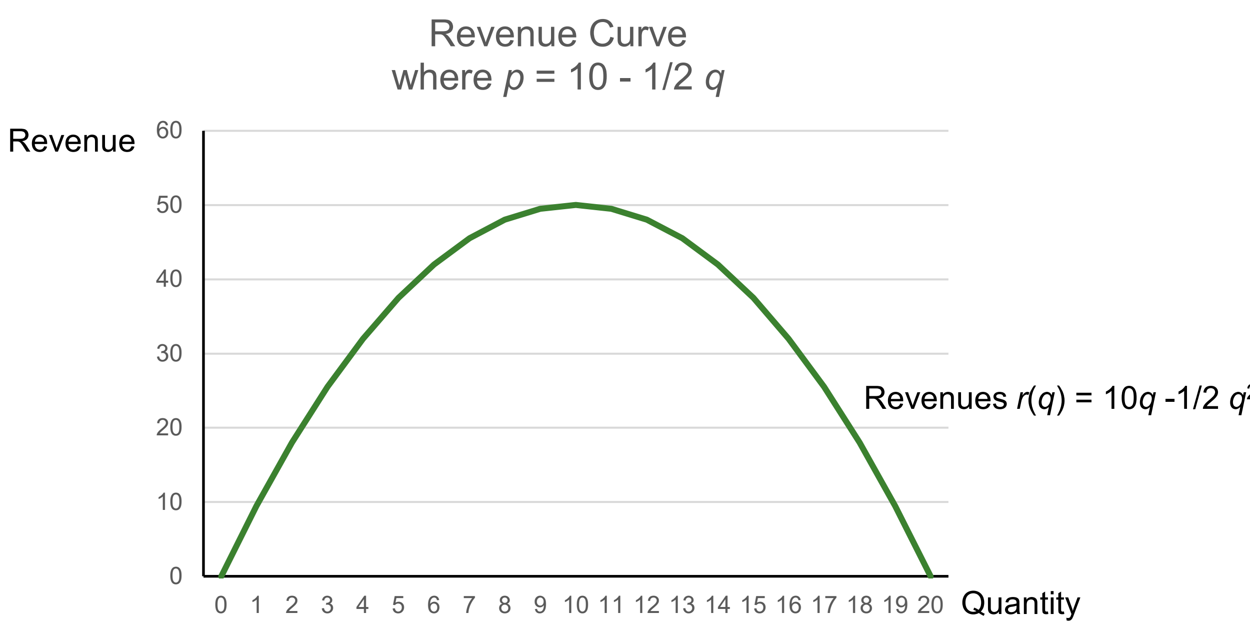


Profits

3. When the firm faces a downward-sloping residual (inverse) demand curve $p = a + bq$:

$$\begin{aligned} r(q) &= pq \\ &= (a + bq)q \\ &= aq + bq^2 \end{aligned}$$

- The graph of the firm's revenues as a function of q is a parabola:



Profits

4. At output q, total costs $t(q)$ are equal to fixed costs f plus variable costs $v(q)$:

$$t(q) = f + v(q)$$

- With *constant marginal costs c*, variable costs $v(q)$ are equal to marginal cost c times output q:

$$v(q) = cq$$

- Then total costs $t(q)$ may be expressed as:

$$\begin{aligned} t(q) &= f + v(q) && \text{generally} \\ &= f + cq && \text{in the case of constant variable costs} \end{aligned}$$

Profits

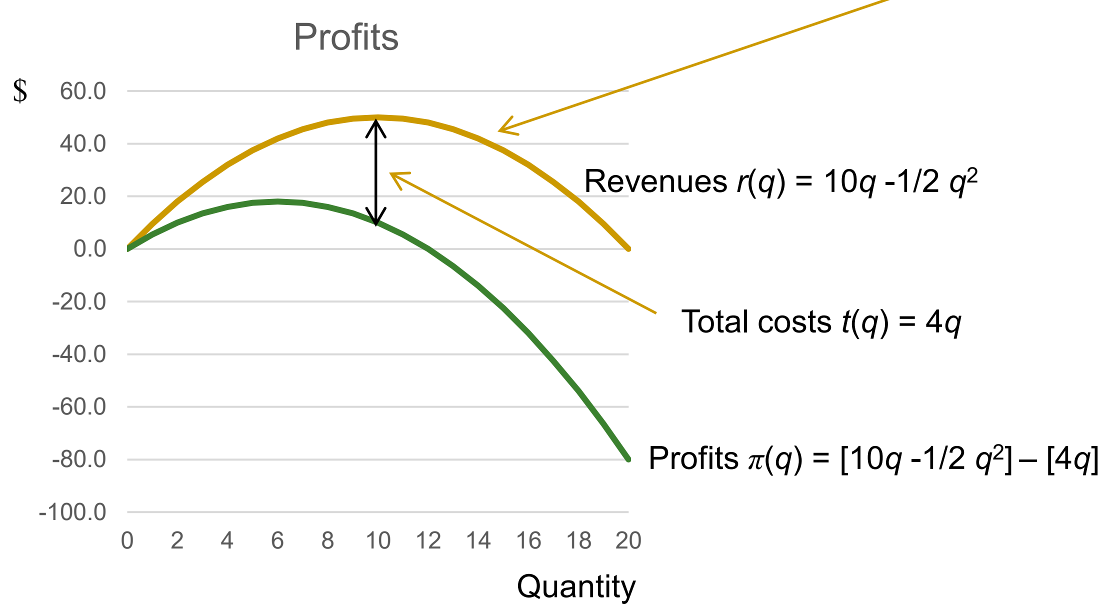
5. Now we can express total profits $\pi(q)$ as:

$$\pi(p) = r(q) - t(q)$$
$$= (a + bq)q - [f + cq]$$
$$= [aq + bq^2] - [f + cq]$$

Since this is a second-order polynomial, its graph is a parabola

- Graphically

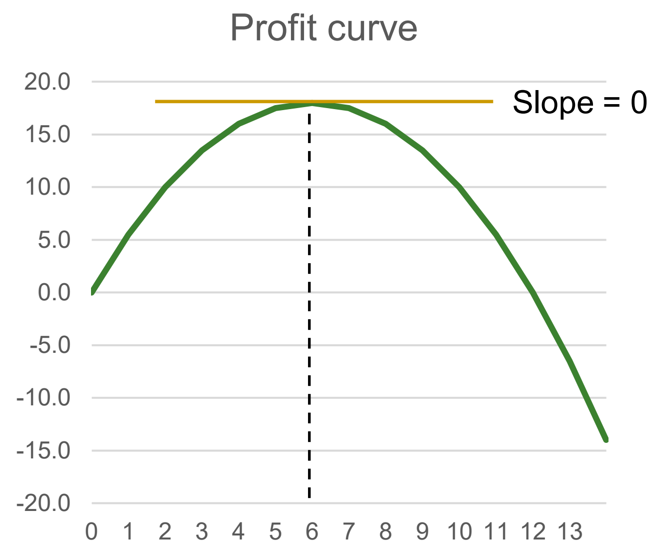
where
$p = 10 - ½ q$
$f = 0$
$c = 4$



Profit maximization

6. The slope at the top of the profit "hill" is zero (a horizontal line):

where
$p = 10 - ½ q$
$f = 0$
$c = 4$



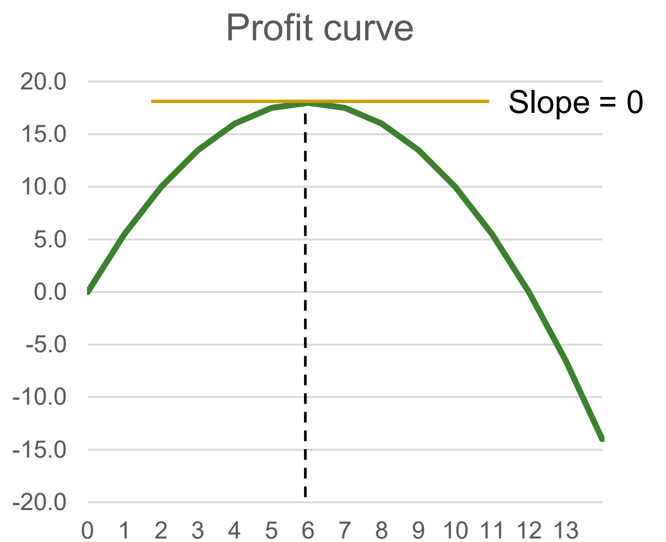
- Definition
  - The *slope of a line* is the change in the *y-values* (Δy) divided by the change in the *x-values* (Δx):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

  - The *slope of a curve* at a point is the slope of the tangent line at that point (as shown above)
    - *For calculus geeks*: The slope of a curve at a point is the *derivative* of the function at that point

Profit maximization

6. The slope at the top of the profit "hill" is zero (a horizontal line):



where
- $p = 10 - \frac{1}{2} q$
- $f = 0$
- $c = 4$

Solve the problem:

- From the chart, we see that the profit-maximizing output q^* is 6
- From the inverse demand curve, we can calculate $p^* = p(6) = 10 - (1/2)(6) = 7$
- $r^* = r(6) = p^*q^* = (7)(6) = 42$
- $f = 0$ (from the hypothetical)
- $v^* = v(6) = cq^* = (4)(6) = 24$
- $t^* = t(q^*) = f + v(q^*) = 0 + 24 = 24$
- $\pi^* = \pi(q^*) = r^* - t^* = 42 - 24 = 18$

Profit maximization

- Marginal analysis—Some definitions
  - The slope of the revenue curve at an output *q* is called the *marginal revenue mr*(*q*)
    - Think of marginal revenue as the revenue the firm would earn if it produced one *additional* unit
    - You can also think of the marginal revenue as the *rate of change* in revenue for an increase in output
    - If $r(q) = aq + bq^2$ (the revenue function for a linear inverse demand curve), then:

$$mr(q) = a + 2bq$$

In the continuous case—think of this as the *instantaneous rate of change* of revenue with respect to output

  - The slope of the total cost curve at an output *q* is called the *marginal cost mc*(*q*)
    - Think of marginal cost as the cost the firm would earn if it produced one *additional* unit
    - If $t(q) = f + cq$ (total costs with constant marginal costs), then:

$$mc(q) = c$$

  - The slope of the profit curve at an output *q* is called the *marginal profit* $m\pi(q)$
    - Think of marginal profit as the profit the firm would earn if it produced one additional unit
    - Marginal profit is marginal revenue minus marginal cost:

$$m\pi(q) = mr(q) - mc(q)$$

For calculus geeks: The marginal function is the derivative of the primary function. So, for example, the marginal revenue function is the derivative of the revenue function.

Profit maximization

OPTIONAL but well worthwhile. You should not be satisfied to be told the formula for the marginal revenue curve. You should want to understand its derivation from the definition of marginal revenue. This provides that explanation.

- Marginal analysis—Deriving the marginal revenue function (continuous case)
  - If $r(q) = aq + bq^2$ (the revenue function for a linear inverse demand curve), then:

    $$mr(q) = a + 2bq$$

    in the continuous case (that is, when one unit is infinitesimally small compared to firm output q)

  - *Proof*: Let q be the firm's output. Then marginal revenue is technically defined as:

    $$mr(q) = \frac{r(q+\Delta q) - r(q)}{\Delta q}, \quad \text{where } \Delta q = 1$$

    Substituting the inverse demand function for r and simplifying:

$$mr(q) = \frac{\left[a(q+\Delta q) + b(q+\Delta q)^2\right] - \left[aq + bq^2\right]}{\Delta q}$$

$$= \frac{\left[(aq + a\Delta q) + \left(bq^2 + 2bq\Delta q + b\Delta q^2\right) - \left[aq + bq^2\right]\right]}{\Delta q}$$

$$= \frac{a\Delta q + 2bq\Delta q + b\Delta q^2}{\Delta q}$$

$$= a + 2bq + b\Delta q$$

But if Δq is small compared to q, it may be ignored. So $mr(q) = a + 2bq$ in the continuous case. Q.E.D.

Profit maximization

- First order condition (FOC)
  - From Slide 22, we know that profits are maximized at the top of the profit "hill," which is where the slope of the profit curve is zero
  - From Slide 24, we know that the slope of the profit curve at an output q is the marginal profit $m\pi(q)$ evaluated at output q
  - From Slide 24, we also know that the marginal profit $m\pi(q)$ is equal to the marginal revenue $mr(q)$ minus the marginal cost $mc(q)$, all evaluated at output q, that is:

$$m\pi(q) = mr(q) - mc(q)$$

  - The *first order condition* for a profit-maximizing level of output q^* is that the marginal profit at q^* equals zero, that is:

$$m\pi(q^*) = mr(q^*) - mc(q^*) = 0$$

or equivalently: $$mr(q^*) = mc(q^*)$$

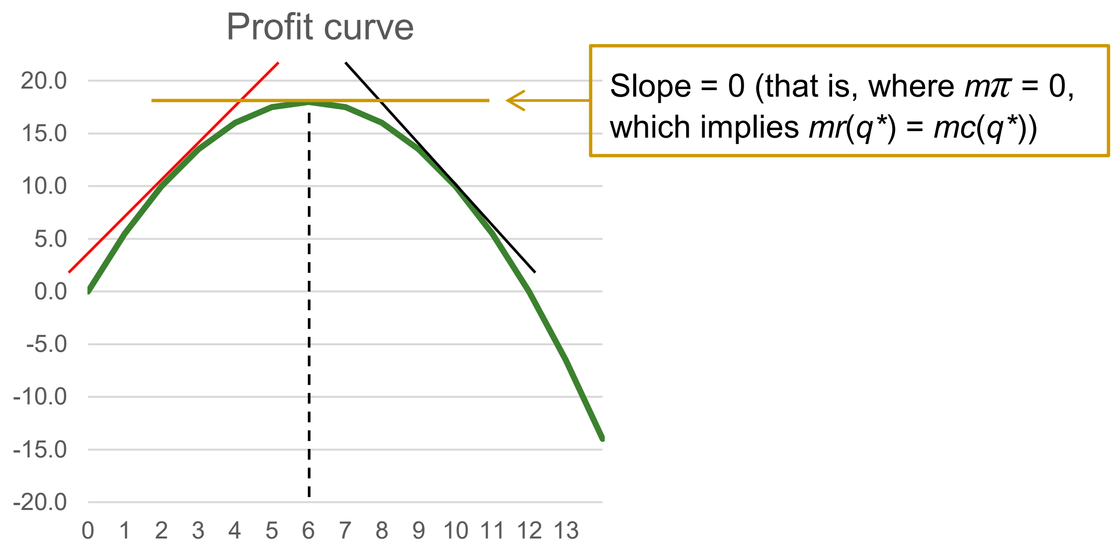
A profit-maximizing firm sets its production level q so that its marginal revenue is equal to its marginal cost

Profit maximization

- First order condition—Example

where

$p = 10 – ½ q$
$f = 0$
$c = 4$



- *Key concept*: Think of the slope as the *instantaneous rate of change* of profits with respect to output
  - If the slope is positive ($m\pi > 0$), then profits are increasing with increases in output
  - If the slope is negative ($m\pi < 0$), then profits are decreasing with increases in output
  - If the slope is zero ($m\pi = 0$), then a change in output in either direction will decrease profits (i.e., the firm is at a profit maximum)

Profit maximization

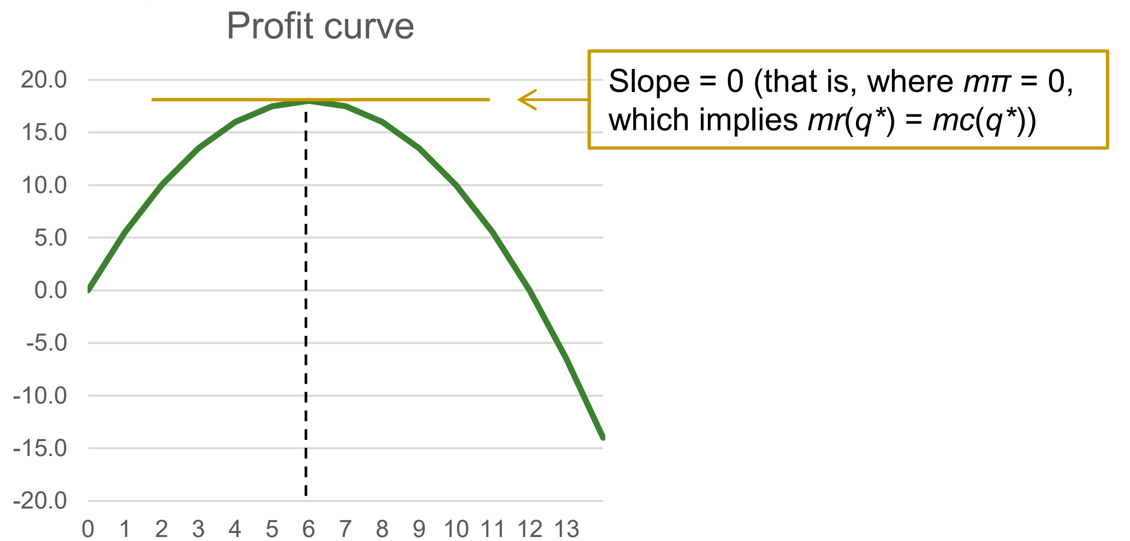
- First order condition—Example

where

$p = 10 - \frac{1}{2} q$

$f = 0$

$c = 4$

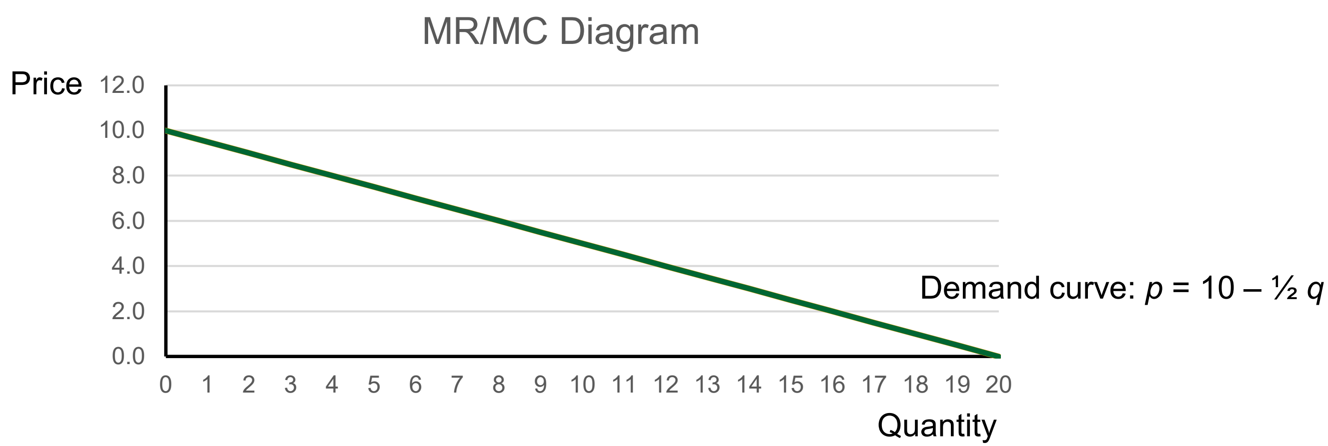


1. $r(q) = p(q)q = (10 - \frac{1}{2} q)q = 10q - \frac{1}{2} q^2$
2. $mr(q) = 10 - q$ (from the formula on Slide 14)
3. $mc(q) = 4$ (from the hypothetical)
4. FOC: $mr(q^*) = mc(q^*)$
   So $10 - q^* = 4$ or $q^* = 6$ (as shown in the diagram)
5. $p^* = p(q^*) = 10 - \frac{1}{2} q^*$
   $= 10 - (\frac{1}{2})(6) = 7$ (from the inverse demand curve)

Profit maximization

- Marginal revenue/marginal cost diagrams
  - Will build this step-by-step in five steps

→ a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$

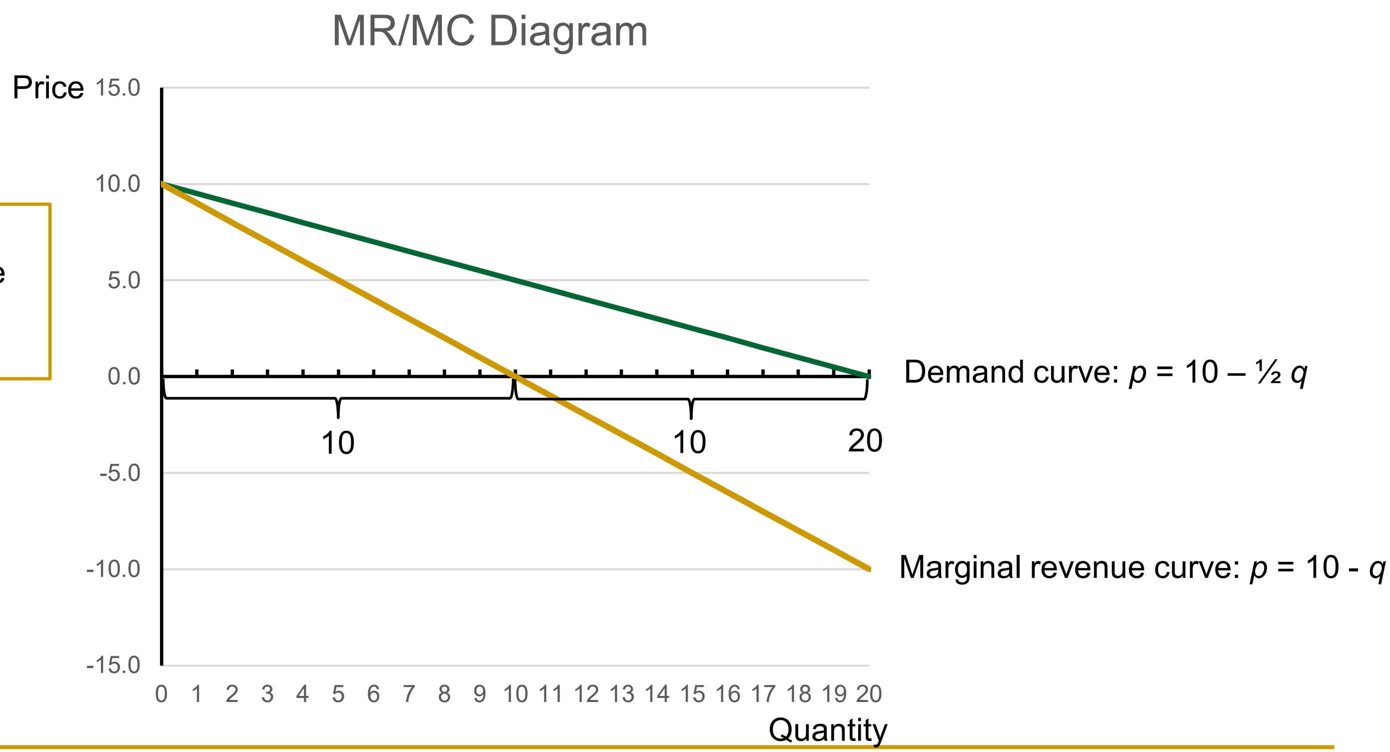


Profit maximization

- Marginal revenue/marginal cost diagrams
  - Will build this step-by-step

a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$

→ b. Add the marginal revenue curve: $p = 10 - q$



Note: With linear demand, the marginal revenue curve falls twice as fast as the inverse demand curve

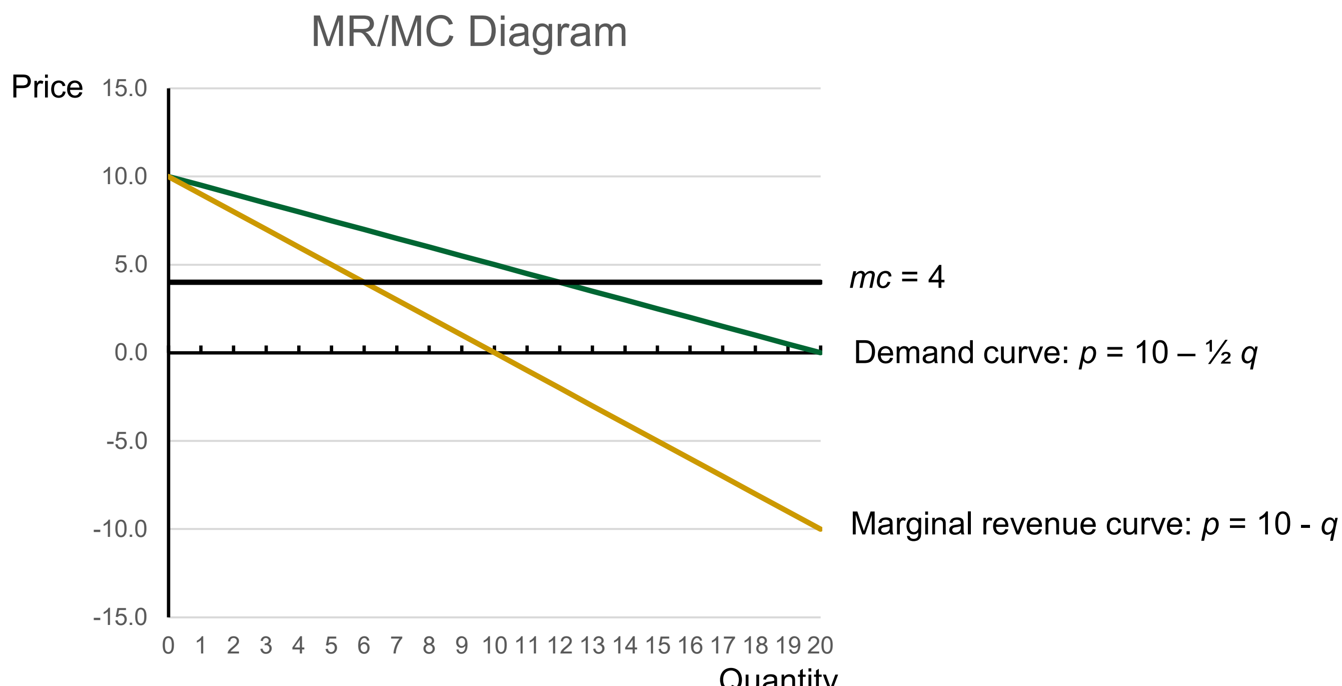
Profit maximization

- Marginal revenue/marginal cost diagrams
  - Will build this step-by-step

a. Consider an (inverse) demand curve: $p = 10 - ½ q$

b. Add the marginal revenue curve: $p = 10 – q$

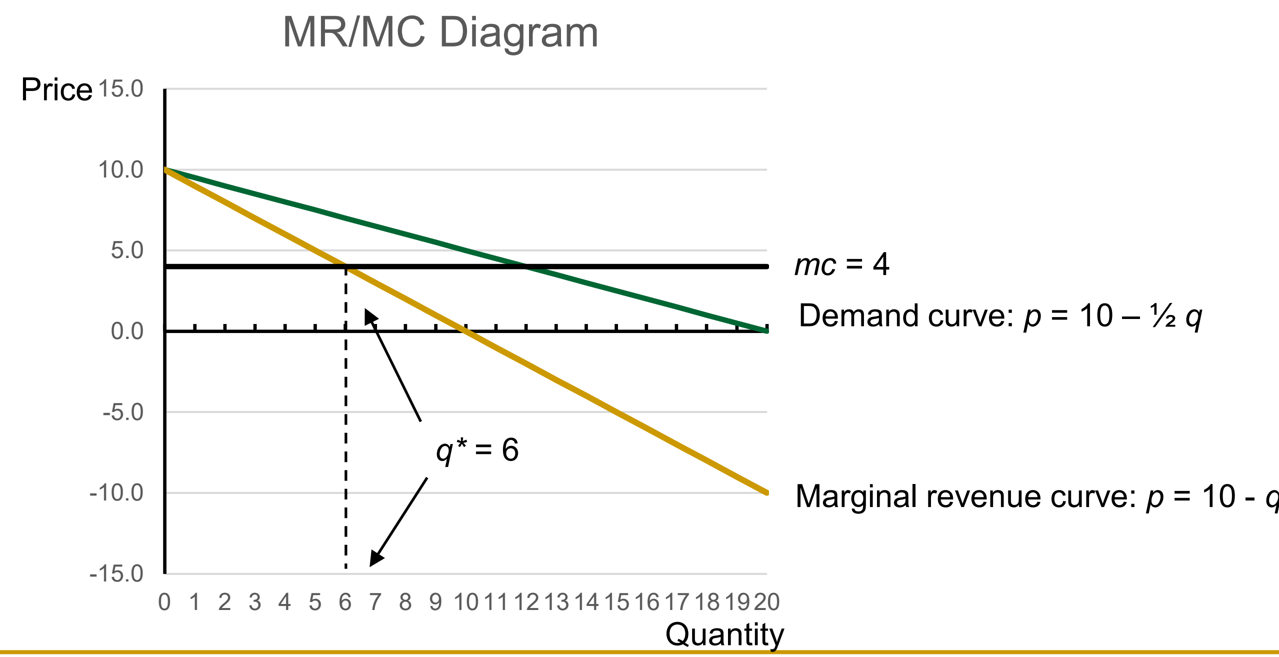
→ c. Add the marginal cost curve: $c = 4$ (constant marginal cost)



Profit maximization

- Marginal revenue/marginal cost diagrams
  - Will build this step-by-step

a. Consider an (inverse) demand curve: $p = 10 - ½ q$
b. Add the marginal revenue curve: $p = 10 – q$
c. Add the marginal cost curve: $c = 4$ (constant marginal cost)
d. Find intersection of *mr* and *mc* curves to determine profit-maximizing q^* (= 6)



Profit maximization

- Marginal revenue/marginal cost diagrams
  - Will build this step-by-step

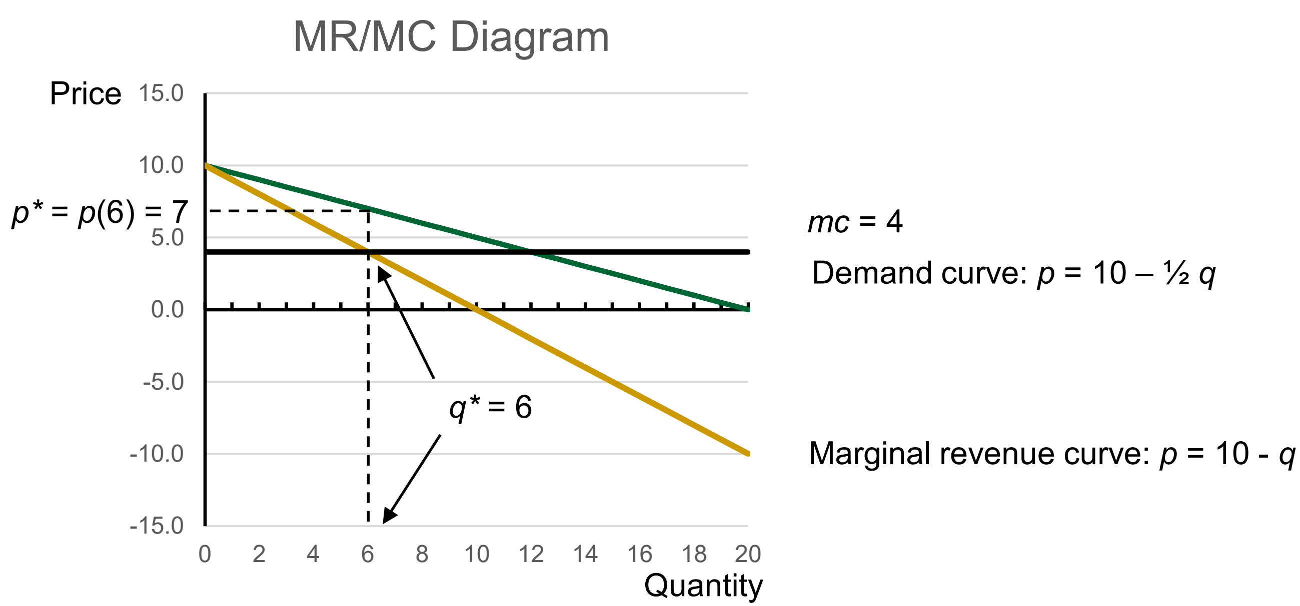
a. Consider an (inverse) demand curve: $p = 10 - ½ q$

b. Add the marginal revenue curve: $p = 10 – q$

c. Add the marginal cost curve: $c = 4$ (constant marginal cost)

d. Find intersection of *mr* and *mc* curves to determine profit-maximizing q^* (= 6)

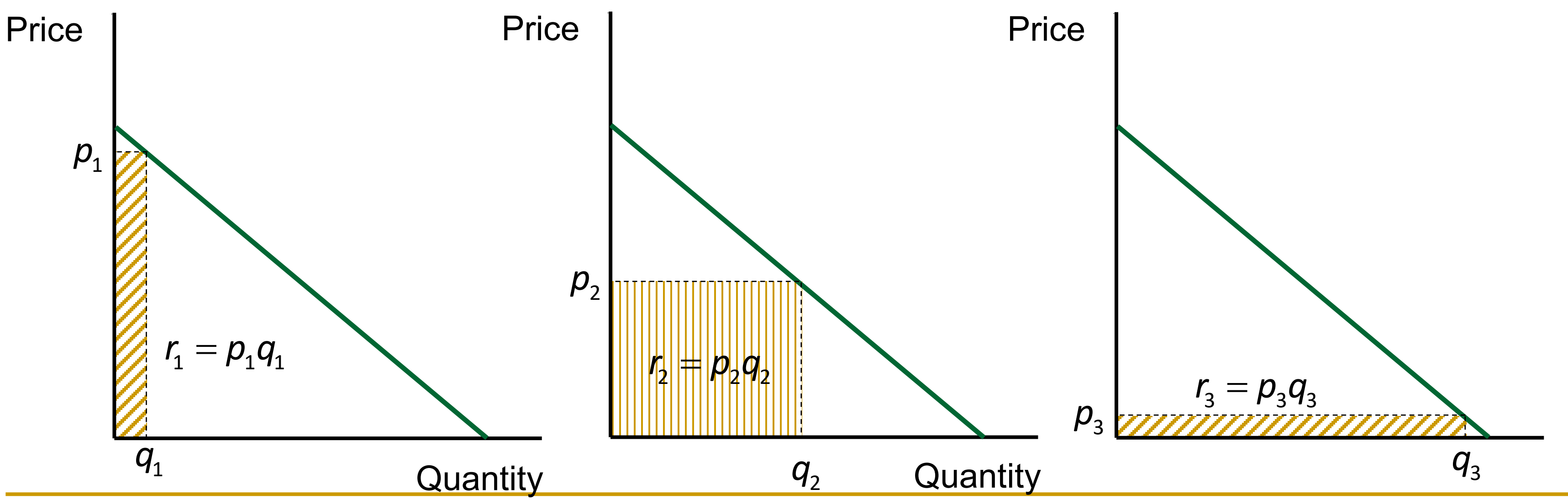
→ e. Find $p^* = p(q^*)$ from the inverse demand curve ($p^* = 7$)



2. Incremental Revenue

Incremental revenue

- Introduction
  - *Incremental revenue* is the net gain in revenue that a firm could earn if it were to increase its product by some discrete amount Δq
  - Incremental revenue is important when determining whether a firm should change its output level to increase its profits
  - Incremental revenue can be positive or negative
    - Moving from q_1 to q_2 increases revenue (incremental revenue is positive)
    - Moving form q_2 to q_3 decreases revenue (incremental revenue is negative)



Incremental revenue

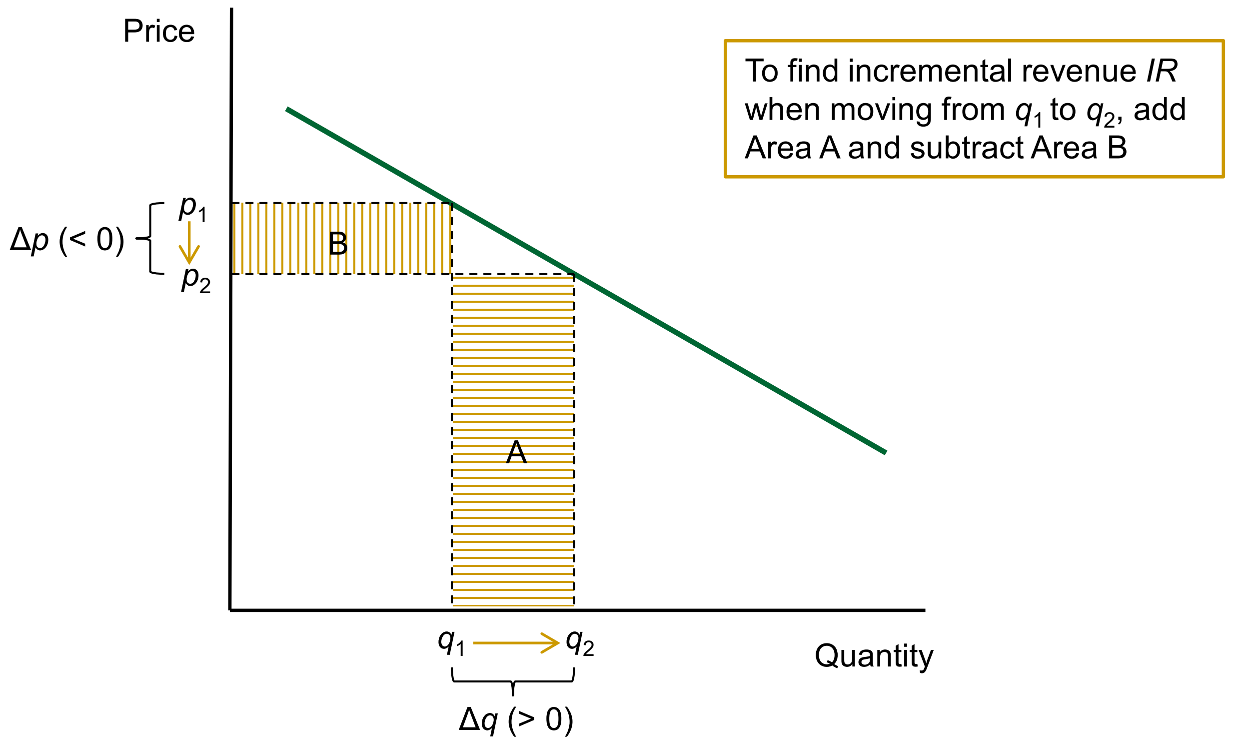
- Think about incremental revenue in two parts:
  1. The *gain* in revenue due to the sale of the additional units at the lower market-clearing price
     - Since there are more units to sell and demand is downward-sloping, the price will drop to clear the market
     - The gain in revenue on the additional sales is equal to $\Delta q(p - \Delta p)$, where—
       - Δq is the additional quantity to be sold
       - Δp is the market price decrease necessary to clear the market with the sale of the additional units
  2. Minus the *loss* of revenue on prior units sold due to the decrease in the market-clearing price
     - This loss of margin is the prior quantity p times the required price decrease, or $[q\Delta p]$
- So

$$IR = \Delta q\left(p - \Delta p\right) - q\Delta p$$

This is the formula for *marginal revenue* in the discrete case when $\Delta q = 1$

Incremental revenue

- Graphically



Area A = $\Delta q(p_1 - \Delta p)$ is the *gain* in revenue from the additional sales Δq at the lower price $p_2 = p_1 - \Delta p$
Area B = $q_1 \Delta p_1$ is the *loss* in revenue due to the sales of q_1 at the lower price p_2

So

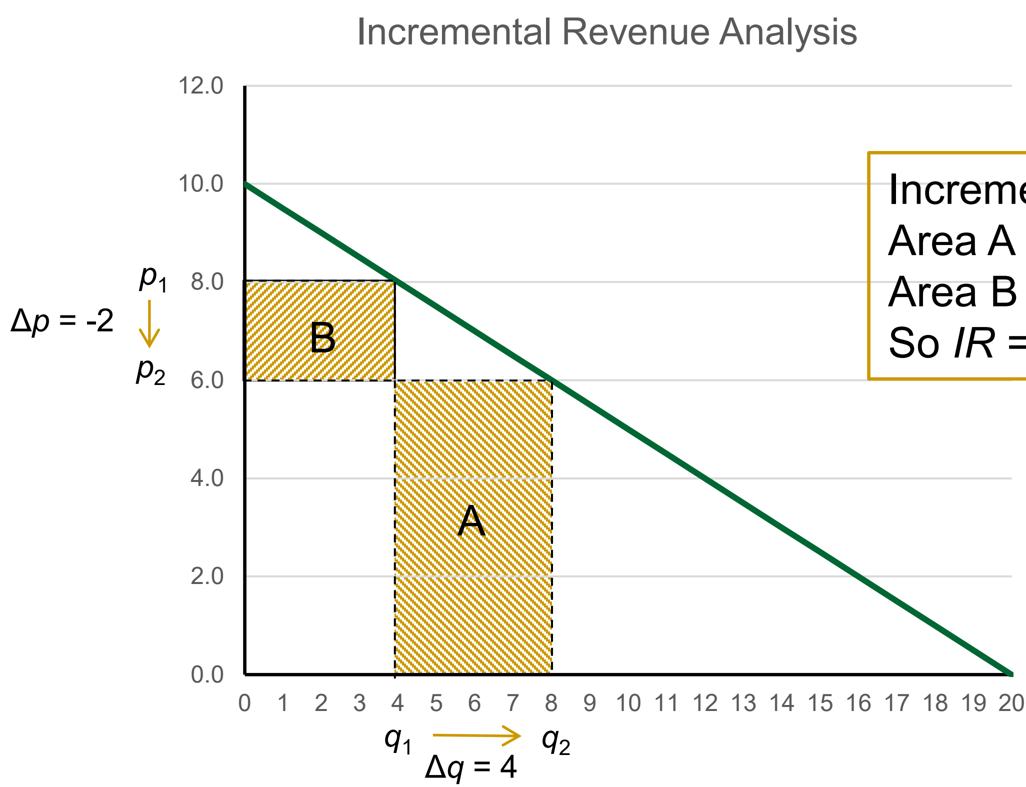
$$IR = \underbrace{\Delta q\left(p - \Delta p\right)}_{\text{Area A}} - \underbrace{q\Delta p}_{\text{Area B}}$$

Incremental revenue

- Example
  - (Inverse) demand: p =10 – ½ q
  - Starting point: q_1 = 4
  - End point: q_2 = 8

You need to calculate these variables:

So $p_1 = 8$ $\Delta q = q_2 - q_1 = 8 - 4 = 4$
So $p_2 = 6$ $\Delta p = p_2 - p_1 = 6 - 8 = -2$



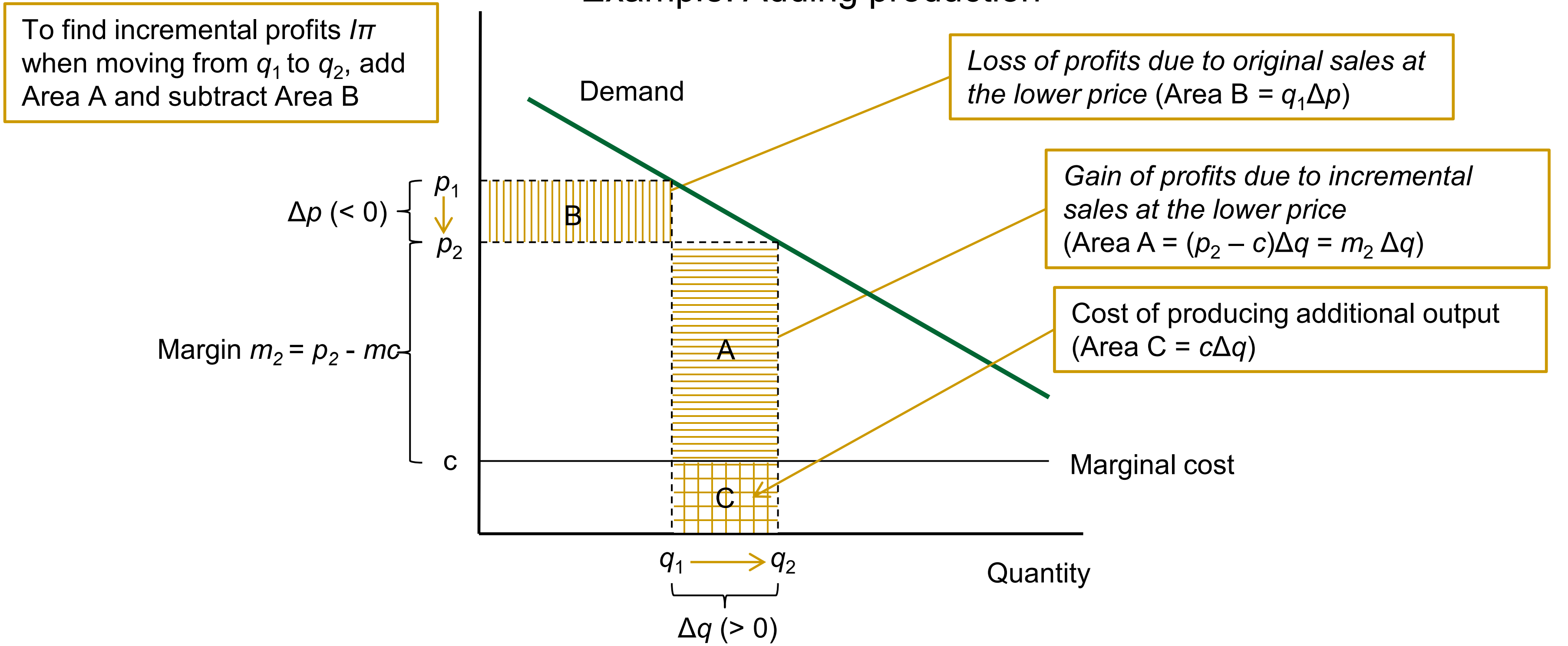
Incremental revenue = Area A – Area B
Area A = $p_2\Delta q$ = (6)(4) = 24
Area B = $q_1\Delta p$ = (4)(-2) = -8
So IR = 32 – 8 = 16

That is, the firm makes $16 more in revenues by moving from q_1 to q_2

Incremental profit

- We can easily extend the analysis of incremental revenues to incremental profits—We just have to:
  - Add the costs of additional production if we are adding to output ($\Delta q > 0$), *or*
  - Subtract the costs if we are reducing output ($\Delta q < 0$)

Example: Adding production



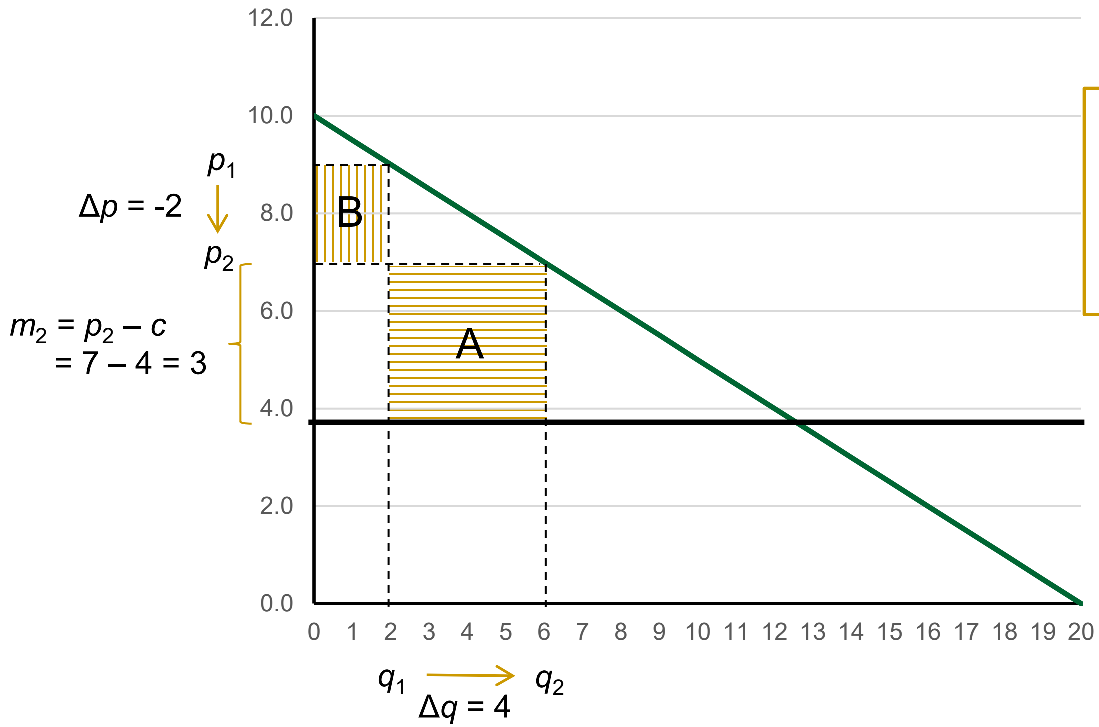
Incremental profit

- *Example*: Output increase
  - (Inverse) demand: $p = 10 - \frac{1}{2} q$
  - Starting point: $q_1 = 2$
  - End point: $q_2 = 6$
  - Constant marginal cost $c = 4$

You need to calculate these variables:

So $p_1 = 9$
So $p_2 = 7$

$\Delta q = q_2 - q_1 = 6 - 2 = 4$
$\Delta p = p_2 - p_1 = 7 - 9 = -2$
Margin $m_2 = p_2 - c$
$= 7 - 4 = 3$



Incremental profits = Area A – Area B
Area A = $m_2 \Delta q = (3)(4) = 12$
Area B = $q_1 \Delta p = (2)(-2) = 4$
So $I\pi = 12 - 4 = 8$

That is, the firm makes $8 more in profits by moving from q_1 to q_2

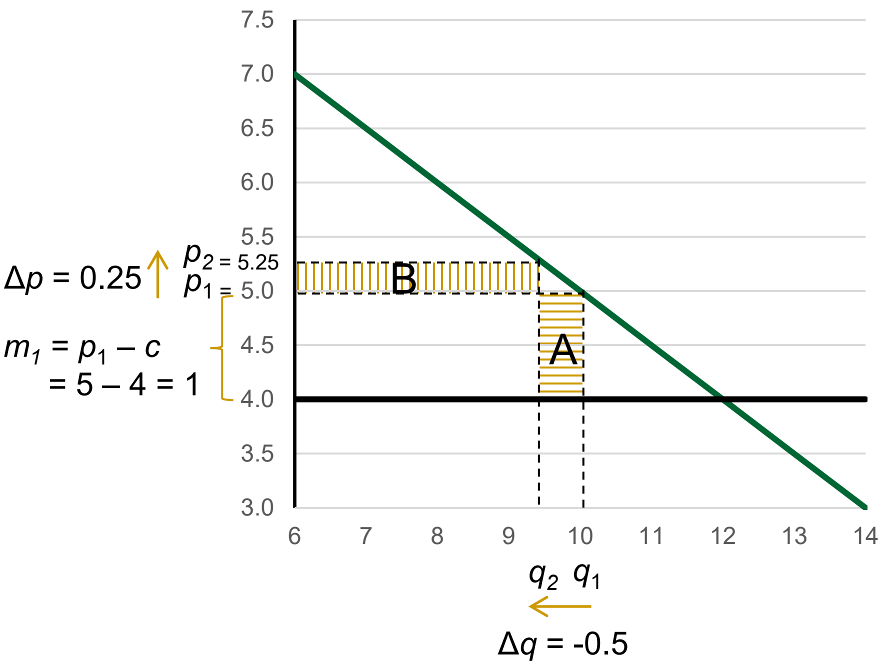
Incremental profit

- *Example*: Price increase (decreasing production)
  - (Inverse) demand: $p = 10 - ½ q$
  - Starting point: $p_1 = 5$
  - End point: $p_2 = 5.25$
  - Constant marginal cost $c = 4$

You need to calculate these variables:

So $q = 20 - 2p$
So $q_1 = 10$
So $q_2 = 9.5$

$\Delta q = q_2 - q_1 = 9.5 - 10 = -0.5$
$\Delta p = p_2 - p_1 = 5.25 - 5 = 0.25$



With an increase price and a concomitant *reduction* in output, the roles of Areas A and B are *reversed*:

Area A now represents the *loss* of profits from lost sales that would have been made at original price p_1 (= $m_1 \Delta q$)

Area B represents the *gain* of profits from the increased price charged on the sales that continue to be made (= $q_2 \Delta p$)

Incremental profits = Area B – Area A
Area B = q2Δp = (9.5)(0.25) = 2.375
Area A = m1Δq = (1)(-0.5) = -0.5
So incremental profits = 2.375 – 0.5 = 1.875

Incremental profit

- Observations
  - The prior example shows that under the conditions of the hypothetical, a 5 percent price increase would be profitable to the firm

This is mathematically identical to the exercise required by the *hypothetical monopolist test*, which is the primary analytical tool used by the agencies and the courts to define relevant markets. The hypothetical monopolist test asks whether a hypothetical monopolist of the candidate market could profitably sustain a “small but significant and nontransitory increase in price” (SSNIP), usually taken to be 5 percent. If so, the candidate market is a relevant market. In the prior example, if we assume that the demand curve is for the candidate market as a whole, this will be the residual demand curve for the hypothetical monopolist. If the original market price was $5 (as in the hypothetical), the hypothetical monopolist would find it profitable to reduce output in order to raise price by a 5 percent SSNIP.

We will confront the hypothetical monopolist test in almost every case study going forward, starting with the H&R Block/TaxAct case study next week. You will have plenty of opportunities to become familiar with the mechanics of the hypothetical monopolist test.

Appendix 1: Inverting Demand and Inverse Demand Functions

Inverting demand and inverse demand functions

- Motivation
  - You will be given either the demand function or the inverse demand function in a problem. But you may need to derive the other function in order to solve the problem.
  - Example
    - In the price increase problem on Slide 41, you were given the inverse demand function:

$$p = 10 - \frac{1}{2}q$$

    - But the problem gave you p_1 and p_2 and required you to calculate q_1 and q_2. To do this, you need to convert the inverse demand function into the demand function, so that you could use the prices to calculate the associated quantities
    - To create the demand function, you need to algebraically manipulate the inverse demand equation to isolate q on the left-hand side, so that quantities (which you need) are expressed in terms of prices (which the problem gives you)

Inverting demand and inverse demand functions

- Mechanics
  - An equality is maintained if you perform the same operation to both sides of the equation
  - Here are the steps to convert the above inverse demand function to a demand function:

Add ½ q to both sides:

$$p + \frac{1}{2}q = 10 - \frac{1}{2}q + \frac{1}{2}q$$
$$= 10$$

Subtract p from both sides:

$$p + \frac{1}{2}q - p = 10 - p$$

Simply:

$$\frac{1}{2}q = 10 - p$$

Multiply both sides by 2:

$$(2)\left(\frac{1}{2}q\right) = (2)(10 - p)$$

Simply:

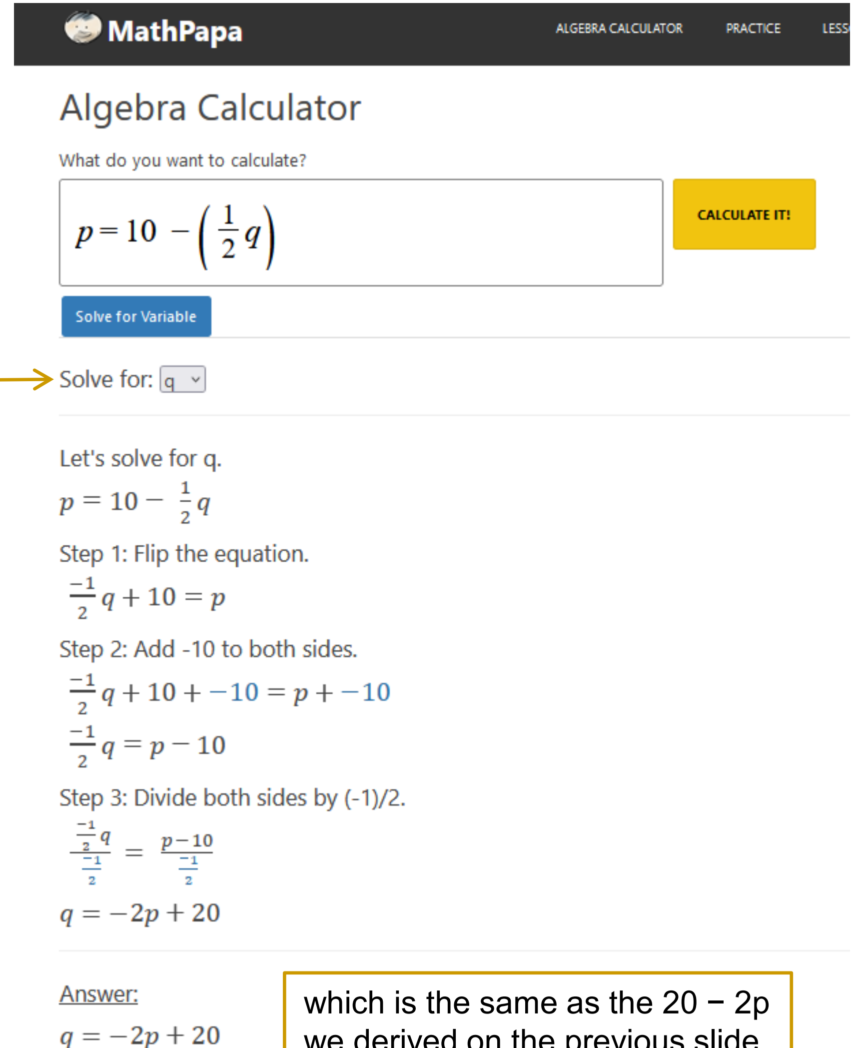
$$q = 20 - 2p$$

This is the demand curve that you would need for the price increase incremental revenue problem

  - The same technique can be used to convert a demand curve into an inverse demand curve

Inverting demand and inverse demand functions

- Or use an algebraic calculator:



We want *q* on the right-hand side, so solve for *q*

which is the same as the 20 − 2p we derived on the previous slide

Unit 7. Competition Economics

Part 2. Markets and Market Equilibria

Professor Dale Collins
Merger Antitrust Law
Georgetown University Law Center

September 27, 2022

Topics

- Substitutes, complements, and elasticities
- Markets and market equilibria
  - Perfectly competitive markets
  - Perfectly monopolized markets
  - Imperfectly competitive markets
    - Cournot oligopoly models
    - Bertrand oligopoly models
    - Dominant firm with a competitive fringe

Substitutes, Complements, and Elasticities

Substitutes/Complements

- Substitutes
  - *Definition*: Two products or services are *substitutes* if, when consumer demand increases for one product, it will decrease for the other product
    - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} < 0$$

Because Δq_1 and Δq_2 move in opposite directions, they will have different signs (i.e., one will be positive and the other will be negative)

  - Examples
    - Coke and Pepsi
    - iPhone and Galaxy S series mobile phones
    - Nike and Adidas shoes
    - Hertz and Avis rental cars
  - *Horizontal mergers* involve combinations of firms that offer substitute products

Substitutes/Complements

- Substitutes
  - Substitutes and prices
    - If products 1 and 2 are substitutes, then as the price of 1 increases, the demand for 2 increases:

A negative number times a negative number is a positive number

$$\overset{(-)}{\frac{\Delta q_2}{\Delta q_1}} \overset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \overset{(+)}{\frac{\Delta q_2}{\Delta p_1}} > 0$$

Slope of the demand curve for product 1 (< 0 since downward sloping)

Substitutes/Complements

- Complements
  - *Definition*: Two products are *complements* if, when consumer demand increases for one product, consumer demand also will increase for the other product
  - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} > 0$$

  - Examples
    - *Vertical mergers* involve complements
      - Television LCD screens and TV sets
      - Car engines and cars
      - Cable TV programming and cable TV distribution (AT&T/Time Warner)
      - Drug manufacture and drug distribution
    - But some conglomerate mergers can also involve complements
      - Printers and ink cartridges
      - Razors and razor blades
      - Computers and computer software

Substitutes/Complements

- Complements
  - Complements and prices
    - If products 1 and 2 are complements, then as the price of 1 increases, the demand for 2 decreases

A positive number times a negative number is a negative number

$$\overset{(+)}{\frac{\Delta q_2}{\Delta q_1}}\ \overset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \overset{(-)}{\frac{\Delta q_2}{\Delta p_1}} < 0$$

Slope of the demand curve for product 1 (< 0 since downward sloping)

Elasticities

- Own-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product

The Greek letter epsilon (ε) is the usual symbol in economics for elasticity

$$\varepsilon \equiv \frac{\%\Delta q_i}{\%\Delta p_i}$$

Percentage change q_i in the quantity of product i demanded

Percentage change p_i in the price of product i

    - This is sometimes called *elasticity of demand* or *price elasticity of demand*
    - Own-elasticities are always *negative in sign* since changes in prices and quantities move in opposite directions along a downward-sloping demand curve
  - Examples:
    - If price increases by 5% and demand decreases by 10%, then the own-elasticity is -2 (= -10%/5%)
    - If price increases by 3% and demand deceases by 1%, then the own-elasticity is -1/3 (= -1%/3%)

Technically, these are *arc elasticities* because they give percentage changes for discrete changes in prices and quantities

Elasticities

- Own-elasticity of demand
  - Relationship to the slope of the demand curve:

Slope of the demand curve

$$\varepsilon \equiv \frac{\%\Delta q}{\%\Delta p} \equiv \frac{\dfrac{\Delta q}{q}}{\dfrac{\Delta p}{p}} = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i},$$

that is, the own-elasticity at a point on the demand curve is equal to the slope of the demand curve at that point times the ratio of price to quantity at that point

---

- *Mathematical note (optional)*
  - *In calculus terms:*

$$\varepsilon \equiv \frac{dq_i}{dp_i}\frac{p_i}{q_i}$$

This deals with the continuous case

Elasticities

For intuition only
(NOT technically correct,
but it is usually the
intuition that is important)

- Some important definitions
  - *Inelastic demand*: Not very price sensitive

$$|\varepsilon| = \left|\frac{\%\text{change in quantity}}{\%\text{change in price}}\right| < 1$$

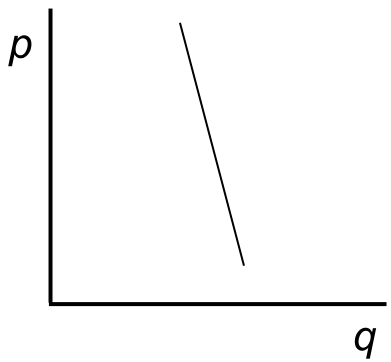
  - *Unit elasticity*:

$$|\varepsilon| = \left|\frac{\%\text{change in quantity}}{\%\text{change in price}}\right| = 1$$

  - *Elastic demand*: Price sensitive

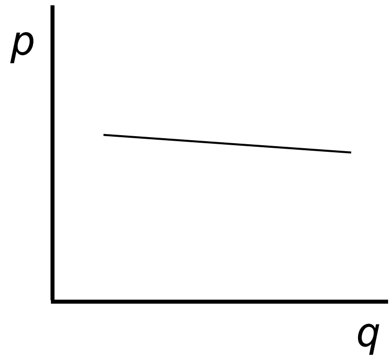
$$|\varepsilon| = \left|\frac{\%\text{change in quantity}}{\%\text{change in price}}\right| > 1$$

Inelastic demand



Little sensitivity to changes in price

Elastic demand



More sensitivity to changes in price

Note: |*x*| is the *absolute value* of *x*, which is the magnitude of *x* without the sign. So |3| = |-3| = 3.

Elasticities

Remember $\varepsilon = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$

- Elasticity of demand and the slope of the demand curve
  - Even when the demand curve is linear (so that the slope is constant), elasticity varies along the demand curve because the ratio of p_i to q_i changes along the curve

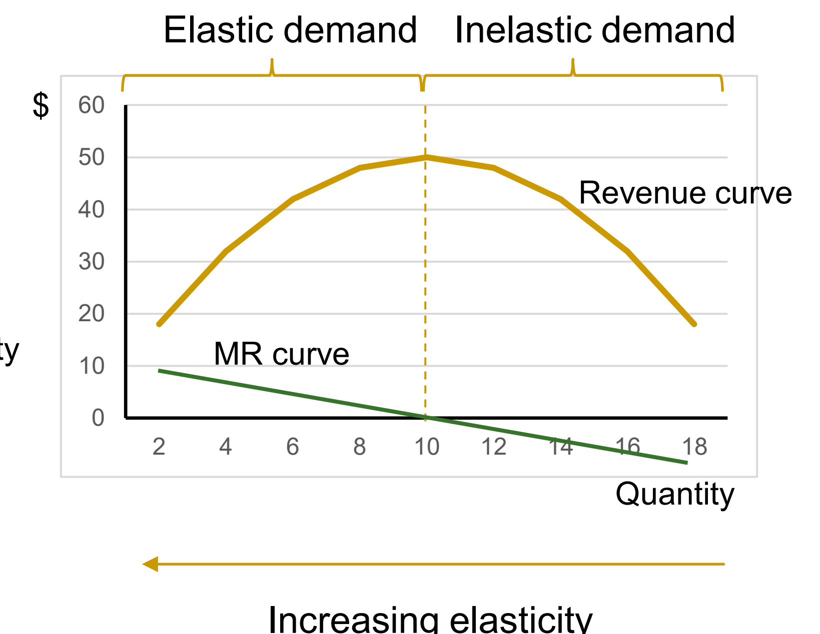
Inverse demand curve:
$p = 20 - 2q$

p	q	Slope	p/q	ε	Total revenue
1	18	-2	0.0556	-0.1111	18
2	16	-2	0.1250	-0.2500	32
3	14	-2	0.2143	-0.4286	42
4	12	-2	0.3333	-0.6667	48
5	10	-2	0.5000	-1.0000	50
6	8	-2	0.7500	-1.5000	48
7	6	-2	1.1667	-2.3333	42
8	4	-2	2.0000	-4.0000	32
9	2	-2	4.5000	-9.0000	18

Inelastic demand |ε| < 1

Unit elasticity |ε| = 1

Elastic demand |ε| > 1



General rules:
Elasticity decreases as quantity increase and prices decrease
Elasticity increases as quantity decrease and prices increase

Elasticities

- Predicting quantity changes for a given price increase
  - An approximation
    - We can approximate a percentage quantity change $\%\Delta q$ for a given percentage price change %Δp by multiplying the own-elasticity ε by the percentage price change:

$$\varepsilon = \frac{\%\Delta q}{\%\Delta p} \Rightarrow \%\Delta q \approx \varepsilon\%\Delta p$$

      - The relationship is not exact since the elasticity can change over the discrete range of the price change (as it does on a linear demand function)
  - For linear demand curves, an exact relationship exists for a price change Δp :

$$\varepsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q} \Rightarrow \Delta q = \varepsilon\frac{q}{p}\Delta p$$

    - Or, if you know the slope b of the demand curve

$$b = \frac{\Delta q}{\Delta p} \Rightarrow \Delta q = b\Delta p$$

These relationships can be important when determining a quantity change associated with a price increase in the hypothetical monopolist test for market definition

Cross-elasticities

- Cross-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} \equiv \frac{\%\Delta q_i}{\%\Delta p_j}$$

Percentage change q_i in the quantity of product *i* demanded

Percentage change p_j in the price of product *j*

  - With a little algebra (as before):

$$\varepsilon_{ij} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$$

Positive for substitutes
Negative for complements

- Mathematical note (optional)
  - In calculus terms:

$$\varepsilon_{ij} \equiv \frac{dq_i}{dp_j}\frac{p_j}{q_i}$$

Cross-elasticities

- Cross-elasticities—More definitions
  - *High cross-elasticity of demand*:
    - A small change in the price of product *i* will cause a large change of demand to product *j*
    - As a result, product *j* brings a lot of competitive pressure on product *i*

Make sure you understand why!

    - *Think of it this way:*
      - In a two-firm market, a high cross-elasticity implies a large number of *marginal customers* who will abandon product *i* when its price increases and will divert to product *j*
      - It also means a correspondingly smaller number of *inframarginal customers* who will stay with product *i* in the wake of a price increase
  - *Low cross-elasticity of demand*:
    - A large change in the price of product *i* will cause only a small change of demand to product *j*
    - As a result, product *j* brings little competitive pressure on product *i*

Make sure you understand why!

This is why antitrust lawyers talk so much about cross-elasticities!

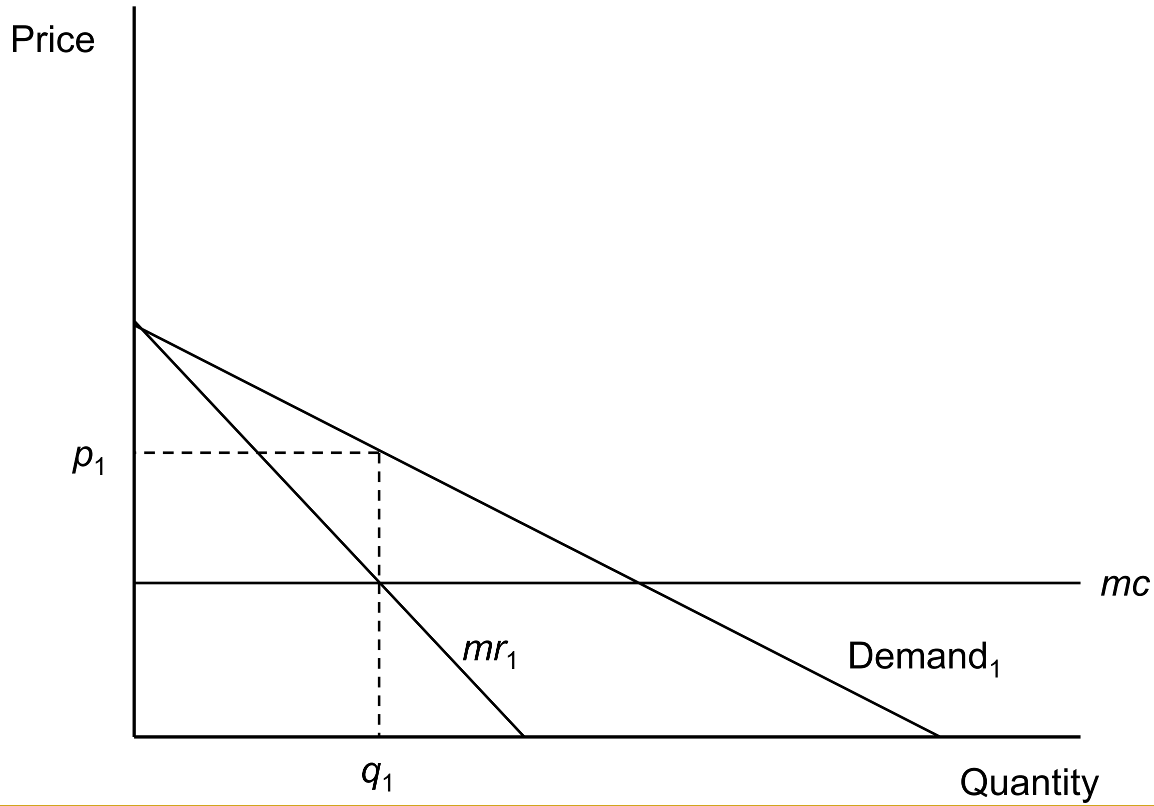
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Intuitively, the higher the cross-elasticities with the other products, the more elastic is the own-elasticity
  - Consequently, if a merger has the effect of decreasing the cross-elasticities of one or more substitute products, then its own-elasticity also decreases
  - *Key result*: All other things being equal, decreasing the cross-elasticity of demand of substitute products shifts the intersection of the marginal revenue curve and the marginal cost curve to the left, leading the firm to decrease output and increase prices

Let's look at a graph to see why

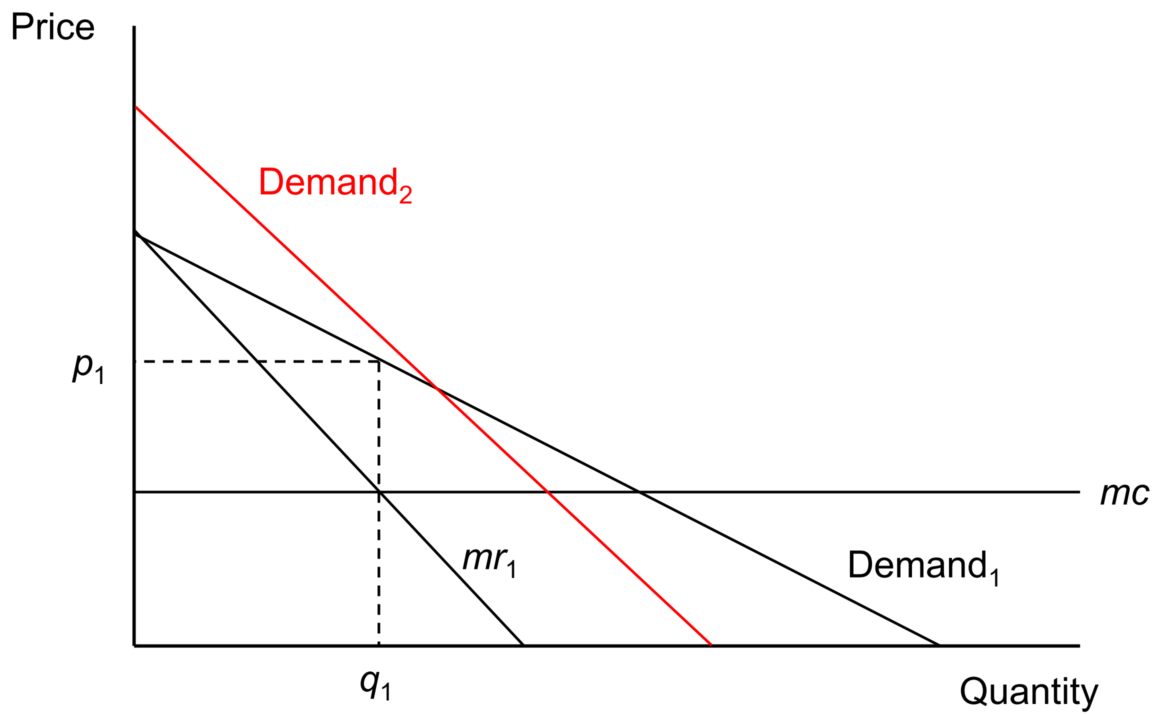
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Premerger profit-maximizing price-quantity equilibrium for the acquiring firm



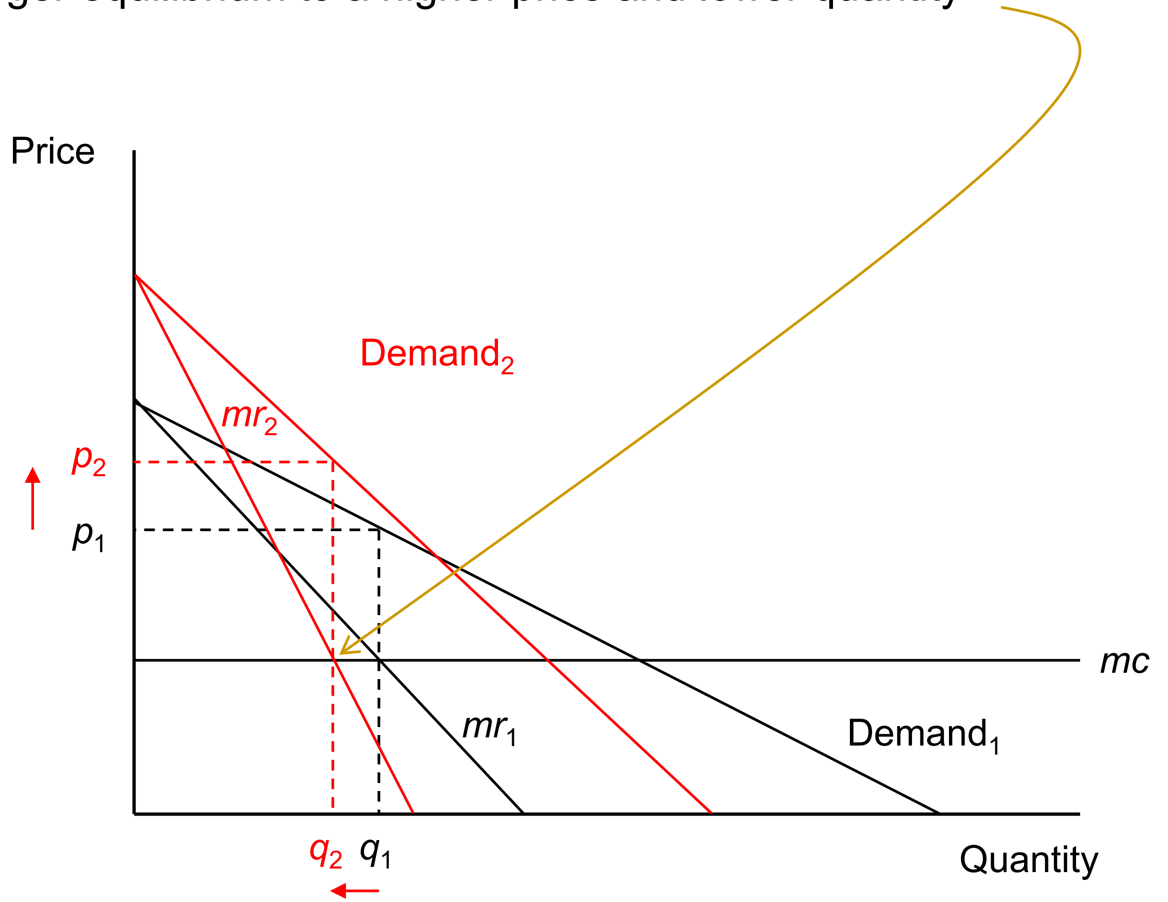
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Postmerger, the acquiring firm increases the acquired firm's price, making the acquired firm's substitute product less attractive and so decreasing the cross-elasticity of demand with the acquiring firm's product
    - The acquiring firm's residual demand curve then becomes more inelastic (steeper) around the premerger equilibrium point



An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Postmerger, the marginal revenue curve also becomes steeper, moving the postmerger equilibrium to a higher price and lower quantity



An important relationship

- Relationship of own-elasticities to cross-elasticities—Equivalent statements:
  - Reducing the attractiveness of substitutes
  - Reducing the cross-elasticities of demand of substitute products
  - Making the demand curve more inelastic
  - Making the demand curve steeper

Around the premerger price-quantity equilibrium

All result in higher prices and lower quantities

An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Technically:

$$|\varepsilon_{11}| = 1 + \frac{1}{s_1}\sum_{i=2}^{n} \varepsilon_{i1} s_i$$

$\varepsilon_{i1} > 0$ if the other products are substitutes for product 1

where ε_{11} is the own-elasticity of product 1 and ε_{i1} is the cross-elasticity of substitute product *i* with respect to the price of product 1 (evaluated at current prices and quantities)

  - Two important takeaways
    1. As the cross-elasticities on the right-hand side decrease, the demand for product 1 becomes more inelastic ($|\varepsilon|$ becomes smaller)
       - This allows Firm 1 to exercise market power and charge higher prices
    2. Competitors with larger market shares have more influence in constraining the price of Firm 1 for any given cross-elasticity (i.e., the cross-elasticities in the formula are weighted by market share)

You do not have to know the formula but you should know the takeaways

Diversion ratios

- *Definition*: Diversion ratio (*D*)

$$D_{12} \equiv \frac{\text{Units captured by Firm 2 as a result of Firm 1's price increase}}{\text{Total units lost by Firm 1 as a result of Firm 1's price increase}} \equiv \left|\frac{\Delta q_2}{\Delta q_1}\right|$$

  - NB: By convention, diversion ratios are *positive*. Since $\Delta q_1/\Delta p_1$ is negative (the demand curve is downward sloping), we need to look at the absolute value of the fraction
- Example
  - Firm 1 increases its price by 5% and loses a total of 20 units to substitute products
  - When Firm 1 increases its price, Firm 2—which maintains its original price—gains 5 units of additional sales
  - So:

$$D_{12} = \left|\frac{\Delta q_2}{\Delta q_1}\right| = \left|\frac{5}{-20}\right| = \frac{5}{20} = 0.25 = 25\%$$

Diversion ratios

- Thinking about diversion ratios
  - Think of D_{12} as $D_{1\rightarrow 2}$, that is—
    1. the number of units lost by Firm 1 that are “diverted” to Firm 2 (which produces a substitute product)
    2. as a result of Firm 1’s price increase
    3. when Firm 2’s price stays constant

    This heuristic assumes that there is a one-to-one substitution between Firm 1’s and Firm 2’s products

Diversion ratios

- Relation to cross-elasticities
  - Diversion ratios are closely related to cross-elasticities: both measure the degree of substitutability between two products when the relative prices change
    - Elasticities measure substitutability in terms of the *percentage* increase in Firm 2's unit sales for a *percentage* increase in Firm 1's price
    - Diversion ratios measure substitutability in terms the increase in Firm 2's unit sales as a percentage of all units lost by Firm 1 as a result of a given increase in Firm 1's price
  - Modern antitrust economics still speaks in terms of cross-elasticities when it often means diversion ratios
    - For example, products with high diversion ratios are said to have high cross-elasticities

We will see diversion ratios again in implementations of the hypothetical monopolist test and in the unilateral effects theory of anticompetitive harm

Markets and Market Equilibria: Perfectly Competitive Markets

Perfectly competitive markets

- *Definition*: A market in which no single firm can affect price, meaning—
  1. The firm perceives its residual demand curve as horizontal
  2. The firm perceives that it can sell any amount of product without affecting the market price
  3. $\frac{dp}{dq_i} = 0$ (as perceived by the firm)
  4. $p = \frac{dc}{dq_i}$ (i.e., price = marginal cost)

These four bullets are just different ways of saying the same thing

- Some more definitions
  - *"Price taking"*: Competitive firms are called *price-takers*, that is, they take market price as given and not something that they can affect
  - *Perfectly competitive equilibrium*: A market equilibrium exists where—
    1. Aggregate supply equals aggregate demand, *and*
    2. Each firm chooses its level of production so that the market-clearing price is equal to the firm's marginal cost of production

Perfectly competitive markets

- What could cause a market to be perfectly competitive?
  - *Traditional theory*: Each individual firm's production is very small compared to aggregate demand at any price, so that individual production changes cannot move materially along the aggregate demand curve
    - This implies that there are a very large number of firms in the market
  - *Modern theory*: Competitors in the marketplace react strategically but non-collusively to price or quantity changes by a firm in ways that maintain the perfectly competitive equilibrium

Competitive firms

- Three take-aways
  1. Competitive firms do not perceive that their output decisions affect the market-clearing price
     - That is, each firm perceives that it faces a horizontal residual demand curve
     - In fact, their individual output decisions do affect the market-clearing price but because the effect is so small no individual firm perceives this
       - In the aggregate, the sum of the output of all competitive firms determines the market-clearing price
  2. Competitive firms chose their output so that $p = mc$
     - Competitive firms, like all other firms, choose output so that marginal revenue is equal to marginal cost ($mr = mc$)
     - Since a competitive firm does not perceive that its output decisions affect the market-clearing price, the firm does not perceive that there is any downward adjustment in market price when it expands its output
     - Therefore, the firm perceives—and makes its output decision—on the premise that its marginal revenue is equal to the market price
     - Hence, the firm selects an output level so that $p = mc$
     - Mathematically:

$$mr(q_i) = p + q_i \frac{\Delta p}{\Delta q_i} = mc(q_i)$$

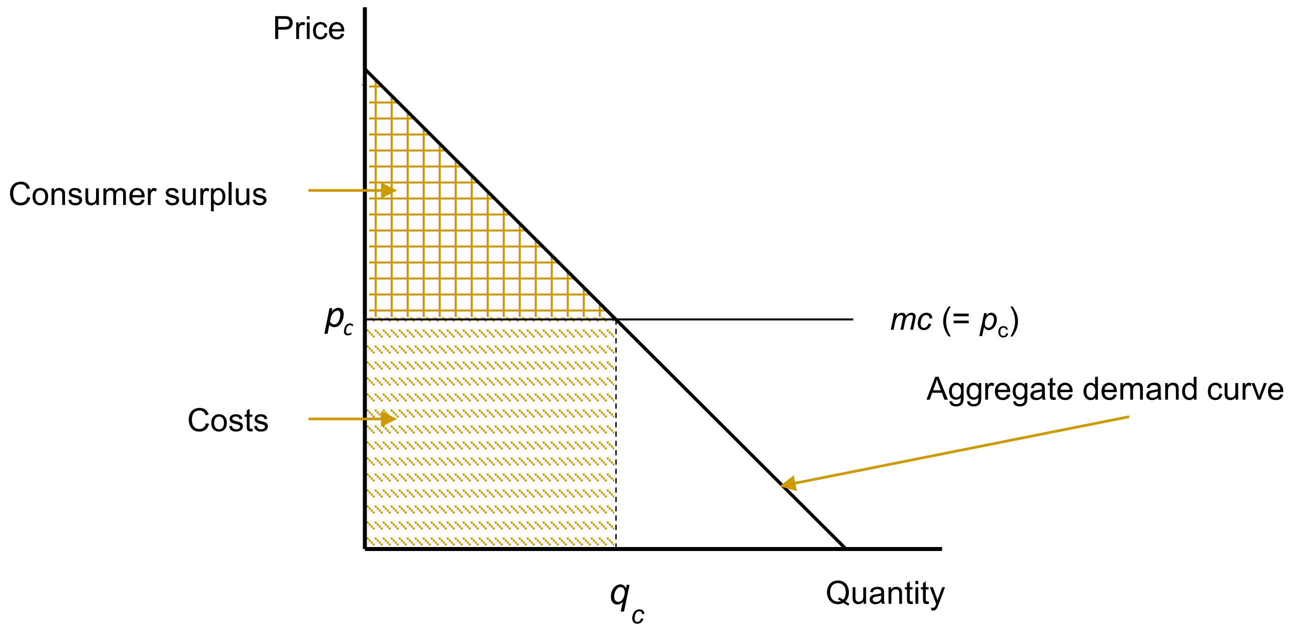
Perceived to be zero since the firm is a price-taker and does not believe that its choice of output affects market price

So:

$$p = mc$$

Competitive firms

- Three take-aways
  3. A competitive market maximizes consumer surplus[1]
     - A competitive market exhausts all gains from trade



[1] We are assuming a simple market where there is only one product that sells at a single uniform price (i.e., there is no price discrimination).

Perfectly Monopolized Markets

Perfect monopoly

- Basic concepts
  - In a perfect monopoly market, there is only one firm that supplies the product
    - This is an economic concept
    - In law, a monopolist need not control 100% of the market
  - Although there is only one firm in the market, it still faces a downward-sloping demand curve
    - There can be some substitutes for the monopolist's product—just not very good ones
  - The aggregate demand curve defines the residual demand curve facing an (economic) monopolist

In economics and in law, a firm that faces a downward-sloping residual demand curve and therefore has some power to influence the market-clearing price for its product is said to have *market power*. In antitrust law, a firm that has very significant power over the market-clearing price is said to have *monopoly power*. In economics, a monopolist is the only firm in the market.

Perfect monopoly

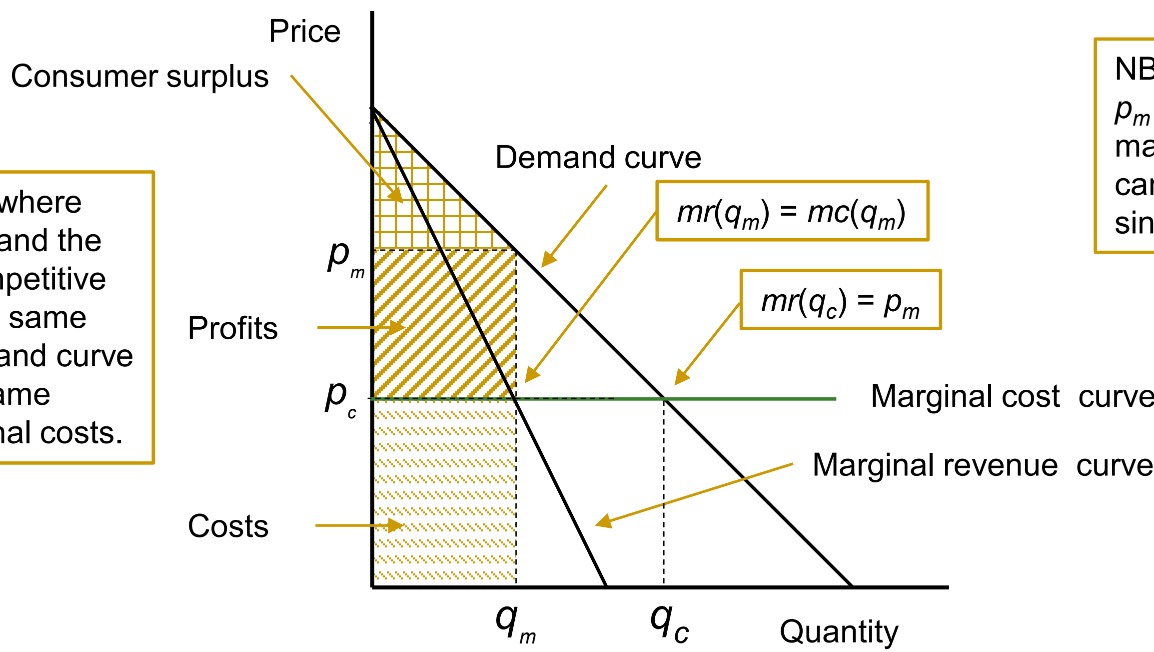
- A monopolist chooses output q_m so that $mr(q_m) = mc(q_m)$
  1. A monopolist charges a higher price than a competitive firm

$$p_m > mr(q_m) = mc(q_m) = mc(q_c) = p_c$$

where marginal costs are constant[1]

  2. A monopolist produces a lower output than would a competitive firm facing the same residual demand curve ($q_m < q_c$)

NB: $q_m = ½\, q_c$, where the monopolist and the firms in the competitive market face the same aggregate demand curve and have the same constant marginal costs.



NB: The monopolist price p_m is the price at which the maximum available profits can be drawn from a single price market.

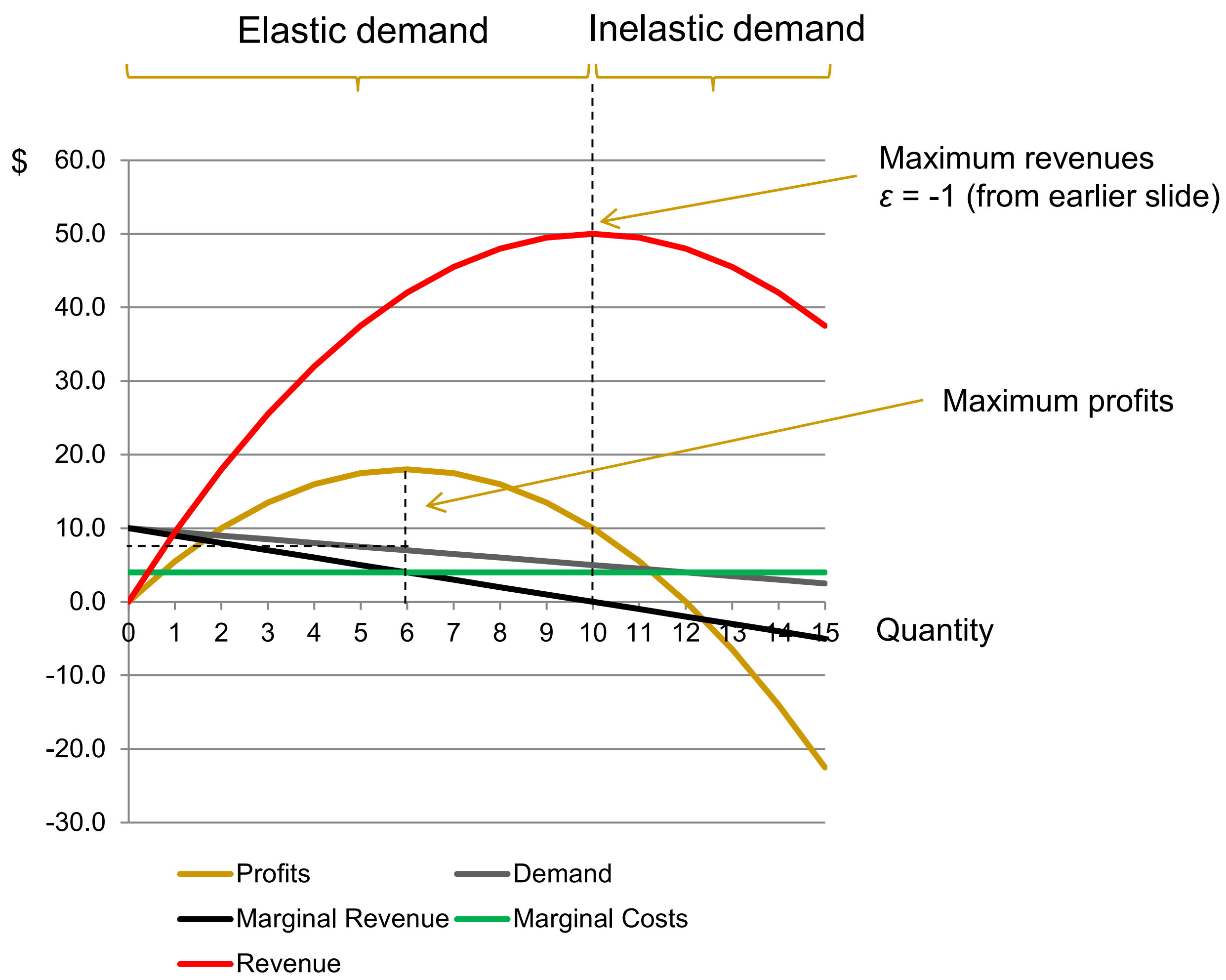
[1] But true whenever marginal costs are constant or increasing.

Monopolists and elasticities

- Proposition
  - A monopolist will not operate in the inelastic portion of its demand curve

Remember:

$$\varepsilon = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$



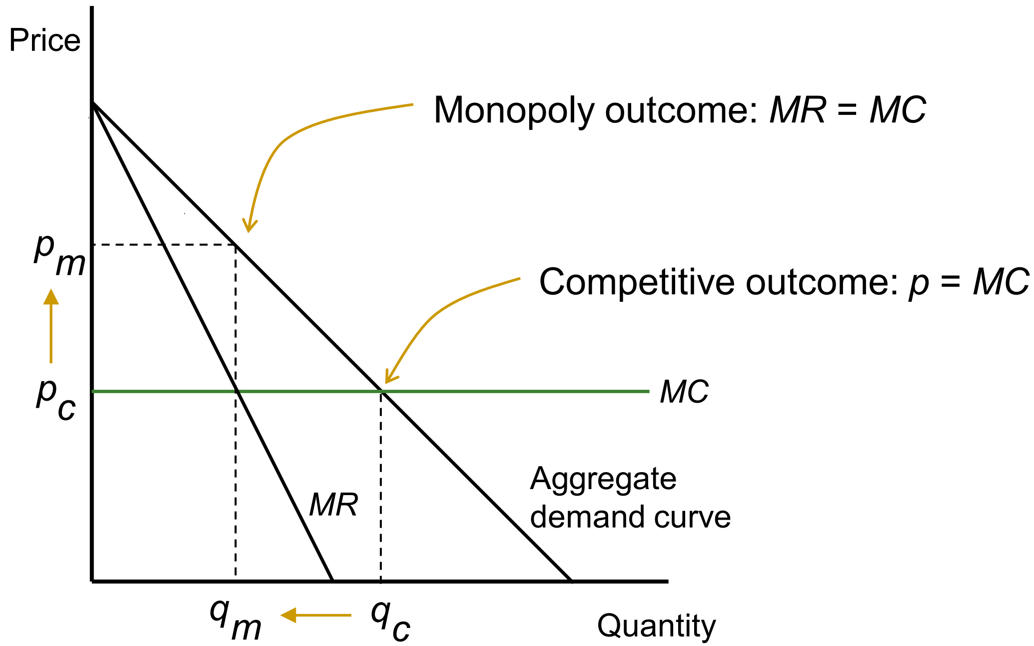
Review: Public policy on monopolies

- Modern view on why monopolies are bad:
  1. Increase price and decrease output
  2. Shift wealth from consumers to producers
  3. Create economic inefficiency ("deadweight loss")

  - May (or may not) have other socially adverse effects
    - Decrease product or service quality
    - Decrease the rate of technological innovation or product improvement
    - Decrease product choice

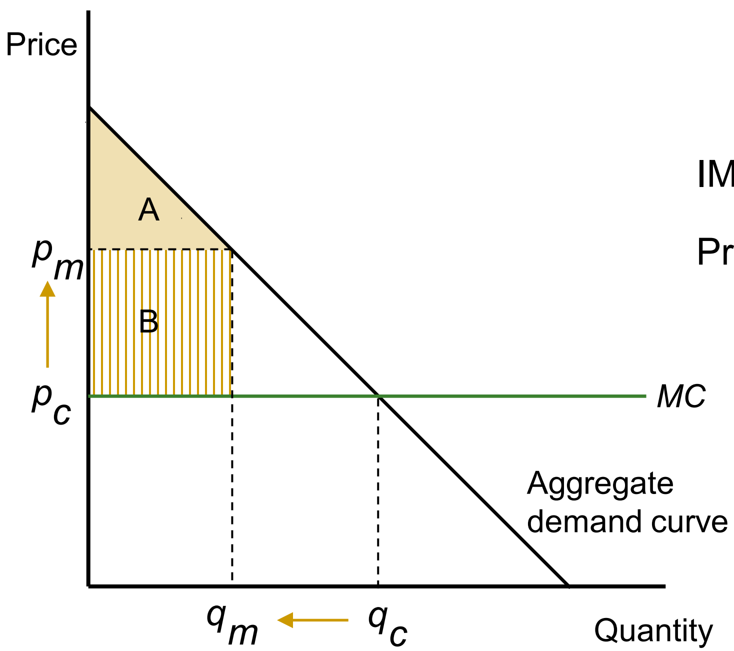
Review: Public policy on monopolies

- Output decreases: $q_c > q_m$
- Prices increase: $p_c < p_m$



Review: Public policy on monopolies

- Shifts wealth from inframarginal consumers to producers*
  - Total wealth created ("surplus"): A + B
  - Sometimes called a "rent redistribution"

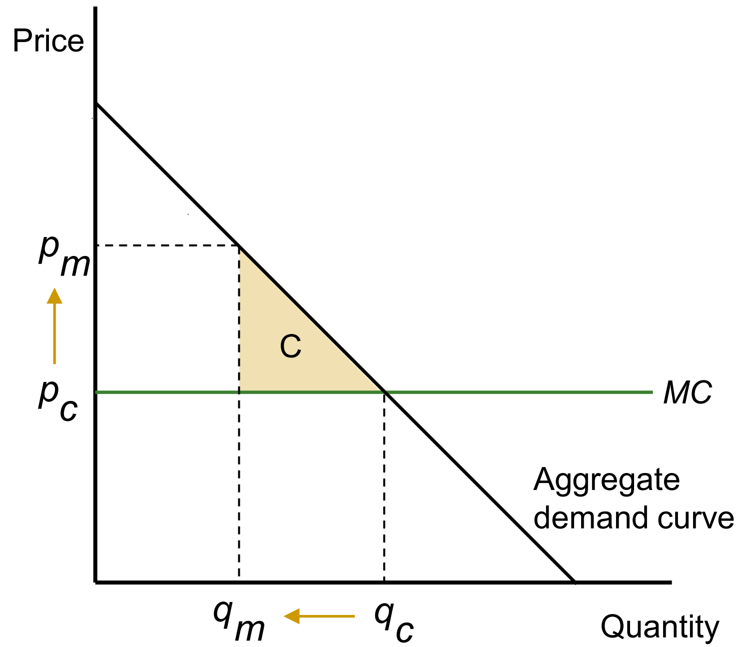


	Competitive	Monopoly
IM consumers	A + B	A
Producers	0	B

* Inframarginal customers here means customers that would purchase at both the competitive price and the monopoly price

Review: Public policy on monopolies

- "Deadweight loss" of surplus of marginal customers*
  - Surplus C just disappears from the economy
  - Creates "allocative inefficiency" because it does not exhaust all gains from trade

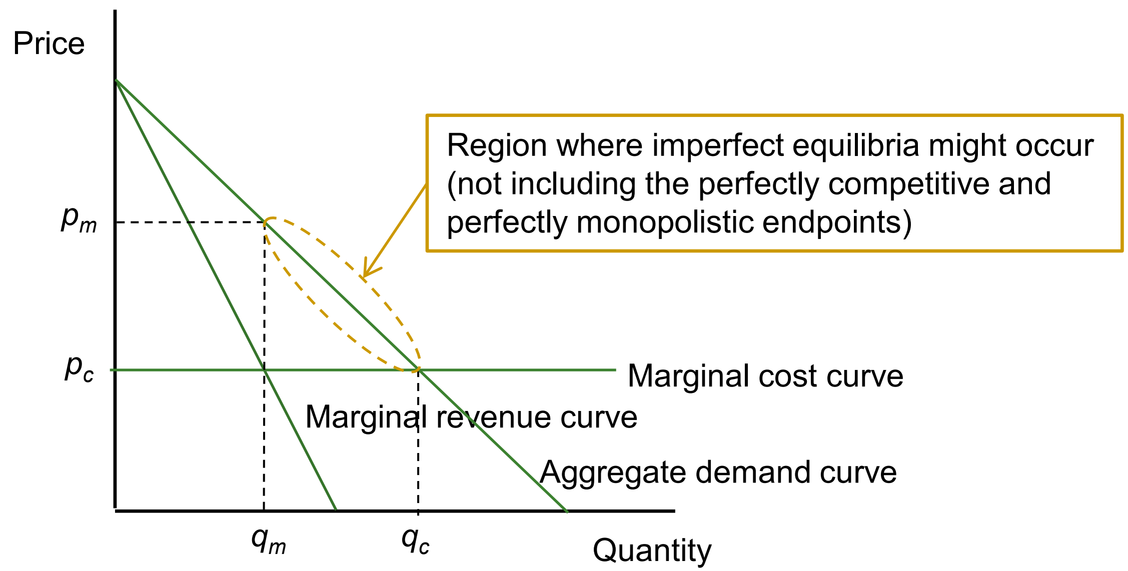


* Marginal customers here means customers that would purchase at both the competitive price and the monopoly price

Imperfectly Competitive Markets

Imperfectly Competitive Markets

- Range of imperfect equilibria
  - An imperfectly competitive equilibrium occurs when the equilibrium price and output on the demand curve falls strictly between the perfect monopoly equilibrium and the perfectly competitive equilibrium

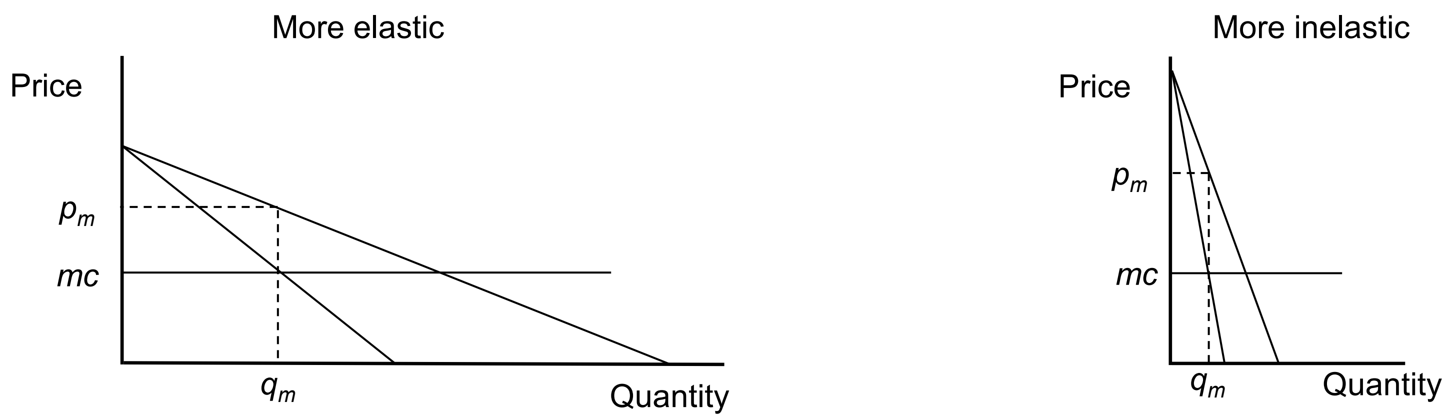


Market power

- Measuring market power
  - Economically, market power is the power of the firm to affect the market-clearing price through its choice of output level
  - The traditional economic measure of market power is the *price-cost margin* or *Lerner index L*, which is a measure of how much price has been marked up as a percentage of price:

$$L = \frac{p - mc}{p}$$

    - In a competitive market, L = 0 since because $p = mc$
    - In a perfectly monopolized market, L increases as the aggregate demand curve becomes steeper (more inelastic):



Homogeneous product models

- Homogeneous product models
  - Assumes that products are undifferentiated (that is, *fungible* or *homogeneous*) in the eyes of the customer
  - Common examples:
    - Ready-mix concrete
    - Winter wheat
    - West Texas Intermediate (WTI) crude oil
    - Wood pulp
  - Two properties of homogeneous products
    1. Customers purchase from the lowest cost supplier → This forces all suppliers in the market to charge the same price (in the absence of search costs)
    2. Since the goods are identical, their quantities can be added:
       - Adding all individual consumer demands at price p gives aggregate demand $Q(p)$
       - Adding all individual firm outputs at price p gives aggregate supply
       - In equilibrium, the amount demanded equals the amount supplied:

$$Q(p) = \sum q_i(p)$$

Cournot oligopoly models

A control variable is the variable the firm can set (control) in its discretion

- The setup
  - The standard homogenous product model is the *Cournot model*
  - In a Cournot model, the firm's control variable is *quantity*
    - The (download-sloping) demand curve gives the relationship between the aggregate quantity produced Q and the market-clearing price p:

$$p = p(Q), \text{ where } Q = \sum_{i=1}^{n} q_i,$$

in a market with n firms

  - The profit equation for firm i is:

$$\pi_i = p(Q)q_i - T_i(q_i), \quad i = 1, 2, \ldots, n$$

Each firm i choses its level of output q_i, but the aggregate level of output determines the market prices

  - First order condition (FOC) for profit-maximizing firm:

$$m\pi_i(q_i) = mr_i(q_i) - mc_i(q_i) = 0$$

This generates n equations in n unknows and can be solved for each q_i

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example
    - Compare the competitive, Cournot, and monopoly outcomes in this example

Demand curve: $Q = 100 - 2p$

	Price	Quantity
Perfectly competitive	5 (= mc)	90
Cournot (n = 2)	20	60
Perfect monopoly	27.5	45

    - Note that the perfect monopoly output is one-half the perfectly competitive output (with linear demand and constant marginal costs)

  - When demand is linear and there are n identical firms in a Cournot model, then:

$$Q_{Cournot} = \frac{n}{n+1} Q_{Competitive}$$

$q_{competitive}$	90	90	90	90	90	90	90	90	90
n	9	8	7	6	5	4	3	2	1
$q_{cournot}$	81	80	78.8	77.1	75	72	67.5	60	45

Cournot oligopoly models

- Relationship of the Lerner index to the HHI
  1. Define the firm *i*'s Lerner index to be:

     $$L_i \equiv \frac{p - mc_i}{p} = \frac{s_i}{\varepsilon}$$

     where s_i is the market share of firm *i* , *ε* is the own-elasticity of demand of the aggregate demand curve, and *p* is the market equilibrium price
  2. Define the market Lerner index as the sum of the share-weighted individual firm Lerner indices. Then:

     $$L \equiv \sum_{i=1}^{n} L_i s_i = \sum_{i=1}^{N} \left(\frac{s_i}{\varepsilon} \right) s_i = \sum_{i=1}^{N} \frac{s_i^2}{\varepsilon} = \frac{HHI}{\varepsilon}$$

     where *L* is the market-share weighted sum of the L_i of the individual firms in the market

     - *Key result*: In a Cournot model, the degree of exercise of market power in the market is a function of market concentration as measured by the HHI
     - The Herfindahl-Hirschman Index (HHI), which is the principal measure of market concentration, is the sum of the squares of the markets shares of the firms in the market. That is:

       $$HHI \equiv s_1^2 + s_2^2 + \cdots + s_N^2 = \sum_{i=1}^{N} s_i^2$$

The only thing on this slide you really need to know

Bertrand oligopoly models

- The setup
  - In a Bertrand model, the firm's control variable is *price*
    - Compare with the Cournot model, where the firm's control variable is *quantity*
    - The (download-sloping) residual demand curve gives the relationship between the firms choice of price and the quantity consumers will demand from the firm at that price
  - The profit equation for firm *i* is:

This is the residual demand function for firm *i*

$$\pi_i(p_i) = p_i \underbrace{q_i(p_i)} - T_i(\underbrace{q_i(p_i)}), \quad i = 1, 2, \ldots, n$$

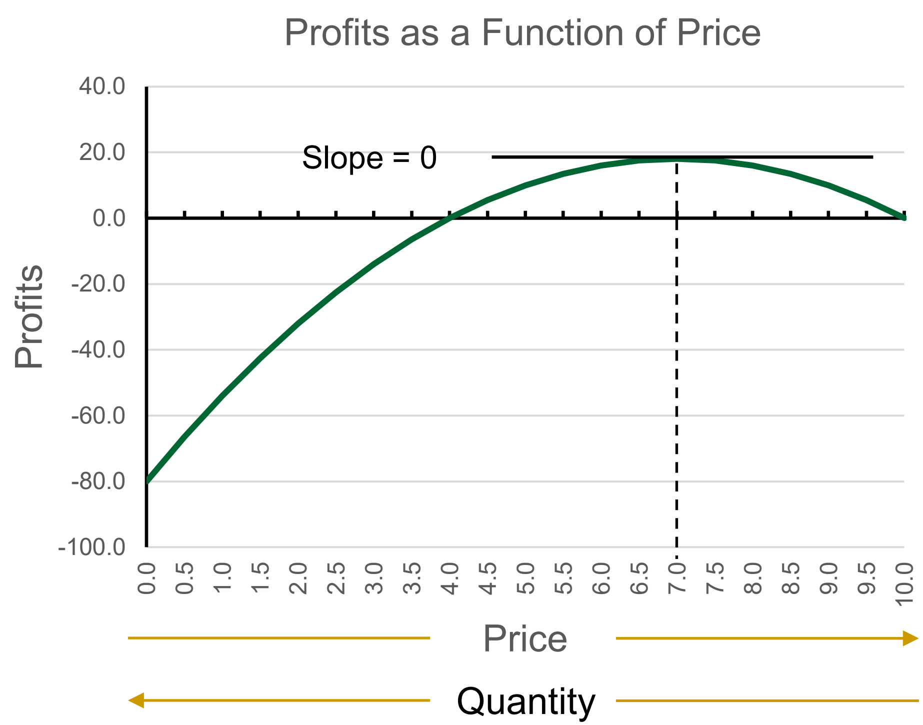
To see the first order conditions in operation, let's first look at profit-maximization for a monopolist whose control variable is price

Bertrand oligopoly models

- Profits as a function of price: Example for a monopolist

Price p	Quantity q	Revenues r	Costs T	Profits Π
0.0	20	0.0	80	-80.0
0.5	19	9.5	76	-66.5
1.0	18	18.0	72	-54.0
1.5	17	25.5	68	-42.5
2.0	16	32.0	64	-32.0
2.5	15	37.5	60	-22.5
3.0	14	42.0	56	-14.0
3.5	13	45.5	52	-6.5
4.0	12	48.0	48	0.0
4.5	11	49.5	44	5.5
5.0	10	50.0	40	10.0
5.5	9	49.5	36	13.5
6.0	8	48.0	32	16.0
6.5	7	45.5	28	17.5
7.0	6	42.0	24	18.0
7.5	5	37.5	20	17.5
8.0	4	32.0	16	16.0

Demand: $q = 20 - 2p$
Fixed costs = 0
Marginal costs = 4



Bertrand oligopoly models

- Observations
  - The profit curve for a monopolist as a function of price is a parabola
    - Although different in shape than the profit curve as a function of quantity
  - The profit maximum is when the slope of the profit curve is zero
  - So:
    - Marginal profits (as a function of price)
    - = Marginal revenues (as a function of price)
      minus marginal costs (as a function of price)
    - = 0

Bertrand oligopoly models

- Profit-maximization when a monopolist sets price: Example

Demand: $q = 20 - 2p$ | Marginal costs ($mc(q)$) = 4
Fixed costs = 0

- Revenues:

$$r(p) = pq(p) = p(20-2p) = 20p - 2p^2$$

This describes the parabola on the prior slide

- Marginal revenues:

$$mr(p) = 20 - 4p$$

Remember, if $y = ax + bx^2$ is the function, then the marginal function is $a + 2bx$

- Cost:

$$C(q(p)) = mc * q(p) = mc(20-2p) = 4(20-2p) = 80 - 8p$$

Constant marginal cost

Note: If $y = a + bx$ is the function, then the marginal function is b

- Marginal cost:

$$mc(p) = -8$$

NB: This is marginal cost as a function of p (not q). Why is it a negative number?

- FOC:

$$mr(p^*) = mc(p^*)$$

$$20 - 4p^* = -8$$

So $p^* = 7$ and $q^* = 6$

Bertrand oligopoly models

- Homogeneous products case with equal cost functions
  - Consider two firms producing homogeneous (identical) products at constant marginal cost *c* and use price p_i as their control variable
  - Consumers also purchase from the lower priced firm
    - If both firms charge the same price, they split equally consumer demand
  - Profit function for firm *i*:

$$\pi(p_i) \begin{cases} = p_i Q(p_i) - c(Q(p_i)) & \text{if } p_i < p_j \\ = \dfrac{p_i Q(p_i) - c(Q(p_i))}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

    - That is, firm *i* gets 100% of market demand $Q(p_i)$ at price p_i if p_i is the lower price of the two firms; the two firms split the market demand if their prices are equal; and firm *i* gets nothing if it has the higher price
    - *Equilibrium*: $p_1 = p_2 = mc$, so that both firms price at marginal cost (i.e., the competitive price) and split equally market demand and total market profits

Bertrand oligopoly models

- Homogeneous products case with asymmetric cost functions
  - Now consider two firms producing homogeneous (identical) products but with different cost functions costs, with firm 1 have lower marginal costs than firm 2 (i.e., $mc_1(q(p)) < mc_2(q(p))$
  - The profit function is the same as before:

$$\pi(p_i) \begin{cases} = p_i Q(p_i) - c(Q(p_i)) & \text{if } p_i < p_j \\ = \dfrac{p_i Q(p_i) - c(Q(p_i))}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

  - *Equilibrium*: Firm 1 prices just below firm 2 and captures 100% of market demand
  - *Idea*: Firm 1 and Firm 2 compete the price down to firm 2’s marginal cost as in the symmetric cost case. Then firm 1 just underprices firm 2 and captures 100% of the market demand

Bertrand oligopoly models

- Differentiated products case
  - When products are differentiated, a lower price charged by one firm will not necessarily move all of the market demand to that firm
    - Consider a market with only red cars and blue cars
    - Some consumers like blue cars so much that even if the price of red cars is lower than the price of blue cars, there will still be positive demand for blue cars
    - Moreover, if the price of blue cars increases, some (inframarginal) blue car customers will purchase blue cars at the higher price, while some (marginal) customers will switch to red cars
    - This means that the demand for red cars (and separately for blue cars) is a function both of the price of red cars and the price of blue cars
    - It also means that the price of blue cars may not equal the price of red cars in equilibrium

Bertrand oligopoly models

- Differentiated products case
  - Simple linear model
    - Firms 1 and 2 produce differentiated products and face the following residual demand curves:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

NB: Each firm's demand decreases with increase in its own price and increases with increases in the price of the other firm

Assume that $b_1 > b_2$, so that each firm's residual demand is more sensitive to its own price than to the other firm's price

    - Assume each firm has a cost function with no fixed costs and the same constant marginal costs:

$$c_i(q_i) = cq_i$$

    - Firm 1's profit-maximization problem:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2)$$

NB: This formulation does not take into account firm 2's reaction to a change in Firm 1's price. It assumes that Firm 2's price is constant.

      - Firm 2 solves an analogous profit-maximization problem
    - Derive the FOCs for each firm and solve for the Bertrand equilibrium:

$$p_1^* = p_2^* = \frac{a + cb_1}{2b_1 - b_2}$$

You do not need to know this. What is important is how the model is set up.

Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with—
    1. a *dominant firm*, with a control variable q and which sees its output decisions as affecting price and so sets output so that $mr = mc$, and
    2. a *competitive fringe* of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., they set their production quantities q_i so that $p = mc(q_i)$)
  - *Decision for the dominant firm*: Pick the profit-maximizing level for its output given the production of the competitive fringe
    - The model requires some constraint on the ability of the competitive fringe to expand its output. Otherwise, the competitive fringe will take over the market.
    - The constraint usually is either limited production capacity or increasing marginal costs

Dominant firm with a competitive fringe

- The model
  - At market price p, let $Q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe.
  - The dominant firm derives its residual demand function $q_d(p)$ starting with the aggregate demand function $Q(p)$ and subtracting the output supplied by the competitive fringe $q_f(p)$ at price p:

$$q_d(p) = Q(p) - q_f(p)$$

  - The dominant firm then maximizes its profit given its residual demand function by solving the following equation for the market price p^* that maximizes the firm's profits:

$$\max_p \pi_D = p \times \left[Q(p) - q_f(p) \right] - T\left(q(p)\right)$$

  - The dominant firm then produces quantity $q^* = q_D(p^*)$

You do not need to know how to solve the dominant firm maximization problem. What is important is the how the model is set up.

Dominant firm with a competitive fringe

- Dominant oligopolies
  - The model can be extended to the case where the dominant firm is replaced by a dominant oligopoly
  - The key is to specify the solution concept for the choice of output by the firms in the oligopoly (e.g., Cournot). You then create a residual demand curve for the oligopoly and apply the solution concept to that demand curve.
- Fringe firms
  - As we saw in Unit 2, the DOJ and the FTC typically ignore fringe firms. The dominant oligopoly model with a competitive fringe provides a theoretical justification.

Appendix

Mathematical notation

- pq: p times q (equivalently, $p \times q$, $p \cdot q$, and (p)(q))
- $p(q)$: p evaluated when quantity is q ("p as a function of q")
- $p(q)q$: p (evaluated at q) times q (i.e., pq)
- Δq: The change in q to the new state from the old state (i.e., $q_2 - q_1$)
- $\sum_{i=1}^{n} a_i$: The sum of the a_i's (i.e., $a_1 + a_2 + \ldots + a_n$)
- $\frac{\Delta y}{\Delta x}$: The change in y divided by the change in x
- $|a|$: The absolute value of a (i.e., a without a positive or negative sign) (e.g., $|3| = |-3| = 3$)
- $\equiv$ Like an equals sign but means a definition

Mathematical notation

Optional calculus terms

- $\dfrac{dy}{dx}$: The derivative of *y* with respect to *x* (where *y* is a function of *x*)
- $\dfrac{\partial y}{\partial x}$: The partial derivative of *y* with respect to *x* (where *y* is a function of *x*)
- Derivatives
  - If $y = a + bx + cx^2$
    then the derivative of *y* with respect to *x* is $\dfrac{dy}{dx} = b + 2cx$