

Class 9 slides

Unit 8. Competition Economics

Part 1. Demand, Costs, and Profits

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0. Opening Thoughts

Economics is common sense made difficult

To hide the fact that their discipline is no more than common sense, economists have created a thicket of esoteric mumbo-jumbo.
—Mail & Guardian (Mar. 13, 1998)

Economic science is but the working of common sense aided by appliances of organized analysis and general reasoning, which facilitate the task of collecting, arranging, and drawing inferences from particular facts.
—Alfred Marshall, *Principles of Economics* (1890)

Antitrust and economics

- The role of economics in antitrust
 - In per se violations, no need to prove actual or likely anticompetitive effect
 - So only the role for economics is proof of damages
 - In rule of reason violations, need to prove actual or likely anticompetitive effect
 - Economics is critical to predicting competitive effects
 - But very few rule of reason cases are investigated or litigated
 - Challenges are to practices that are already in place and can observe competitive effects
 - But still need economics for assessing the “but for” world
 - In monopolization or attempted monopolization cases, need to prove anticompetitive exclusionary conduct
 - Some role for economics in identifying anticompetitive exclusionary conduct
 - But relatively few Section 2 cases are investigated or litigated
 - Challenges are to practices that are already in place and can observe competitive effects
 - But still need economics for assessing the “but for” world
 - In merger cases, need to prove actual or likely anticompetitive effect
 - Economics is essential (under current law)
 - Many mergers are investigated and challenged
 - With the HSR Act, almost all are investigated prior to closing when likely effects cannot be observed and must be predicted
 - Economics provides the principal tool for predicting likely future competitive effects both with and without the merger

More on motivation

- The purpose of merger antitrust law

- Section 7 of the Clayton Act prohibits mergers and acquisitions that “may be substantially to lessen competition, or to tend to create a monopoly”¹
- In modern terms, a transaction may substantially lessen competition when it threatens, with a reasonable probability, to create or facilitate the exercise of market power to the harm of consumers
- Operationally, a transaction harms consumer when it result in—
 - Higher prices
 - Reduced market output
 - Reduced product or service quality in the market as a whole, *or*
 - Reduced rate of technological innovation or product improvement in the market

} Merger antitrust analysis typically focuses on price effects (see Unit 2)

compared to what would have been the case in the absence of the transaction (the “but for” world) and without any offsetting consumer benefits

Consequently, a central focus in merger antitrust law is the effect a merger is likely to have on the profit-maximizing incentives and ability of the merged firm to raise price in the wake of the transaction. In the first instance, this requires us to know how a profit-maximizing firm operates. The basic tools to enable us to do this analysis is the subject of this unit. These same tools are also fundamental to an understanding of merger antitrust law defenses.

¹ 15 U.S.C. § 18.

Antitrust economics

- Two starting points
 1. The *law of demand*: Demand curves are downward sloping
 2. *Profit maximization*: Firms act to maximize their profits
- With these starting points, economics enables us to—
 1. Analyze the incentives and abilities of a profit-maximizing firm given the demand curve facing the firm (the *residual demand curve*)
 2. Analyze how the firm's residual demand curve might change with a merger
 3. Predict how the merged firm might act differently postmerger from the two merging firms premerger
 4. Predict how other firms inside and outside the market may react to the merger
 5. Predict the consumer welfare consequences of this change in behavior

Profit maximization

To begin the analysis, we must understand how a firm makes its choices of price, production level, and other operating variables to maximize its profits

To keep things simple, we will look at a firm that produces only a single undifferentiated product

Profit maximization

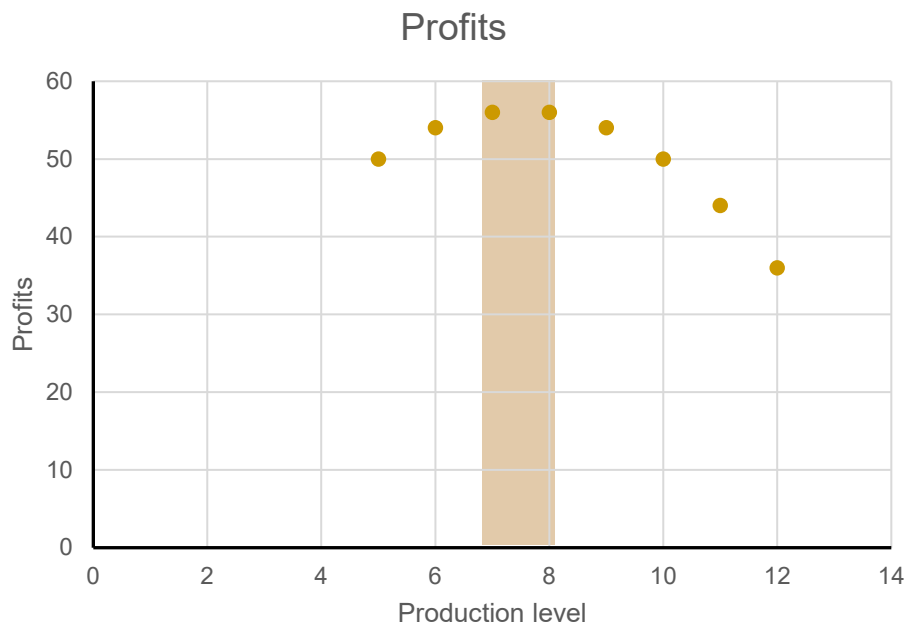
- Consider a very simple problem:
 - Avco makes widgets at a (constant) cost of \$5 each
 - When Avco makes 5 widgets, it can sell out at a price of \$15 per widget. Since Avco makes \$10 on each widget, Avco makes profits of \$50
 - Avco is thinking of increasing its production—it will do so only if this will increase its profits
 - If Avco makes 6 widgets, it must drop its price to \$14 to sell out. Since Avco makes \$9 on each widget, Avco would now make profits of \$54. Avco should increase its production
 - Should Avco increase its production even more?
 - If Avco makes 7 widgets, it must drop its price to \$13 to sell out. Since Avco makes \$8 on each widget, Avco would now make profits of \$56
 - If Avco makes 8 widgets, it must drop its price to \$12 to sell out. Since Avco makes \$7 on each widget, Avco would now make profits of \$56
 - If Avco makes 9 widgets, it must drop its price to \$11 to sell out. Since Avco makes \$6 on each widget, Avco would now make profits of \$54
 - If Avco makes 10 widgets, it must drop its price to \$10 to sell out. Since Avco makes \$5 on each widget, Avco would now make profits of \$50
 - If Avco makes 11 widgets, it must drop its price to \$9 to sell out. Since Avco makes \$4 on each widget, Avco would now make profits of \$44

So Avco should increase its production to 7 (or 8) widgets in order to maximize its profits

Profit maximization

- We can see this on a graph:

Quantity	Price	Revenues	Cost	Profits
5	15	75	25	50
6	14	84	30	54
7	13	91	35	56
8	12	96	40	56
9	11	99	45	54
10	10	100	50	50
11	9	99	55	44
12	8	96	60	36



Profit maximization

- Let's look at this in another way that better illustrates the underlying economics
- *Example 1.* If Avco were to increase its production from 5 units to 6 units and drop its price from \$15 to \$14, two things would happen:
 1. Avco would gain an additional sale, *and*
 - 2. Avco would have to lower its price on all the units it would sell to clear the market
- These two effects would have two consequences for Avco's profits:
 1. On the one customer Avco gained, Avco would make an additional profit of \$9
 - Additional sale of 1 unit times the profit margin of \$9 (at a sales price of \$14 and a unit cost of \$5)
 2. On its original sales of 5 units, Avco would have to lower its price by \$1 and so reduce its profits on those sales by \$5 (since each unit still costs \$5 to make)
 - Original sale price of \$15 minus the new sales price of \$14 equals a \$1 loss on each original sale
 - Five original sales times a \$1 loss on each sale equals a \$5 profit loss
- The change in Avco's profits is then:
 - The gain in profits from the additional sales at the new price (\$9)
 - *Minus* the loss in profits from lowering the price on the original sales (\$5)
 - For a net profit gain of \$4 (this is called the *incremental profit*)

The result of the firm's downward-sloping residual demand curve

Rule: Avco should increase its production whenever the incremental profit gain is positive

Profit maximization

- Let's look at this in another way that better illustrates the underlying economics
 - *Example 2.* Now if Avco were to increase its production from 10 units to 11 units and drop its price from \$10 to \$9, the same two things would happen:
 1. Avco would gain an additional sale
 2. Avco would have to lower its price on all the units it would sell
 - As before, these two effects would have two consequences for Avco's profits:
 1. On the customer Avco gained, Avco would make an additional profit of \$4
 - Additional sale of 1 unit times the profit margin of \$4 (at a sales price of \$9 and a unit cost of \$5) equals \$4 profit gain
 2. On its original sale, it would have to lower the price by \$1 and so reduce profits on those sales by \$10
 - Original sale price of \$10 minus the new sales price of \$9 equals \$1 loss on each original sale
 - Ten original sales times \$1 loss on each sale equals a \$10 profit loss
 - The change in Avco's profits is then:
 - The gain in profits from the additional sales at the new price (\$4)
 - Minus the loss in profits from lowering the price on the original sales (\$10)
 - For a net profit loss of \$6
 - Indeed, running the same analysis on a decrease in production from 10 units to 9 units would show that Avco would increase its profits

*Rule: Avco should decrease its production
whenever the incremental profit gain is negative*

Profit maximization

- Bottom line:

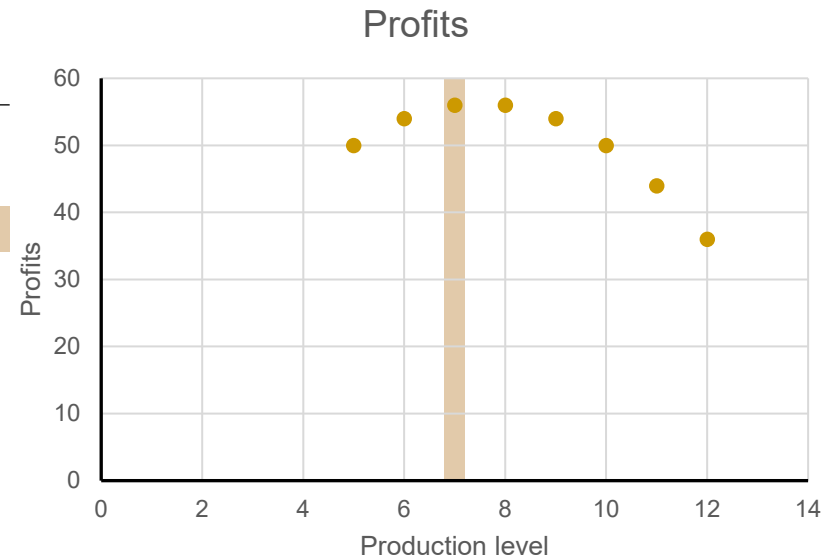
Avco maximizes its profit when its incremental profit is zero

- *Incremental profit* is the profit earned on selling the *next* unit

This is important:
Incremental profit looks
to the *next* sale, not the
last sale

- We can see this on the chart:

Quantity	Price	Revenues	Cost	Profits	Incremental Profit
5	15	75	25	50	4
6	14	84	30	54	2
7	13	91	35	56	0
8	12	96	40	56	-2
9	11	99	45	54	-4
10	10	100	50	50	-6
11	9	99	55	44	-8
12	8	96	60	36	



Profit maximization

- Some definitions
 - *Marginal sales*: Sales that are lost with an increase of one unit of output
 - *Marginal customers* are the customers connected with marginal sales
 - *Inframarginal sales*: Original sales that are retained when the price increases
 - *Inframarginal customers* are the customers connected with inframarginal sales
 - *Marginal profit*: The net profits a firm would make by increasing its production by one unit
 - May be positive or negative
 - *Incremental profits* are the net profits a firm would make increasing its production by some specified amount (which may be more than one unit)
 - *Marginal revenue*: The net revenue a firm would earn by increasing its production by one unit
 - May be positive or negative
 - *Incremental revenue* are the net revenues a firm would earn increasing its production by some specified amount (which may be more than one unit)
 - *Marginal cost*: The net cost to the firm of increasing its production by one unit
 - Always positive
 - *Incremental costs* are the costs a firm would incur by increasing its production by some specified amount (which may be more than one unit)

Profit maximization

- Some important relationships
 1. At a profit maximum, marginal profits are zero
 2. Marginal profit is equal to marginal revenue minus marginal cost
 3. Therefore, to maximize profits, a firm operates so as to set its

marginal revenue equal to its marginal cost

$$mr = mc$$

4. For a linear inverse demand curve of the form $p = a + bq$, the marginal revenue curve is $mr = a + 2bq$
 - The parameter b will always be negative (since the demand curve is downward sloping)
5. Marginal revenue can be decomposed into two parts:
 - a. The gross gain in profits from the sale of an additional unit at the new price (called the *gain on the marginal sale*)
 - b. The gross loss in the profit margin from the sale of the inframarginal units at the new lower price (called the *loss on the inframarginal sales*)

What you should be able to do after Part 1

For a firm—

- ❑ Facing a downward sloping residual (inverse) demand curve $p = a + bq$
- ❑ With fixed costs f and constant marginal costs c

1. Determine and graph the profit-maximizing levels of—

- ❑ Output q^*
- ❑ Price p^*
- ❑ Profits π^*

“*” (star) indicates that the variable is at its profit-maximizing level

“ Δ ” (delta) indicates the change in the variable (read this term as “delta q”)

2. Determine and graph the net incremental revenue for a firm increasing output by some amount Δq , including—

- ❑ The gross gain in revenues from the increase in output, and
- ❑ The gross loss in revenues from the reduction of price for sales at the original price

3. Derive and graph an inverse demand curve given a demand curve

1. Profit Maximization

An observation by Dave Berry

Later on, Newton also invented calculus, which is defined as “the branch of mathematics that is so scary it causes everybody to stop studying mathematics.” That's the whole point of calculus. At colleges and universities, on the first day of calculus, professors go to the board and write huge, incomprehensible “equations” that they make up right on the spot, knowing that this will cause all the students to drop the course and never return to the mathematics building. This frees the professors to spend the rest of the semester playing cards and regaling one another with stories about the “mathematical symbols” they've invented over the years. (“Remember the time Professor Hinkwattle drew a ‘cosine derivative’ that was actually a picture of a squid?” “Yes! Students were diving out the windows! From the fourth floor!”)¹

¹ Dave Berry, *Up in the Air on the Question of Gravity*, Baltimore Sun, Mar. 16, 1997, at 3J.

Profits

1. When the firm produces output q , its profits $\pi(q)$ are equal to its revenues $r(q)$ minus its total costs $t(q)$:

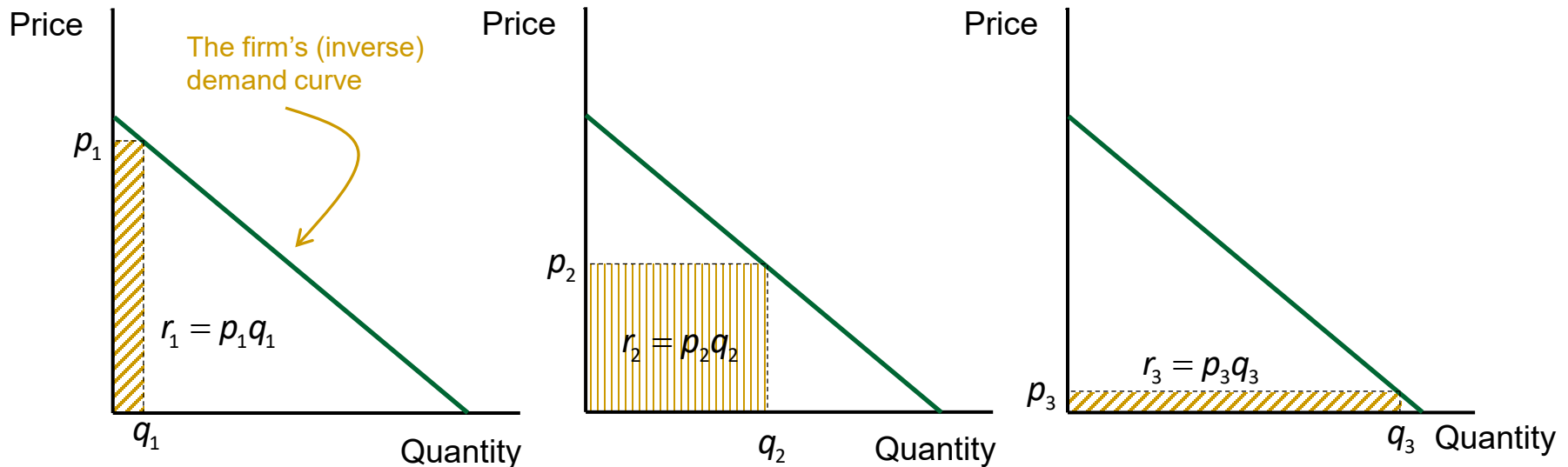
$$\pi(q) = r(q) - t(q)$$

We write $\pi(q)$ rather than just π to remind us that profit is a function of the quantity the firm sells

2. Revenues $r(q)$ are equal to price p times output q :

$$r(q) = pq$$

3. Revenues can be shown as a rectangle in a price-quantity chart:



Profits

4. When the firm faces a linear downward-sloping residual (inverse) demand curve $p = a + bq$:

The parameter b will be negative since the inverse demand curve is downward sloping

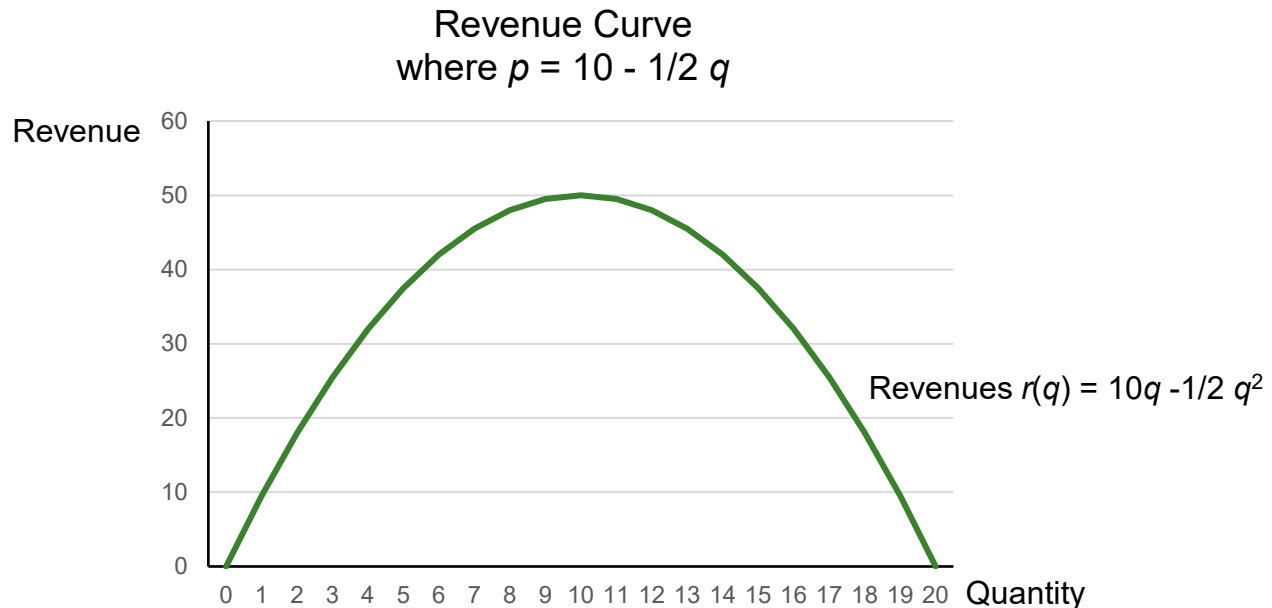
$$r(q) = pq$$

$$= (a + bq)q$$

$$= aq + bq^2$$

Since this is a second-order polynomial, its graph is a parabola

- The graph of the firm's revenues as a function of q is a parabola:



Profits

5. At output q , total costs $t(q)$ are equal to fixed costs f plus variable costs $v(q)$:

$$t(q) = f + v(q)$$

Note that fixed costs f are NOT a function of production quantity q

- With *constant marginal costs* c , variable costs $v(q)$ are equal to marginal cost c times output q :

$$v(q) = cq$$

- Then total costs $t(q)$ may be expressed as:

$$\begin{aligned} t(q) &= f + v(q) && \text{generally} \\ &= f + cq && \text{in the case of constant variable costs} \end{aligned}$$

Profits

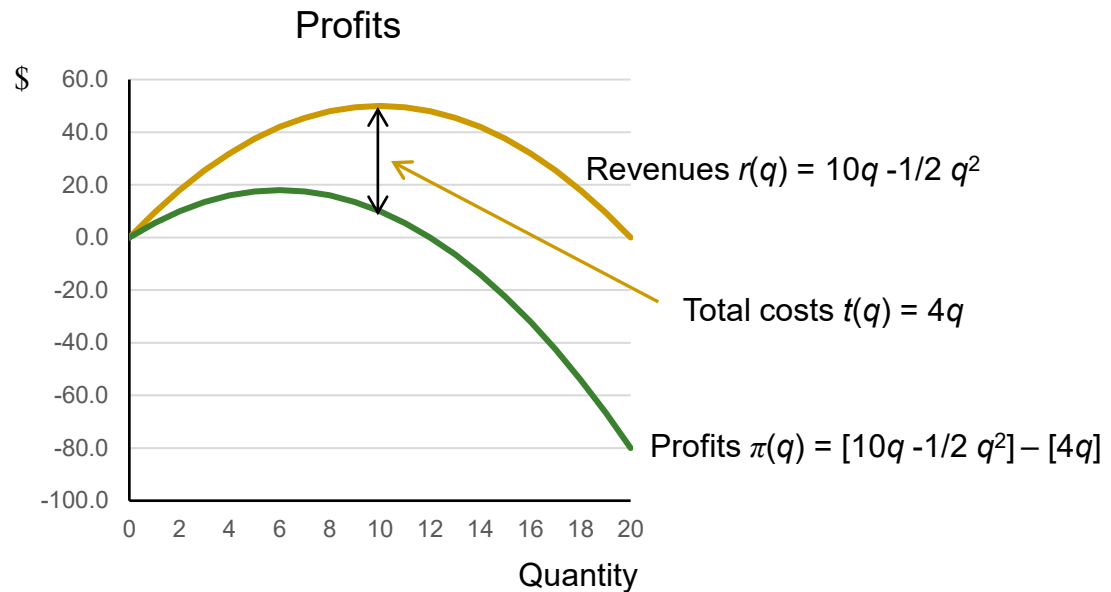
6. Now we can express total profits $\pi(q)$ as:

$$\begin{aligned}\pi(p) &= r(q) - t(q) \\ &= (a + bq)q - [f + cq] \\ &= [aq + bq^2] - [f + cq]\end{aligned}$$

Since this is a second-order polynomial, its graph is a parabola

□ Graphically:

where
 $p = 10 - \frac{1}{2}q$
 $f = 0$
 $c = 4$



Profit maximization

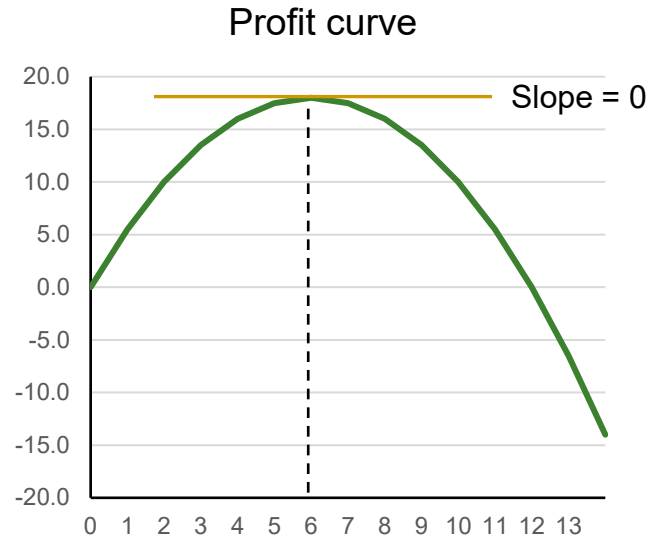
7. The slope at the top of the profit “hill” is zero (a horizontal line):

where

$$p = 10 - \frac{1}{2} q$$

$$f = 0$$

$$c = 4$$



□ Definition

- The *slope of a line* is the change in the *y-values* (Δy) divided by the change in the *x-values* (Δx):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- The *slope of a curve* at a point is the slope of the tangent line at that point (as shown above)
 - For calculus geeks: The slope of a curve at a point is the *derivative* of the function at that point

Profit maximization

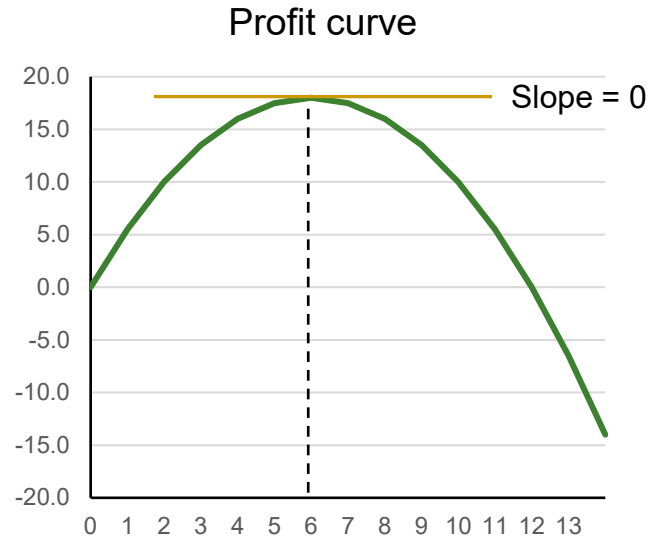
8. The slope at the top of the profit “hill” is zero (a horizontal line):

where

$$p = 10 - \frac{1}{2} q$$

$$f = 0$$

$$c = 4$$



Solve the problem:

- ❑ From the chart, we see that the profit-maximizing output q^* is 6
- ❑ From the inverse demand curve, we can calculate $p^* = p(6) = 10 - (1/2)(6) = 7$
- ❑ $r^* = r(6) = p^*q^* = (7)(6) = 42$
- ❑ $f = 0$ (from the hypothetical)
- ❑ $v^* = v(6) = cq^* = (4)(6) = 24$
- ❑ $t^* = t(q^*) = f + v(q^*) = 0 + 24 = 24$
- ❑ $\pi^* = \pi(q^*) = r^* - t^* = 42 - 24 = 18$

Profit maximization

■ Marginal analysis—Some definitions

- The slope of the revenue curve at an output q is called the *marginal revenue* $mr(q)$
 - Think of marginal revenue as the revenue the firm would earn if it produced one *additional* unit
 - You can also think of the marginal revenue as the *rate of change* in revenue for an increase in output
 - If $r(q) = aq + bq^2$ (the revenue function for a linear inverse demand curve), then:

$$mr(q) = a + 2bq$$

In the continuous case—think of this as the *instantaneous rate of change* of revenue with respect to output

- The slope of the total cost curve at an output q is called the *marginal cost* $mc(q)$
 - Think of marginal cost as the cost the firm would earn if it produced one *additional* unit
 - If $t(q) = f + cq$ (total costs with constant marginal costs), then:

$$mc(q) = c$$

- The slope of the profit curve at an output q is called the *marginal profit* $m\pi(q)$
 - Think of marginal profit as the profit the firm would earn if it produced one additional unit
 - Marginal profit is marginal revenue minus marginal cost:

$$m\pi(q) = mr(q) - mc(q)$$

For calculus geeks: The marginal function is the derivative of the primary function. So, for example, the marginal revenue function is the derivative of the revenue function.

Profit maximization

OPTIONAL but well worthwhile. You should not be satisfied to be told the formula for the marginal revenue curve. You should want to understand its derivation from the definition of marginal revenue. This provides that explanation.

- Marginal analysis—Deriving the marginal revenue function (continuous case)

- If $r(q) = aq + bq^2$ (the revenue function for a linear inverse demand curve), then:

$$mr(q) = a + 2bq$$

in the continuous case (that is, when one unit is infinitesimally small compared to firm output q)

- *Proof:* Let q be the firm's output. Then marginal revenue is technically defined as:

$$mr(q) = \frac{r(q + \Delta q) - r(q)}{\Delta q}, \text{ where } \Delta q = 1$$

Substituting the inverse demand function for r and simplifying:

$$\begin{aligned} mr(q) &= \frac{[a(q + \Delta q) + b(q + \Delta q)^2] - [aq + bq^2]}{\Delta q} \\ &= \frac{[(aq + a\Delta q) + (bq^2 + 2bq\Delta q + b\Delta q^2)] - [aq + bq^2]}{\Delta q} \\ &= \frac{a\Delta q + 2bq\Delta q + b\Delta q^2}{\Delta q} \\ &= a + 2bq + b\Delta q \end{aligned}$$

But if Δq is very small compared to q , it may be ignored. So $mr(q) = a + 2bq$ in the continuous case. Q.E.D.

Profit maximization

- First order condition (FOC)
 - From Slide 22, we know that profits are maximized at the top of the profit “hill,” which is where the slope of the profit curve is zero
 - From Slide 24, we know that the slope of the profit curve at an output q is the marginal profit $m\pi(q)$ evaluated at output q
 - From Slide 24, we also know that the marginal profit $m\pi(q)$ is equal to the marginal revenue $mr(q)$ minus the marginal cost $mc(q)$, all evaluated at output q , that is:

$$m\pi(q) = mr(q) - mc(q)$$

- The *first order condition* for a profit-maximizing level of output q^* is that the marginal profit at q^* equals zero, that is:

$$m\pi(q^*) = mr(q^*) - mc(q^*) = 0$$

or equivalently: $mr(q^*) = mc(q^*)$

A profit-maximizing firm sets its production level q so that its marginal revenue is equal to its marginal cost

Profit maximization

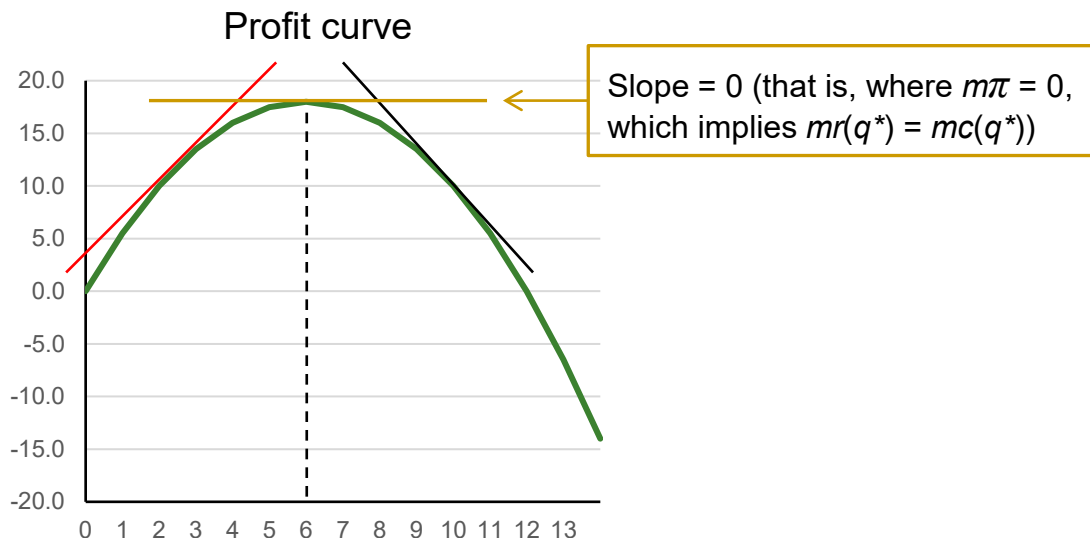
- First order condition—Example

where

$$p = 10 - \frac{1}{2} q$$

$$f = 0$$

$$c = 4$$



- **Key concept:** Think of the slope as the *instantaneous rate of change* of profits with respect to output
 - If the slope is positive ($m\pi > 0$), then profits are increasing with increases in output
 - If the slope is negative ($m\pi < 0$), then profits are decreasing with increases in output
 - If the slope is zero ($m\pi = 0$), then a change in output in either direction will decrease profits (i.e., the firm is at a profit maximum)

Profit maximization

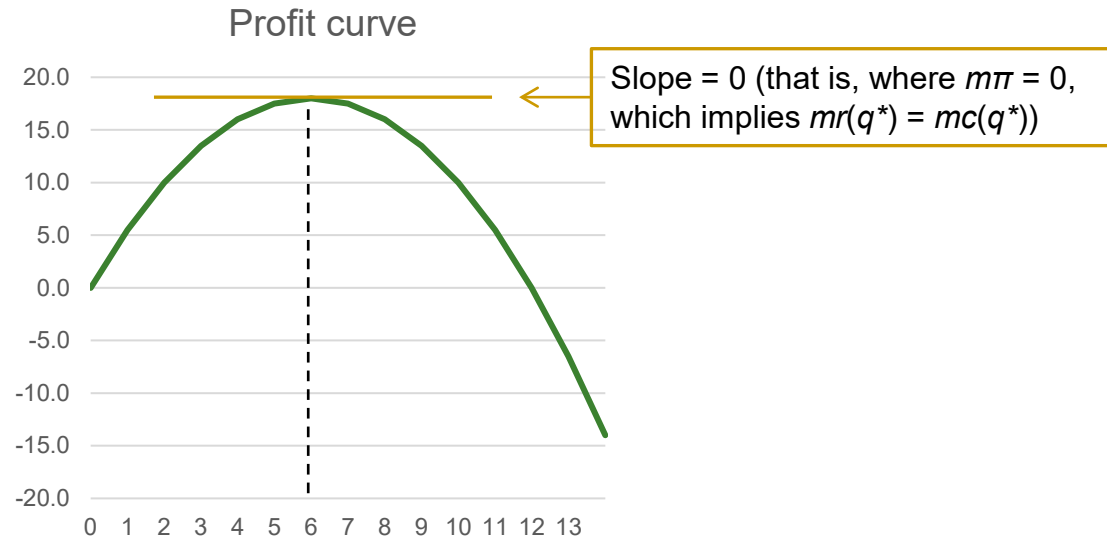
■ First order condition—Example

where

$$p = 10 - \frac{1}{2} q$$

$$f = 0$$

$$c = 4$$



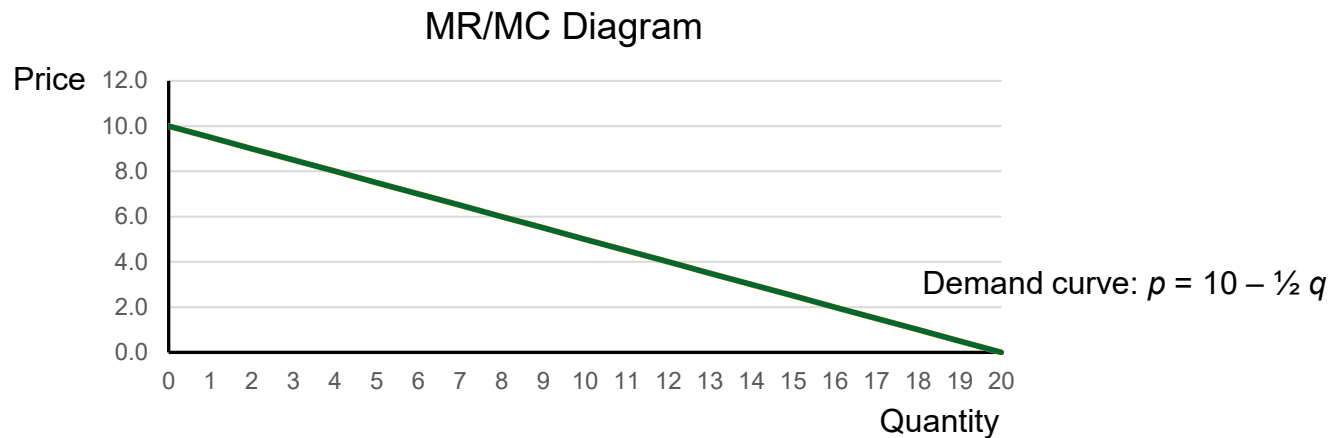
1. $r(q) = p(q)q = (10 - \frac{1}{2} q)q = 10q - \frac{1}{2} q^2$
2. $mr(q) = 10 - q$ (from the formula on Slide 14)
3. $mc(q) = 4$ (from the hypothetical)
4. FOC: $mr(q^*) = mc(q^*)$
So $10 - q^* = 4$ or $q^* = 6$ (as shown in the diagram)
5. $p^* = p(q^*) = 10 - \frac{1}{2} q^*$
 $= 10 - (\frac{1}{2})(6) = 7$ (from the inverse demand curve)

Profit maximization

- Marginal revenue/marginal cost diagrams

- Will build this step-by-step in five steps

→ a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2} q$



Profit maximization

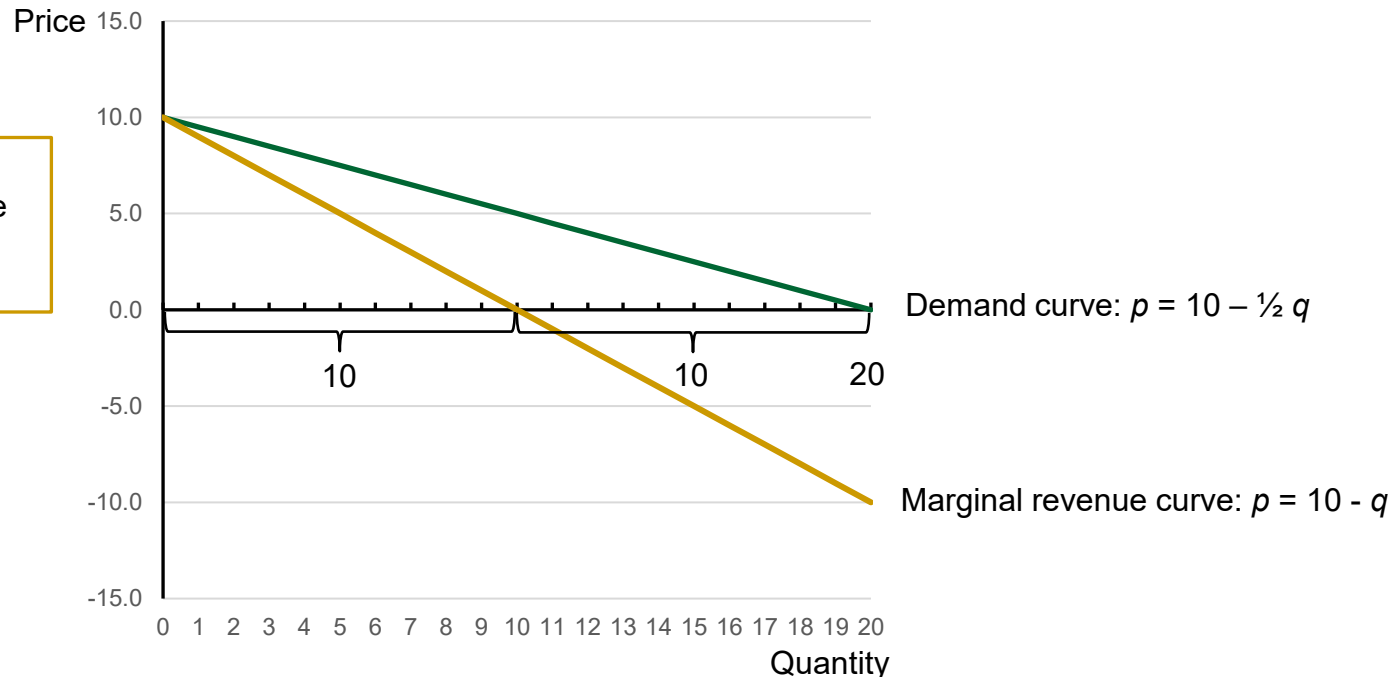
■ Marginal revenue/marginal cost diagrams

- Will build this step-by-step

a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2}q$

→ b. Add the marginal revenue curve: $p = 10 - q$

MR/MC Diagram



Note: With linear demand, the marginal revenue curve falls twice as fast as the inverse demand curve

Profit maximization

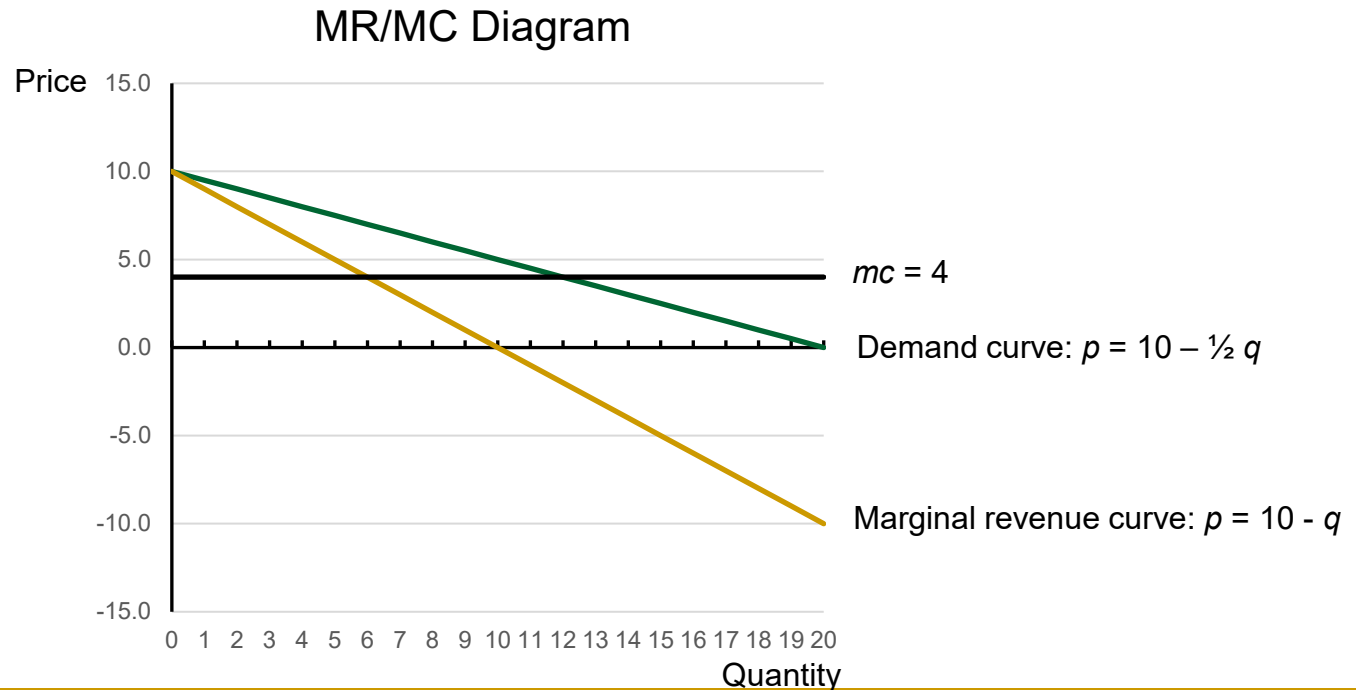
■ Marginal revenue/marginal cost diagrams

- Will build this step-by-step

- a. Consider an (inverse) demand curve: $p = 10 - \frac{1}{2}q$

- b. Add the marginal revenue curve: $p = 10 - q$

- c. Add the marginal cost curve: $c = 4$ (constant marginal cost)



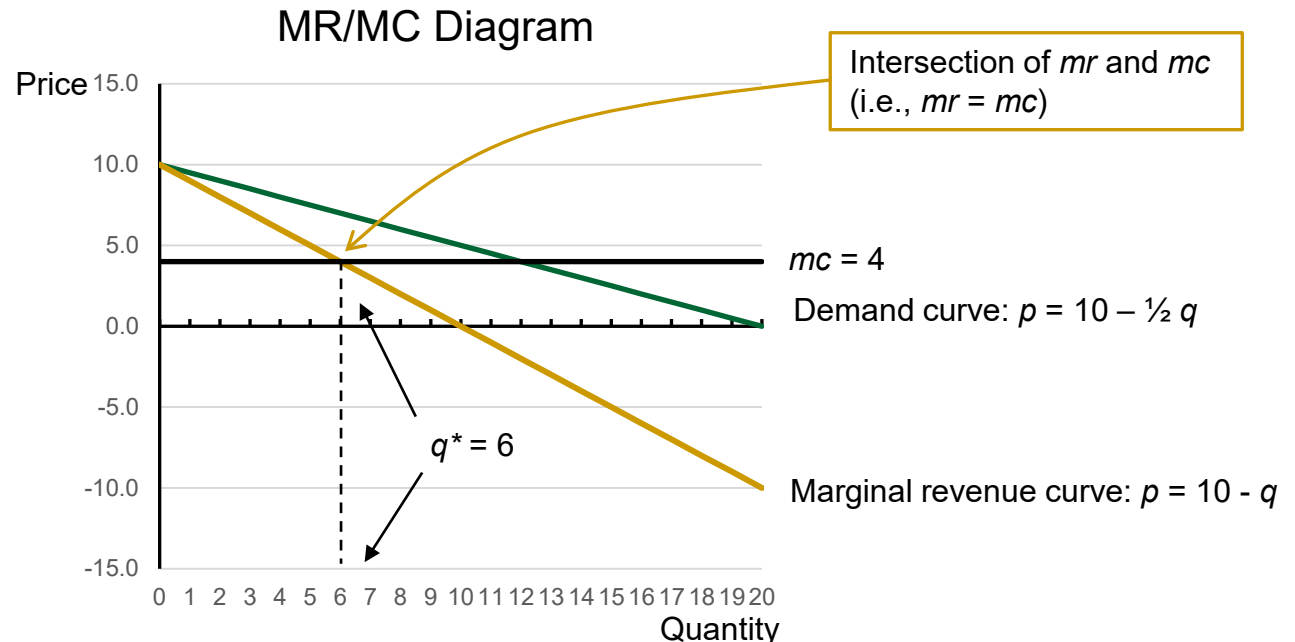
Profit maximization

■ Marginal revenue/marginal cost diagrams

- Will build this step-by-step

- Consider an (inverse) demand curve: $p = 10 - \frac{1}{2}q$
- Add the marginal revenue curve: $p = 10 - q$
- Add the marginal cost curve: $c = 4$ (constant marginal cost)

→ d. Find intersection of mr and mc curves to determine profit-maximizing q^* (= 6)



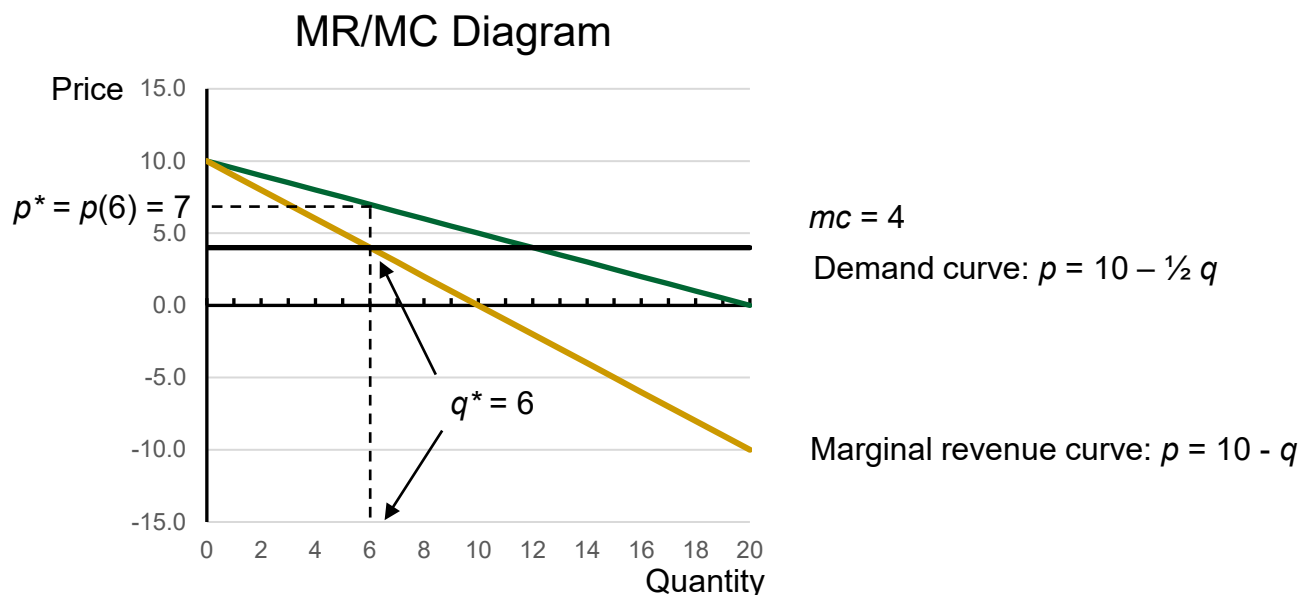
Profit maximization

■ Marginal revenue/marginal cost diagrams

- Will build this step-by-step

- Consider an (inverse) demand curve: $p = 10 - \frac{1}{2}q$
- Add the marginal revenue curve: $p = 10 - q$
- Add the marginal cost curve: $c = 4$ (constant marginal cost)
- Find intersection of mr and mc curves to determine profit-maximizing q^* ($= 6$)

→ e. Find $p^* = p(q^*)$ from the inverse demand curve ($p^* = 7$)

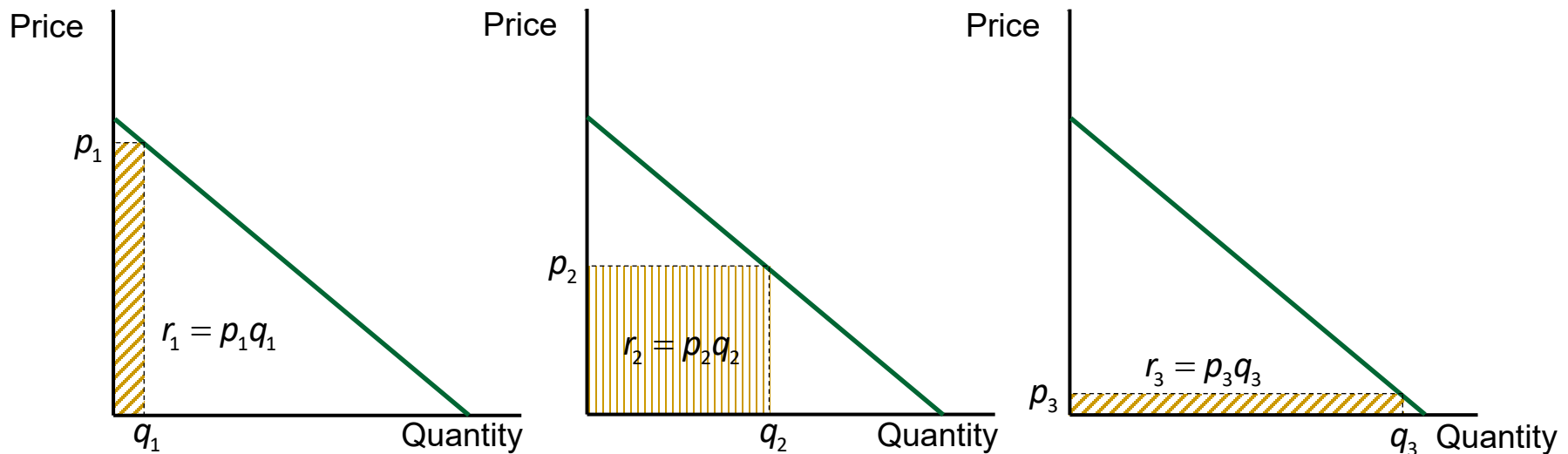


2. Incremental Revenue and Profits

Incremental revenue

■ Introduction

- *Incremental revenue* is the net gain in revenue that a firm could earn if it were to increase its product by some discrete amount Δq
- Incremental revenue is important when determining whether a firm should change its output level to increase its profits
- Incremental revenue can be positive or negative
 - Moving from q_1 to q_2 increases revenue (incremental revenue is positive)
 - Moving from q_2 to q_3 decreases revenue (incremental revenue is negative)



Incremental revenue

- Think about incremental revenue (IR) in two parts:
 1. The *gain* in revenue due to the sale of the additional (marginal) units at the lower market-clearing price
 2. Minus the revenue loss on the inframarginal units due to the lower price
- We can express this mathematically:
 - Let p and q be the starting price and quantity
 - Let Δq be the additional quantity to be sold (the marginal units)
 - Let Δp is the market price decrease necessary to clear the market with the sale of the additional units (let Δp be the absolute value of the price decrease, so that it is a positive number that we subtract from p to find the new price)

Then:

- $\Delta q(p - \Delta p)$ is the revenue gain on sale of the additional (marginal) units
 - = marginal units times the new price
- $q\Delta p$ is the revenue loss on the sale of the inframarginal units
 - = original (inframarginal) units times the price decrease

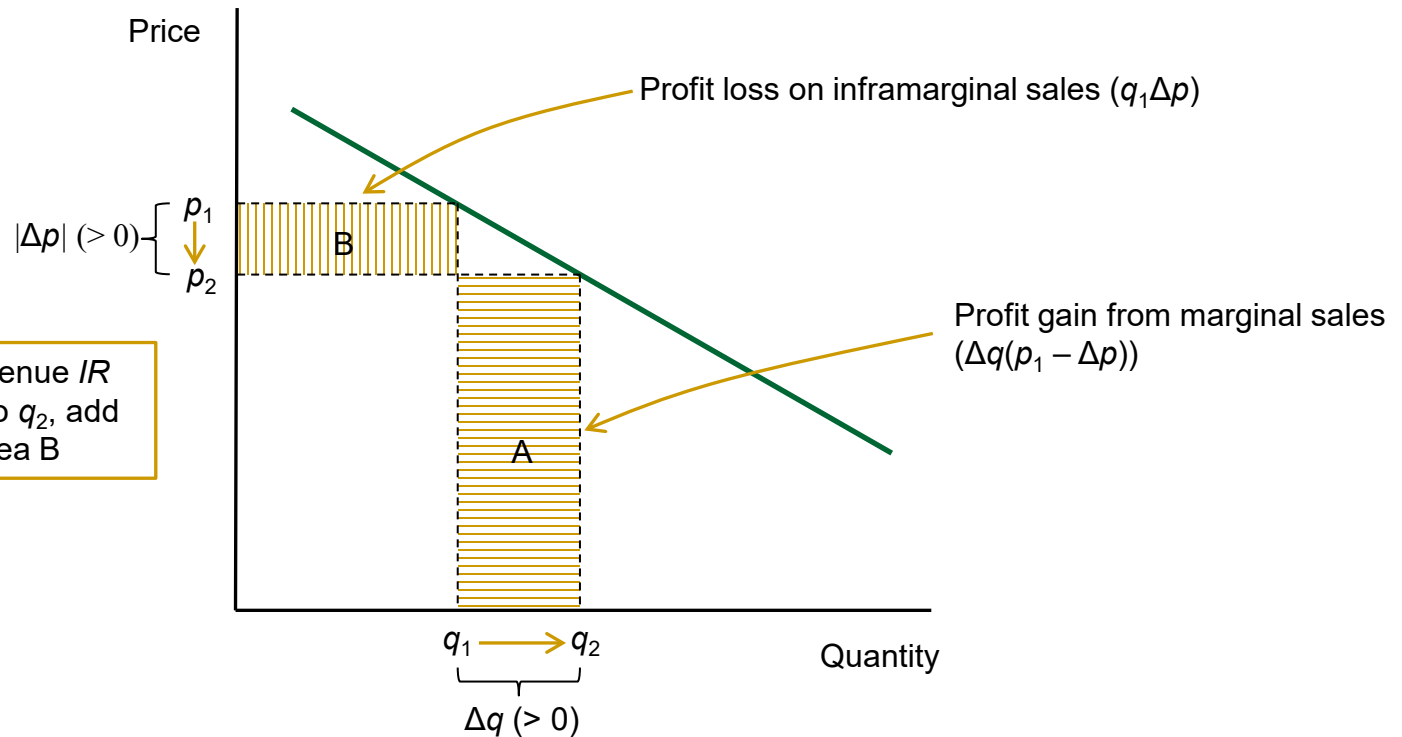
■ So:

$$IR = \underbrace{\Delta q(p - \Delta p)}_{\text{Profit gain on marginal sales}} - \underbrace{q\Delta p}_{\text{Profit loss on inframarginal sales}}$$

This is the formula for *marginal revenue* in the discrete case when $\Delta q = 1$

Incremental revenue

- We can see this graphically:



To find incremental revenue IR when moving from q_1 to q_2 , add Area A and subtract Area B

Area A = $\Delta q(p_1 - \Delta p)$ is the *gain* in revenue from the additional sales Δq at the lower price $p_2 = p_1 - \Delta p$
 Area B = $q_1 \Delta p$ is the *loss* in revenue due to the sales of q_1 at the lower price p_2

So

$$IR = \overbrace{\Delta q(p_1 - \Delta p)}^{\text{Area A}} - \overbrace{q_1 \Delta p}^{\text{Area B}}$$

Incremental revenue

■ Example

- (Inverse) demand: $p = 10 - \frac{1}{2}q$
- Starting point: $q_1 = 4$
- End point: $q_2 = 8$

You need to calculate these variables:

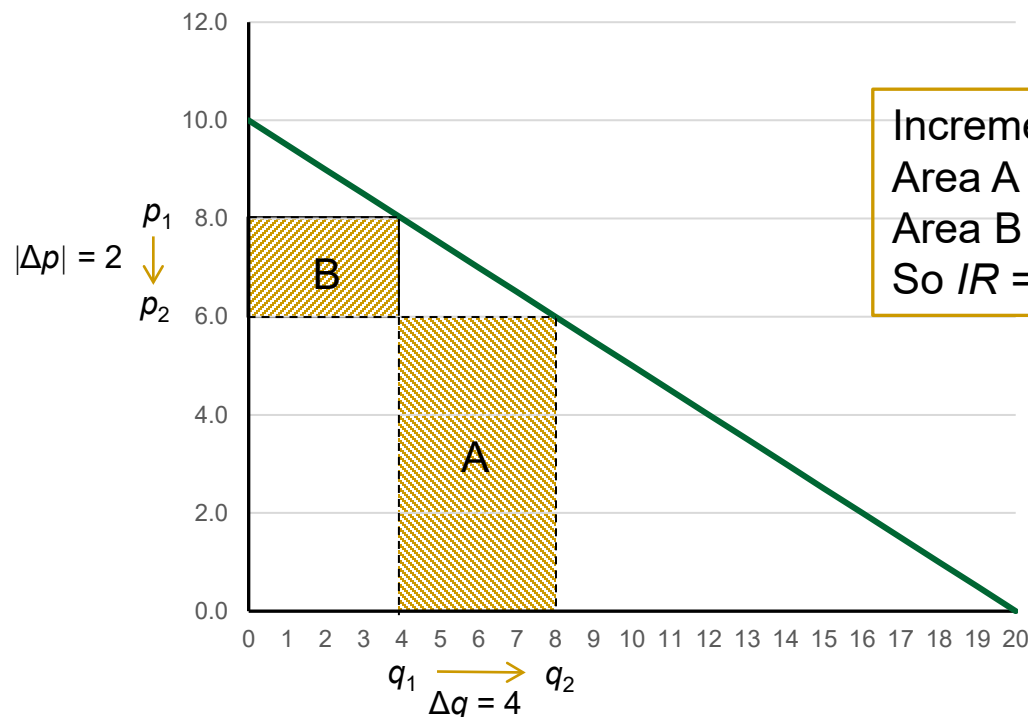
$$\text{So } p_1 = 8$$

$$\text{So } p_2 = 6$$

$$\Delta q = q_2 - q_1 = 8 - 4 = 4$$

$$|\Delta p| = |p_2 - p_1| = |6 - 8| = 2$$

Incremental Revenue Analysis



Incremental revenue = Area A – Area B

$$\text{Area A} = p_2 \Delta q = (6)(4) = 24$$

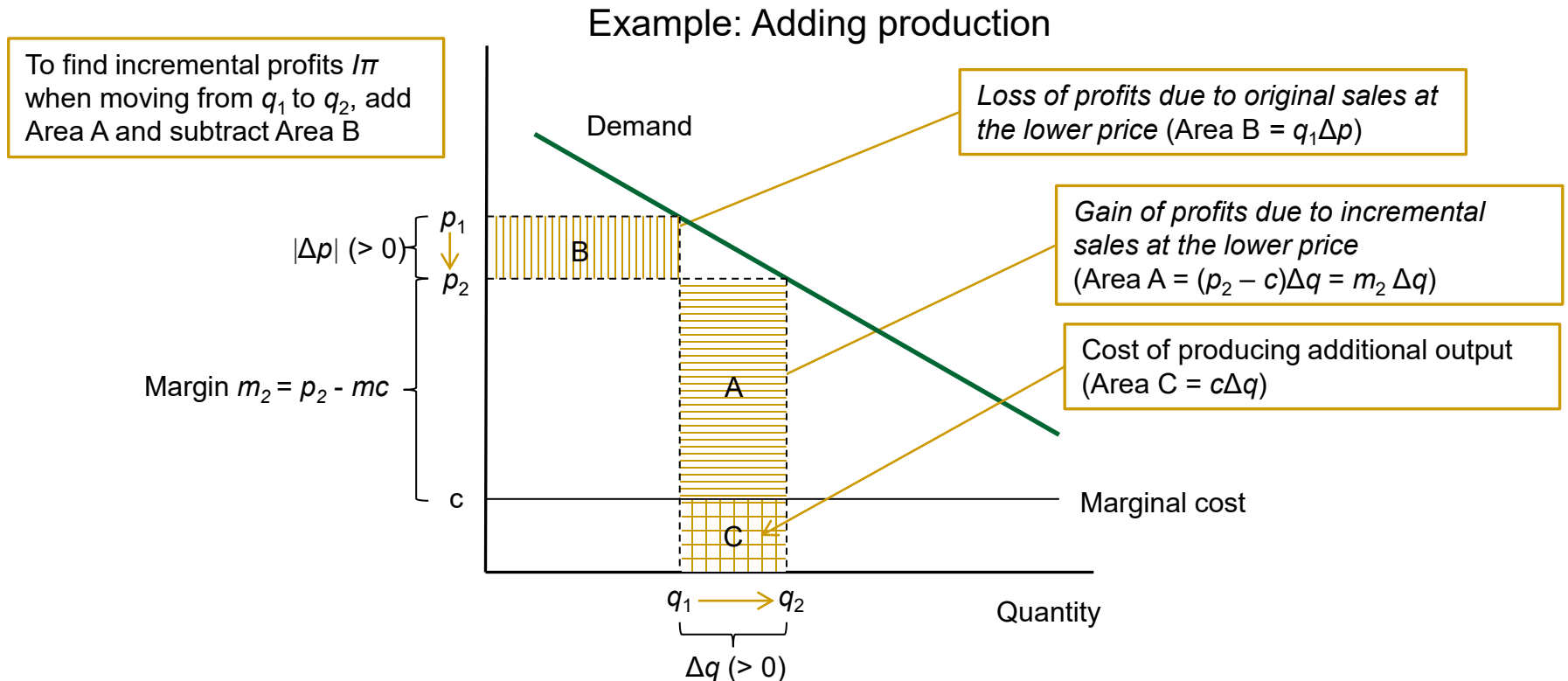
$$\text{Area B} = q_1 \Delta p = (4)(2) = 8$$

$$\text{So } IR = 24 - 8 = 16$$

That is, the firm makes \$16 more in revenues by moving from q_1 to q_2

Incremental profit

- We can easily extend the analysis of incremental revenues to incremental profits—We just have to:
 - Add the costs of additional production if we are adding to output ($\Delta q > 0$), or
 - Subtract the costs if we are reducing output ($\Delta q < 0$)



Incremental profit

- **Example: Output increase**
 - (Inverse) demand: $p = 10 - \frac{1}{2}q$
 - Starting point: $q_1 = 2$
 - End point: $q_2 = 6$
 - Constant marginal cost $c = 4$

You need to calculate these variables:

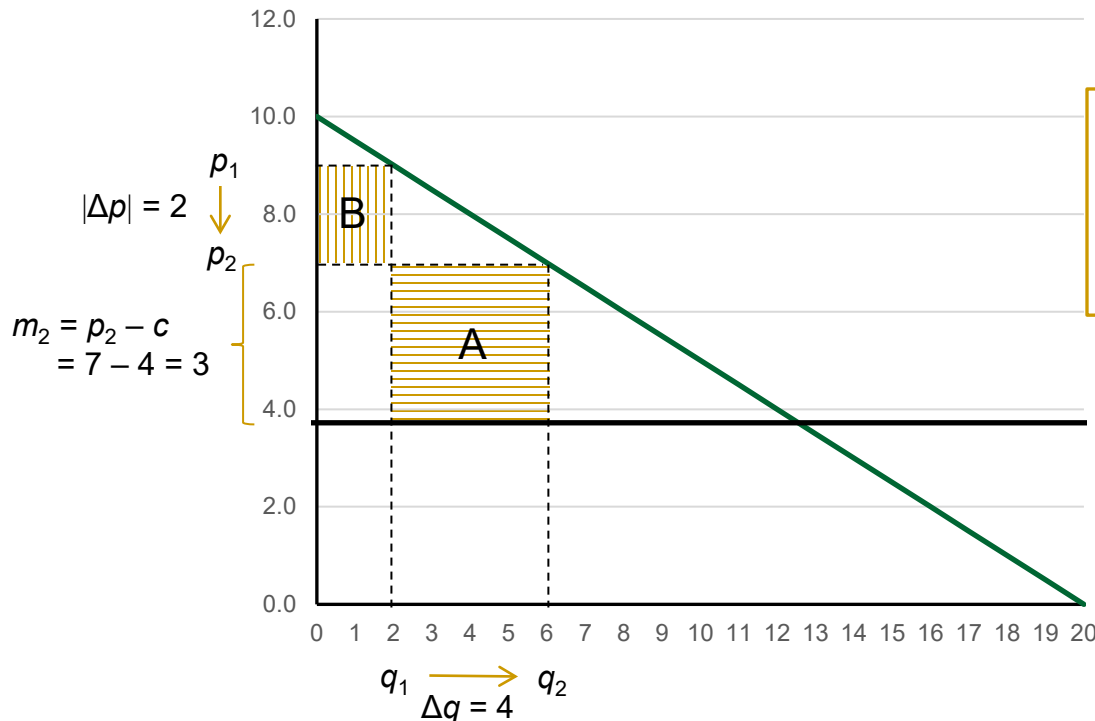
$$\text{So } p_1 = 9$$

$$\text{So } p_2 = 7$$

$$\Delta q = q_2 - q_1 = 6 - 2 = 4$$

$$|\Delta p| = |p_2 - p_1| = |7 - 9| = 2$$

$$\begin{aligned} \text{Margin } m_2 &= p_2 - c \\ &= 7 - 4 = 3 \end{aligned}$$



Incremental profits = Area A – Area B
 Area A = $m_2 \Delta q = (3)(4) = 12$
 Area B = $q_1 \Delta p = (2)(2) = 4$
 So $I\pi = 12 - 4 = 8$

That is, the firm makes \$8 more in profits by moving from q_1 to q_2

Incremental profit

■ Example: Price increase (decreasing production)

- (Inverse) demand: $p = 10 - \frac{1}{2}q$
- Starting point: $p_1 = 5$
- End point: $p_2 = 5.25$
- Constant marginal cost $c = 4$

You need to calculate these variables:

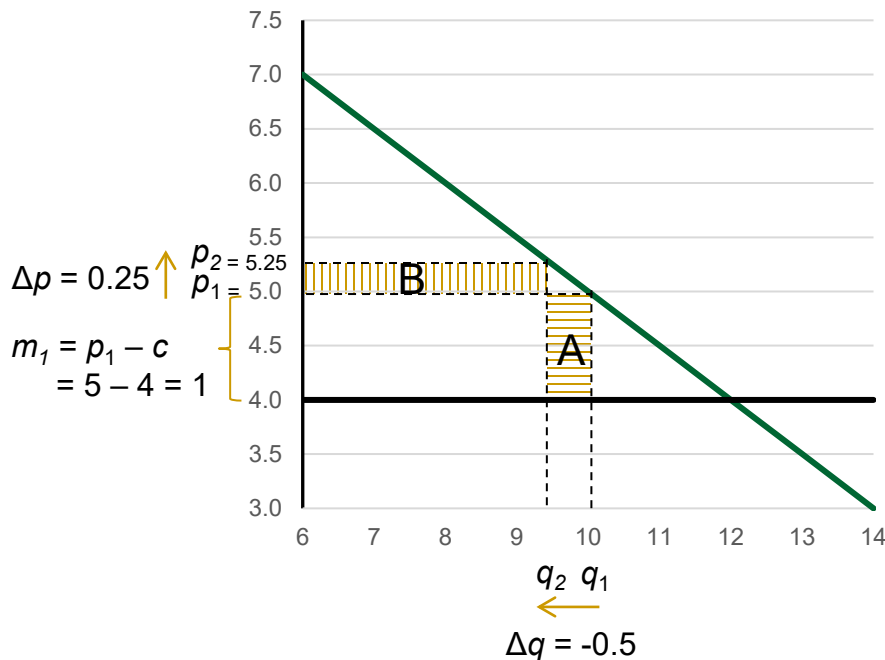
$$\text{So } q = 20 - 2p$$

$$\text{So } q_1 = 10$$

$$\text{So } q_2 = 9.5$$

$$\Delta q = q_2 - q_1 = 9.5 - 10 = -0.5$$

$$\Delta p = p_2 - p_1 = 5.25 - 5 = 0.25$$



With an increase price and a concomitant *reduction* in output, the roles of Areas A and B are *reversed*:

Area A now represents the *loss* of profits from lost sales that would have been made at original price p_1 ($= m_1 \Delta q$)

Area B represents the *gain* of profits from the increased price charged on the sales that continue to be made ($= q_2 \Delta p$)

Incremental profits = Area B – Area A
 Area B = $q_2 \Delta p = (9.5)(0.25) = 2.375$
 Area A = $m_1 \Delta q = (1)(-0.5) = -0.5$
 So incremental profits = $2.375 - 0.5 = 1.875$

Incremental profit

■ Observations

- The prior example shows that under the conditions of the hypothetical, a 5 percent price increase would be profitable to the firm

This is mathematically identical to the exercise required by the *hypothetical monopolist test*, which is the primary analytical tool used by the agencies and the courts to define relevant markets. The hypothetical monopolist test asks whether a hypothetical monopolist of the candidate market could profitably sustain a “small but significant and nontransitory increase in price” (SSNIP), usually taken to be 5 percent. If so, the candidate market is a relevant market. In the prior example, if we assume that the demand curve is for the candidate market as a whole, this will be the residual demand curve for the hypothetical monopolist. If the original market price was \$5 (as in the hypothetical), the hypothetical monopolist would find it profitable to reduce output in order to raise price by a 5 percent SSNIP.

We will confront the hypothetical monopolist test in almost every case study going forward, starting with the H&R Block/TaxAct case study next week. You will have plenty of opportunities to become familiar with the mechanics of the hypothetical monopolist test.

Appendix 1: Inverting Demand and Inverse Demand Functions

Inverting demand and inverse demand functions

■ Motivation

□ You will be given either the demand function or the inverse demand function in a problem. But you may need to derive the other function in order to solve the problem.

□ Example

■ In the price increase problem on Slide 41, you were given the inverse demand function:

$$p = 10 - \frac{1}{2}q$$

■ But the problem gave you p_1 and p_2 and required you to calculate q_1 and q_2 . To do this, you need to convert the inverse demand function into the demand function, so that you could use the prices to calculate the associated quantities

■ To create the demand function, you need to algebraically manipulate the inverse demand equation to isolate q on the left-hand side, so that quantities (which you need) are expressed in terms of prices (which the problem gives you)

Inverting demand and inverse demand functions

■ Mechanics

- An equality is maintained if you perform the same operation to both sides of the equation
- Here are the steps to convert the above inverse demand function to a demand function:

Add $\frac{1}{2}q$ to both sides:

$$p + \frac{1}{2}q = 10 - \frac{1}{2}q + \frac{1}{2}q$$
$$= 10$$

Subtract p from both sides:

$$p + \frac{1}{2}q - p = 10 - p$$

Simply:

$$\frac{1}{2}q = 10 - p$$

Multiply both sides by 2:

$$(2)\left(\frac{1}{2}q\right) = (2)(10 - p)$$

Simply:

$$q = 20 - 2p$$

This is the demand curve that you would need for the price increase incremental revenue problem

- The same technique can be used to convert a demand curve into an inverse demand curve

Inverting demand and inverse demand functions

- Or use an algebraic calculator:

The screenshot shows the MathPapa Algebra Calculator interface. At the top, there is a navigation bar with "MathPapa" and links for "ALGEBRA CALCULATOR", "PRACTICE", and "LESS". Below this, the title "Algebra Calculator" is displayed. A text input field contains the equation $p = 10 - \left(\frac{1}{2}q\right)$. To the right of the input field is a yellow button labeled "CALCULATE IT!". Below the input field is a blue button labeled "Solve for Variable". Underneath, there is a "Solve for:" dropdown menu with "q" selected. The main content area shows the following steps:
Let's solve for q.
$$p = 10 - \frac{1}{2}q$$

Step 1: Flip the equation.
$$\frac{-1}{2}q + 10 = p$$

Step 2: Add -10 to both sides.
$$\frac{-1}{2}q + 10 + -10 = p + -10$$

$$\frac{-1}{2}q = p - 10$$

Step 3: Divide both sides by (-1)/2.
$$\frac{\frac{-1}{2}q}{\frac{-1}{2}} = \frac{p-10}{\frac{-1}{2}}$$

$$q = -2p + 20$$

At the bottom, the "Answer:" is given as $q = -2p + 20$. A yellow box highlights the text "which is the same as the $20 - 2p$ we derived on the previous slide".

We want q on the right-hand side, so solve for q