Unit 8. Competition Economics

Part 2. Markets and Market Equilibria

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September 26, 2024

Topics

- Substitutes, complements, and elasticities
- Markets and market equilibria
  - Perfectly competitive markets
  - Perfectly monopolized markets
  - Imperfectly competitive markets
    - Cournot oligopoly models
    - Bertrand oligopoly models
    - Dominant firm with a competitive fringe

Substitutes, Complements, Elasticities, and Diversion Ratios

Substitutes/Complements

- Substitutes
  - *Definition*: Two products or services are *substitutes* if, when consumer demand increases for one product, it will decrease for the other product
    - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} < 0$$

> Because Δq_1 and Δq_2 move in opposite directions, they will have different signs (i.e., one will be positive and the other will be negative) and the fraction will be negative

  - Examples
    - Coke and Pepsi
    - iPhone and Galaxy S series mobile phones
    - Nike and Adidas shoes
    - Hertz and Avis rental cars
  - *Horizontal mergers* involve combinations of firms that offer substitute products

Substitutes/Complements

- Substitutes
  - Substitutes and prices
    - If products 1 and 2 are substitutes, then as the price of product 1 increases, the demand for product 2 increases:

A negative number times a negative number is a positive number

$$\overset{(-)}{\frac{\Delta q_2}{\Delta q_1}}\ \overset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \overset{(+)}{\frac{\Delta q_2}{\Delta p_1}} > 0$$

Slope of the demand curve for product 1 (< 0 since downward sloping)

Substitutes/Complements

- Complements
  - *Definition*: Two products are *complements* if, when consumer demand increases for one product, consumer demand also will increase for the other product
  - Symbolically:

$$\frac{\Delta q_2}{\Delta q_1} > 0$$

  - Examples
    - *Vertical mergers* involve complements
      - Television LCD screens and TV sets
      - Car engines and cars
      - Cable TV programming and cable TV distribution (AT&T/Time Warner)
      - Drug manufacture and drug distribution
    - But some conglomerate mergers can also involve complements
      - Printers and ink cartridges
      - Razors and razor blades
      - Computers and computer software

Substitutes/Complements

- Complements
  - Complements and prices
    - If products 1 and 2 are complements, then as the price of product 1 increases, the demand for product 2 decreases

A positive number times a negative number is a negative number

$$\overset{(+)}{\frac{\Delta q_2}{\Delta q_1}}\;\overset{(-)}{\frac{\Delta q_1}{\Delta p_1}} = \overset{(-)}{\frac{\Delta q_2}{\Delta p_1}} < 0$$

Slope of the demand curve for product 1 (< 0 since downward sloping)

Elasticities

- Own-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded divided by the percentage change in the price of that *same* product

The Greek letter epsilon (ε) is the usual symbol in economics for elasticity

$$\varepsilon \equiv \frac{\%\Delta q_i}{\%\Delta p_i}$$

Percentage change q_i in the quantity of product i demanded

Percentage change p_i in the price of product i

  - This is sometimes called *elasticity of demand* or *price elasticity of demand*
  - Own-elasticities are always *negative in sign* since changes in prices and quantities move in opposite directions along a downward-sloping demand curve
- Examples:
  - If price increases by 5% and demand decreases by 10%, then the own-elasticity is -2 (= -10%/5%)
  - If price increases by 3% and demand deceases by 1%, then the own-elasticity is -1/3 (= -1%/3%)

Technically, these are *arc elasticities* because they give percentage changes for discrete changes in prices and quantities

Elasticities

- Own-elasticity of demand: Some numerical estimates

Product	ε	Product	ε
Salt	0.1	Movies	0.9
Matches	0.1	Shellfish, consumed at home	0.9
Toothpicks	0.1	Tires, short-run	0.9
Airline travel, short-run	0.1	Oysters, consumed at home	1.1
Residential natural gas, short-run	0.1	Private education	1.1
Gasoline, short-run	0.2	Housing, owner occupied, long-run	1.2
Automobiles, long-run	0.2	Tires, long-run	1.2
Coffee	0.25	Radio and television receivers	1.2
Legal services, short-run	0.4	Automobiles, short-run	1.2-1.5
Tobacco products, short-run	0.45	Restaurant meals	2.3
Residential natural gas, long-run	0.5	Airline travel, long-run	2.4
Fish (cod) consumed at home	0.5	Fresh green peas	2.8
Physician services	0.6	Foreign travel, long-run	4.0
Taxi, short-run	0.6	Chevrolet automobiles	4.0
Gasoline, long-run	0.7	Fresh tomatoes	4.6

Source: Preston McAfee & Tracy R. Lewis, Introduction to Economic Analysis ch. 3.1 (2009)

Elasticities

- Own-elasticity of demand
  - Relationship to the slope of the residual demand curve:

$$\varepsilon_i \equiv \frac{\%\Delta q_i}{\%\Delta p_i} \equiv \frac{\frac{\Delta q_i}{q_i}}{\frac{\Delta p_i}{p_i}} = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i},$$

Slope of the demand curve

Rearranging terms

that is, the own-elasticity at a point on the firm's residual demand curve is equal to the slope of the residual demand curve at that point times the ratio of price to quantity at that point

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  - *Mathematical note (optional)*
    - *In calculus terms:*

$$\varepsilon_i \equiv \frac{dq_i}{dp_i}\frac{p_i}{q_i}$$

This deals with the continuous case

Elasticities

For intuition only (NOT technically correct, but it is usually the intuition that is important)

- Some important definitions
  - *Inelastic demand*: Not very price sensitive

$$|\varepsilon| = \left| \frac{\%\text{change in quantity}}{\%\text{change in price}} \right| < 1$$

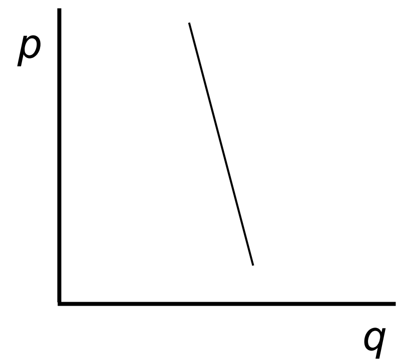
  - *Unit elasticity*:

$$|\varepsilon| = \left| \frac{\%\text{change in quantity}}{\%\text{change in price}} \right| = 1$$

  - *Elastic demand*: Price sensitive

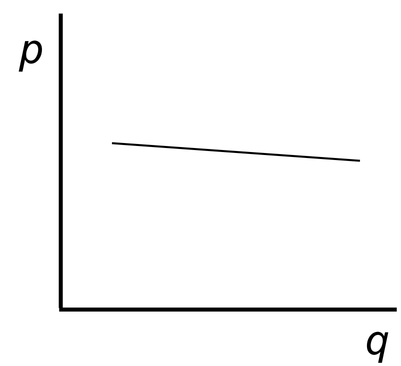
$$|\varepsilon| = \left| \frac{\%\text{change in quantity}}{\%\text{change in price}} \right| > 1$$

Inelastic demand



Little sensitivity to changes in price

Elastic demand



More sensitivity to changes in price

Note: |*x*| is the *absolute value* of *x*, which is the magnitude of *x* without the sign. So |3| = |-3| = 3.

Elasticities

Remember $\varepsilon = \dfrac{\Delta q_i}{\Delta p_i}\dfrac{p_i}{q_i}$

- Elasticity of demand and the slope of the demand curve
  - Even when the demand curve is linear (so that the slope is constant), elasticity varies along the demand curve because the ratio of p_i to q_i changes along the curve

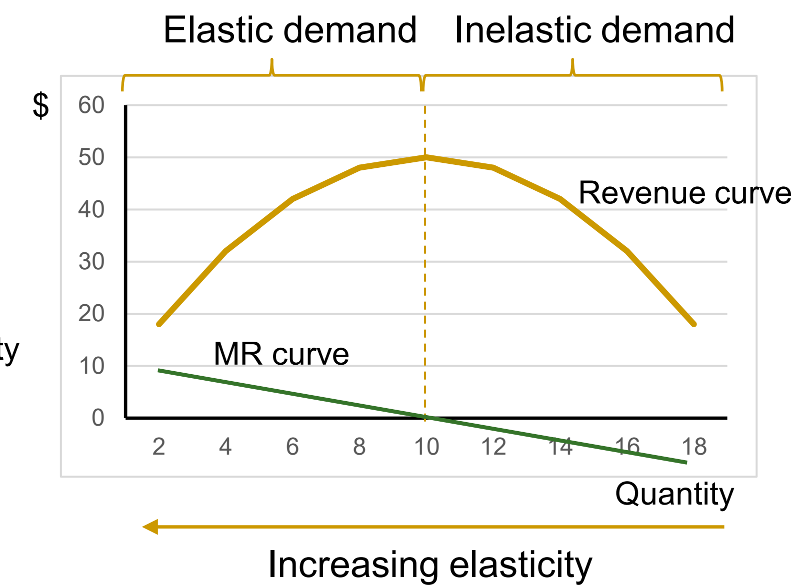
Inverse demand curve:
$p = 20 - 2q$

p	*q*	Slope	*p*/*q*	ε	Total revenue
1	18	-2	0.0556	-0.1111	18
2	16	-2	0.1250	-0.2500	32
3	14	-2	0.2143	-0.4286	42
4	12	-2	0.3333	-0.6667	48
5	10	-2	0.5000	-1.0000	50
6	8	-2	0.7500	-1.5000	48
7	6	-2	1.1667	-2.3333	42
8	4	-2	2.0000	-4.0000	32
9	2	-2	4.5000	-9.0000	18

Inelastic demand $|\varepsilon| < 1$

Unit elasticity $|\varepsilon| = 1$

Elastic demand $|\varepsilon| > 1$



General rules:
Elasticity decreases as quantity increases and prices decreases → lower *p*/*q* ratios
Elasticity increases as quantity decrease and prices increase → higher *p*/*q* ratios

Elasticities

- Predicting quantity changes for a given price increase
  - An approximation
    - We can approximate a percentage quantity change $\%\Delta q$ for a given percentage price change $\%\Delta p$ by multiplying the own-elasticity ε by the percentage price change:

$$\varepsilon = \frac{\%\Delta q}{\%\Delta p} \Rightarrow \%\Delta q \approx \varepsilon \%\Delta p$$

      - The relationship is not exact since the elasticity can change over the discrete range of the price change (as it does on a linear demand function)
  - For linear demand curves, an exact relationship exists for a price change Δp :

$$\varepsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q} \Rightarrow \underbrace{\Delta q = \varepsilon \frac{q}{p}\Delta p}_{\text{For predicting unit quantity changes}} \text{ and } \underbrace{\frac{\Delta q}{q} = \varepsilon \frac{\Delta p}{p}}_{\text{For predicting percentage quantity changes}}$$

These relationships can be important when determining a quantity change associated with a price increase in the hypothetical monopolist test for market definition

Elasticities

- The *Lerner condition* for profit-maximizing firms
  - *Proposition*: When a firm *i* maximizes its profits, at the profit-maximum levels of price and output the firm's own elasticity ε_i is equal to $1/m_i$:

$$\left|\varepsilon_i\right| = \frac{1}{m_i},$$

where *m* is the *gross margin*:

$$m_i \equiv \frac{p_i - c}{p_i}$$

Proof (optional): The firm's first order condition for a profit-maximum:

Marginal revenue = Marginal cost

Mathematically $$p_i + \frac{dp}{dq} q_i = c_i$$

Rearranging and dividing by *p*: $$\frac{p_i - c_i}{p} = -\frac{dp}{dq}\frac{q_i}{p_i}$$

$$m_i = \frac{1}{\left|\varepsilon_i\right|}, \text{ so } \left|\varepsilon_i\right| = \frac{1}{m_i} \qquad Q.E.D.$$

Cross-elasticities

- Cross-elasticity of demand
  - *Definition*: The percentage change in the quantity demanded for product *j* divided by the percentage change in the price of product *i*.

$$\varepsilon_{ij} \equiv \frac{\%\Delta q_i}{\%\Delta p_j}$$

Percentage change q_i in the quantity of product *i* demanded

Percentage change p_j in the price of product *j*

- With a little algebra (as before):

$$\varepsilon_{ij} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$$

Positive for substitutes
Negative for complements

- Mathematical note (optional)
  - In calculus terms:

$$\varepsilon_{ij} \equiv \frac{dq_i}{dp_j}\frac{p_j}{q_i}$$

Cross-elasticities

- Cross-elasticities—More definitions
  - *High cross-elasticity of demand*:
    - A small change in the price of product *i* will cause a large change of demand to product *j*
    - As a result, product *j* brings a lot of competitive pressure on product *i*

Make sure you understand why!

    - *Think of it this way:*
      - In a two-firm market, a high cross-elasticity implies a large number of *marginal customers* who will abandon product *i* when its price increases and will divert to product *j*
      - It also means a correspondingly smaller number of *inframarginal customers* who will stay with product *i* in the wake of a price increase
  - *Low cross-elasticity of demand*:
    - A large change in the price of product *i* will cause only a small change of demand to product *j*
    - As a result, product *j* brings little competitive pressure on product *i*

Make sure you understand why!

This is why antitrust lawyers talk so much about cross-elasticities!

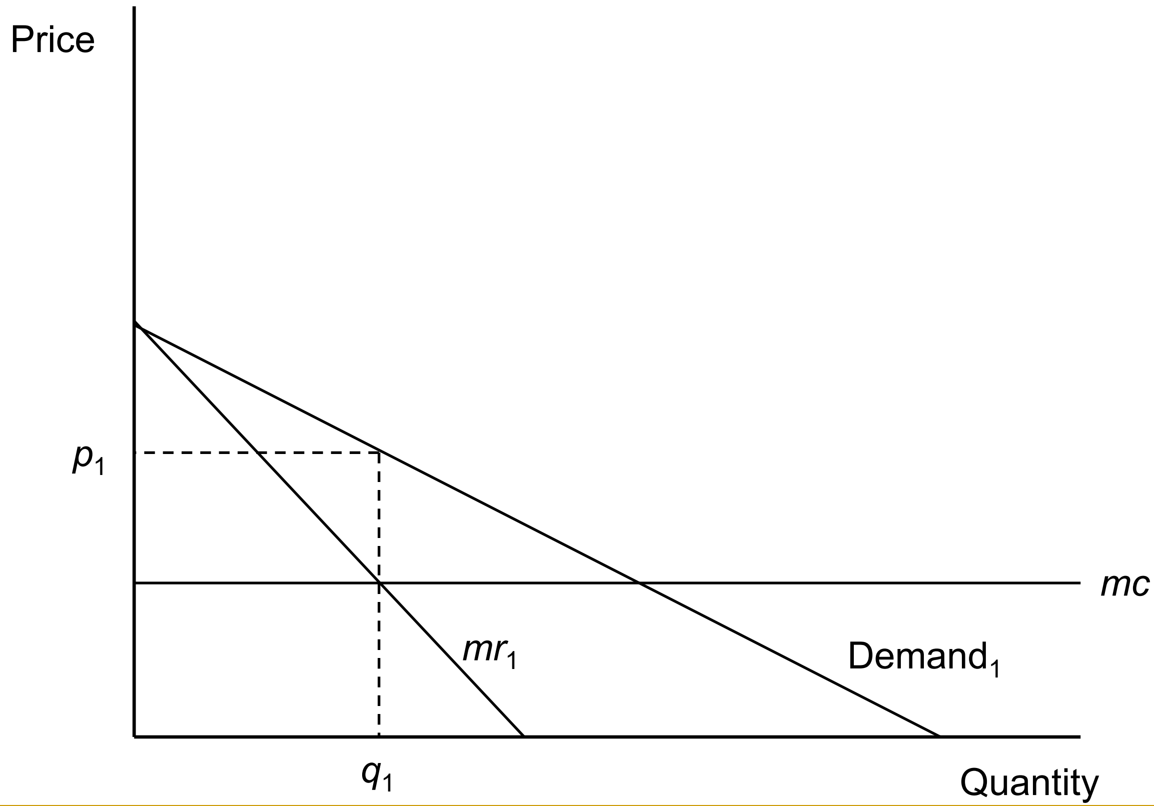
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Intuitively, the higher the cross-elasticities of product A with the other products, the more elastic is product A's own-elasticity
  - Consequently, if a merger has the effect of decreasing the cross-elasticities of product A (say an overlap product of one of the merging firms) with one or more substitute products, then product A's own-elasticity also decreases
  - *Key result*: All other things being equal, decreasing the cross-elasticity of demand of substitute products shifts the intersection of the marginal revenue curve and the marginal cost curve to the left, leading the firm to decrease output and increase prices

Let's look at the next three graphs to see why

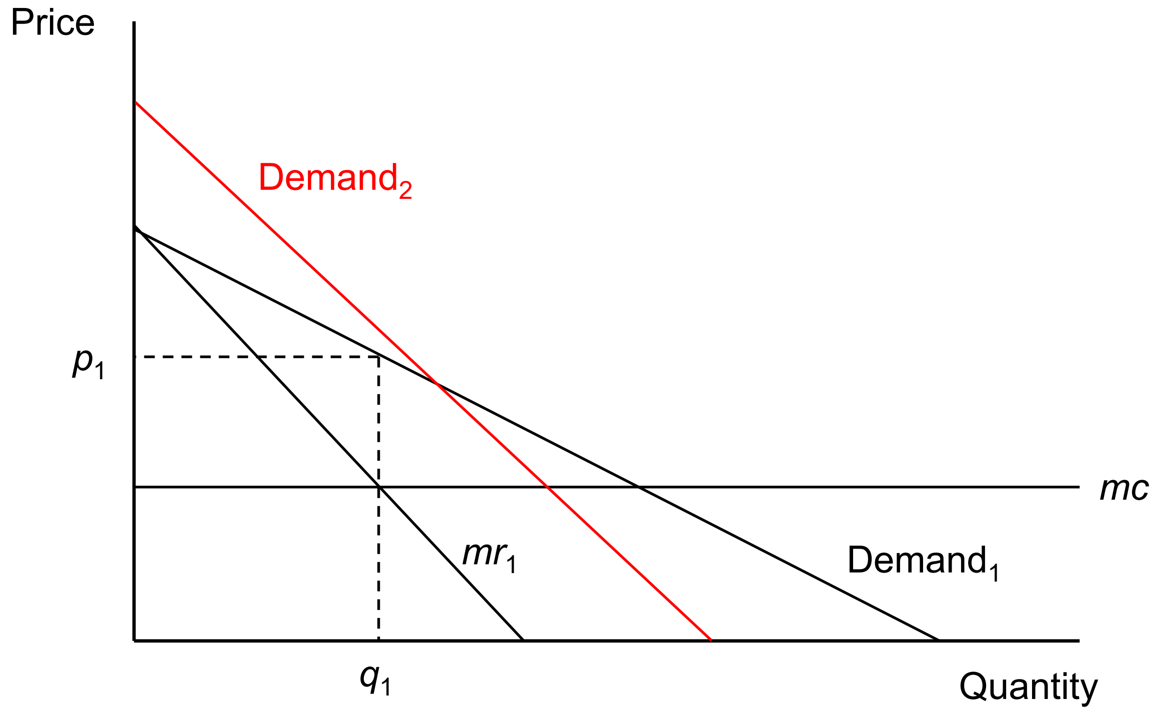
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Premerger profit-maximizing price-quantity equilibrium for the acquiring firm



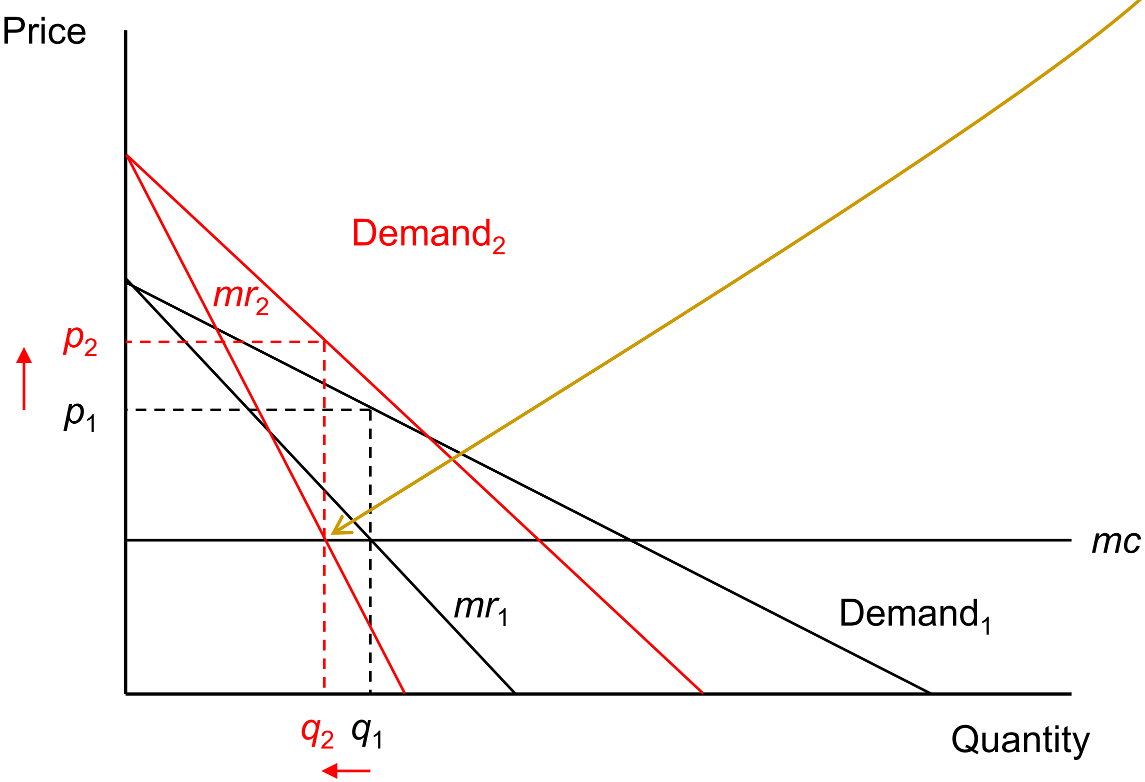
An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Postmerger, the acquiring firm increases the acquired firm's price, making the acquired firm's substitute product less attractive and so decreasing the cross-elasticity of demand with the acquiring firm's product
    - The acquiring firm's residual demand curve then becomes more inelastic (steeper) around the premerger equilibrium point (q_1, p_1)



An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Postmerger, the marginal revenue curve also becomes steeper, moving the postmerger equilibrium to a higher price and lower quantity (q_2, p_2)



An important relationship

- Relationship of own-elasticities to cross-elasticities—
  Equivalent statements:
  - Reducing the attractiveness of substitutes
  - Reducing the cross-elasticities of residual demand of substitute products
  - Making the residual demand curve more inelastic
  - Making the residual demand curve steeper
  - Reducing the residual own-elasticity of demand

Around the premerger price-quantity equilibrium

All result in higher prices and lower quantities

- NB: At this point in the analysis, these relationships are only directional
  - They tell us the *direction* equilibrium price and quantity move
  - But so far, they do not tell us the *magnitude* of the changes
  - So we cannot yet determine whether the change in the cross-elasticities yields a substantial lessening of competition

An important relationship

- Relationship of own-elasticities to cross-elasticities
  - Technically:

$$|\varepsilon_{11}| = 1 + \frac{1}{s_1}\sum_{i=2}^{n} \varepsilon_{i1} s_i$$

$\varepsilon_{i1} > 0$ if the other products are substitutes for product 1

where ε_{11} is the own-elasticity of product 1 and ε_{i1} is the cross-elasticity of substitute product *i* with respect to the price of product 1 (evaluated at current prices and quantities)

  - Two important takeaways
    1. As the cross-elasticities on the right-hand side decrease, the demand for product 1 becomes more inelastic ($|\varepsilon|$ becomes smaller)
       - This allows Firm 1 to exercise market power and charge higher prices
    2. Competitors with larger market shares have more influence in constraining the price of Firm 1 for any given cross-elasticity (i.e., the cross-elasticities in the formula are weighted by market share)

You do not have to know the formula, but you should know the takeaways

Diversion ratios

- *Definition*: Diversion ratio (D)

$$D_{12} \equiv \frac{\text{Units captured by Firm 2 as a result of Firm 1's price increase}}{\text{Total units lost by Firm 1 as a result of Firm 1's price increase}} \equiv \left|\frac{\Delta q_2}{\Delta q_1}\right|$$

  - NB: By convention, diversion ratios are *positive*. Since $\Delta q_1/\Delta p_1$ is negative (the demand curve is downward sloping), we need to look at the absolute value of the fraction

- Example
  - Firm 1 increases its price by 5% and loses a total of 20 units to substitute products
  - When Firm 1 increases its price, Firm 2—which maintains its original price—gains 5 units of additional sales
  - So:

$$D_{12} = \left|\frac{\Delta q_2}{\Delta q_1}\right| = \left|\frac{5}{-20}\right| = \frac{5}{20} = 0.25 = 25\%$$

Diversion ratios

- Thinking about diversion ratios
  - Think of D_{12} as $D_{1\rightarrow2}$, that is—
    1. the number of units lost by Firm 1 that are "diverted" to Firm 2 (which produces a substitute product)
    2. as a result of Firm 1's price increase
    3. when Firm 2's price stays constant

    NB: This heuristic assumes that there is a one-to-one substitution between Firm 1's and Firm 2's products

Diversion ratios

- Relation to cross-elasticities
  - Diversion ratios are closely related to cross-elasticities: both measure the degree of substitutability between two products when the relative prices change
    - Elasticities measure substitutability in terms of the *percentage* increase in Firm 2's unit sales for a *percentage* increase in Firm 1's price
    - Diversion ratios measure substitutability in terms the increase in Firm 2's unit sales as a percentage of all units lost by Firm 1 as a result of a given increase in Firm 1's price
  - Modern antitrust economics still speaks in terms of cross-elasticities when it often means diversion ratios
    - For example, products with high diversion ratios are said to have high cross-elasticities

We will see diversion ratios again in implementations of the hypothetical monopolist test and in the unilateral effects theory of anticompetitive harm

Perfectly Competitive Markets

Perfectly competitive markets

- *Definition*: A market in which no single firm can affect price, meaning—
  1. The firm perceives its residual demand curve as horizontal
  2. The firm perceives that it can sell any amount of product without affecting the market price
  3. $\frac{dp}{dq_i} = 0$ (as perceived by the firm)
  4. $p = \frac{dc}{dq_i}$ (i.e., price = marginal cost)

These four bullets are just different ways of saying the same thing

- Some more definitions
  - *"Price taking"*: Competitive firms are called *price-takers*, that is, they take market price as given and not something that they can affect
  - *Perfectly competitive equilibrium*: A market equilibrium exists when—
    1. Aggregate supply equals aggregate demand, *and*
    2. Each firm chooses its level of production so that the market-clearing price is equal to the firm's marginal cost of production

Perfectly competitive markets

- What could cause a market to be perfectly competitive?
  - *Traditional theory*: Each individual firm's production is very small compared to aggregate demand at any price, so that individual production changes cannot move materially along the aggregate demand curve
    - This implies that there are a very large number of firms in the market
  - *Modern theory*: Competitors in the marketplace react strategically but non-collusively to price or quantity changes by a firm in ways that maintain the perfectly competitive equilibrium

Competitive firms

- Three take-aways
  1. Competitive firms do not perceive that their output decisions affect the market-clearing price
     - That is, each firm perceives that it faces a horizontal residual demand curve
     - In fact, their individual output decisions do affect the market-clearing price but because the effect is so small no individual firm perceives this
       - In the aggregate, the sum of the output of all competitive firms determines the market-clearing price
  2. Competitive firms chose their output so that $p = mc$
     - Competitive firms, like all other firms, choose output so that marginal revenue is equal to marginal cost ($mr = mc$)
     - Since a competitive firm does not perceive that its output decisions affect the market-clearing price, the firm does not perceive that there is any downward adjustment in market price when it expands its output
     - Therefore, the firm perceives—and makes its output decision—on the premise that its marginal revenue is equal to the market price
     - Hence, the firm selects an output level so that $p = mc$
     - Mathematically:

$$mr(q_i) = p + q_i \frac{\Delta p}{\Delta q_i} = mc(q_i)$$

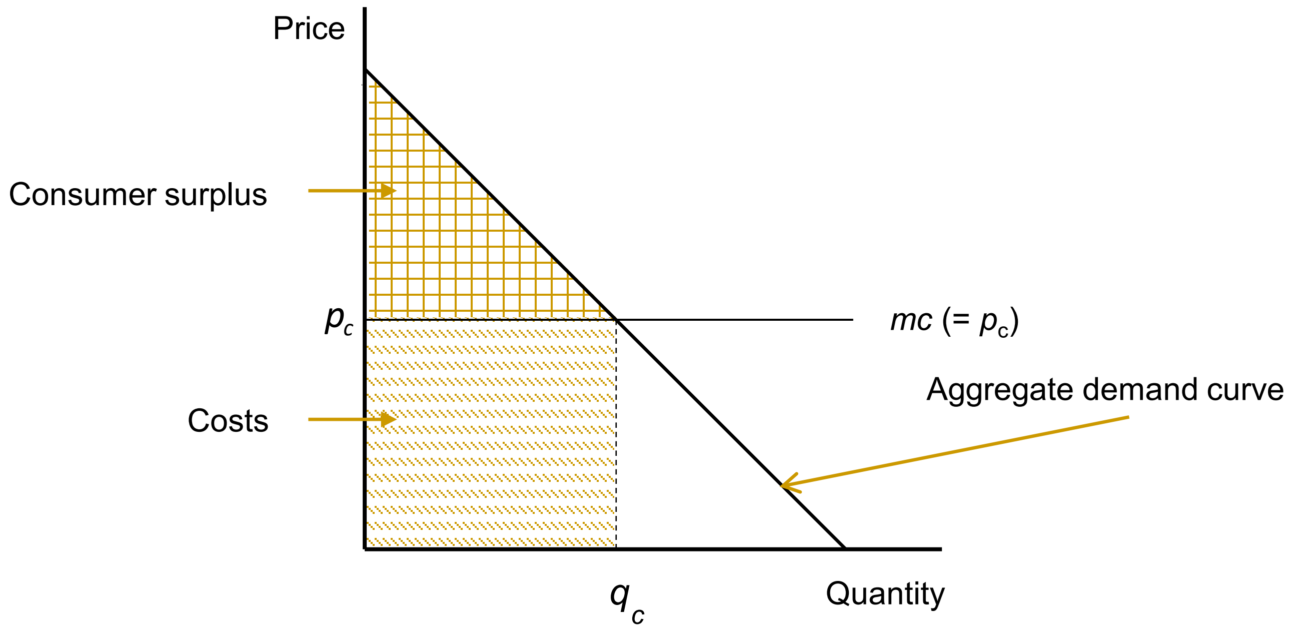
Perceived to be zero since the firm is a price-taker and does not believe that its choice of output affects market price

So:

$$p = mc$$

Competitive firms

- Three take-aways
  3. A competitive market maximizes consumer surplus[1]
     - A competitive market exhausts all gains from trade



[1] We are assuming a simple market where there is only one product that sells at a single uniform price (i.e., there is no price discrimination).

Perfectly Monopolized Markets

Perfect monopoly

- Basic concepts
  - In a perfect monopoly market, there is only one firm that supplies the product
    - This is an economic concept
    - In law, a monopolist need not control 100% of the market
  - Although there is only one firm in the market, it still faces a downward-sloping demand curve
    - There can be some substitutes for the monopolist's product—just not very good ones
  - The aggregate demand curve defines the residual demand curve facing an (economic) monopolist

In economics and in law, a firm that faces a downward-sloping residual demand curve and therefore has some power to influence the market-clearing price for its product is said to have *market power*. In antitrust law, a firm that has very significant power over the market-clearing price is said to have *monopoly power*. In economics, a monopolist is the only firm in the market.

Perfect monopoly

A consequence of the monopolist's downward-sloping demand curve

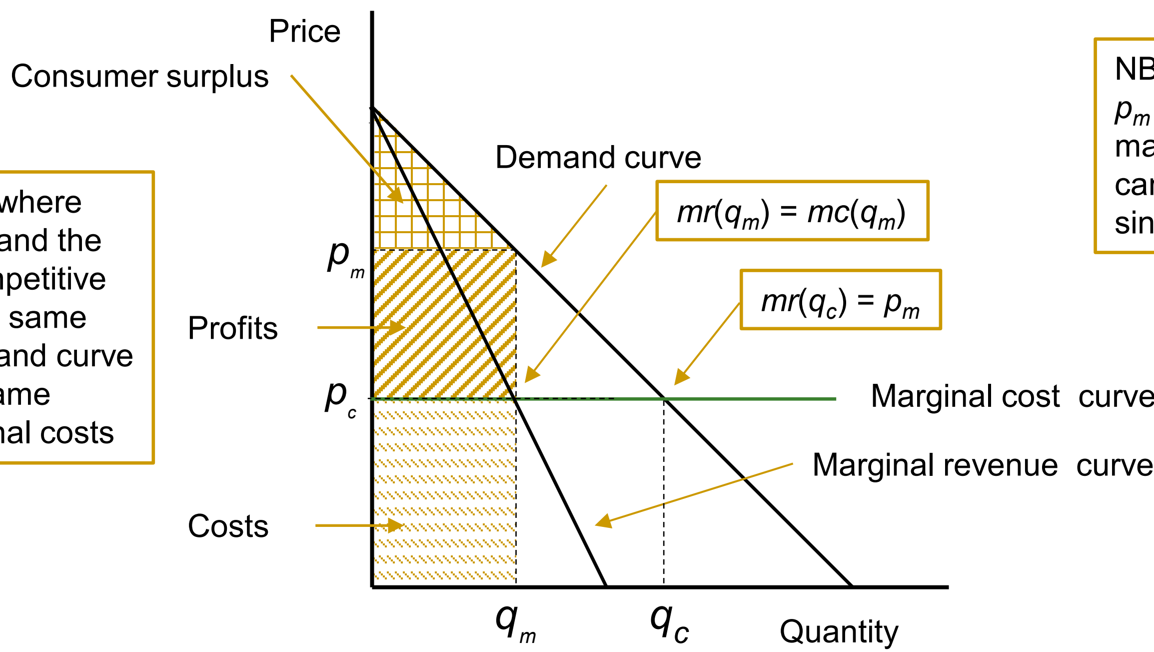
- A monopolist chooses output q_m so that $mr(q_m) = mc(q_m)$
  1. A monopolist charges a higher price than a competitive firm

$$p_m > mr(q_m) = mc(q_m) = mc(q_c) = p_c$$

where marginal costs are constant[1]

  2. A monopolist produces a lower output than would a competitive firm facing the same residual demand curve ($q_m < q_c$)

NB: $q_m = ½\, q_c$, where the monopolist and the firms in the competitive market face the same aggregate demand curve and have the same constant marginal costs

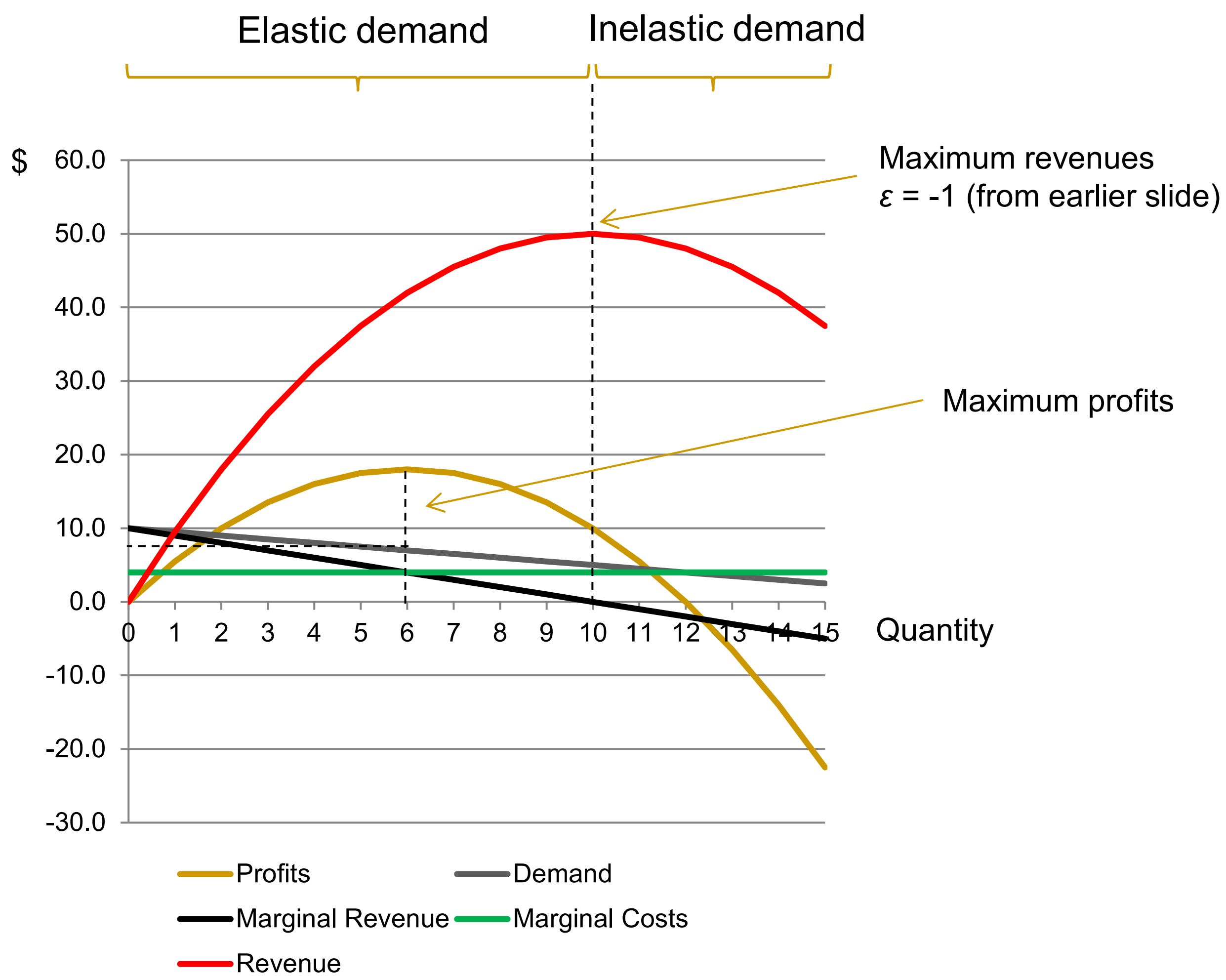


NB: The monopolist price p_m is the price at which the maximum available profits can be drawn from a single price market

[1] But true whenever marginal costs are constant or increasing.

Monopolists and elasticities

- Proposition
  - A monopolist will not operate in the inelastic portion of its demand curve



Remember:

$$\varepsilon = \frac{\Delta q_i}{\Delta p_i}\frac{p_i}{q_i}$$

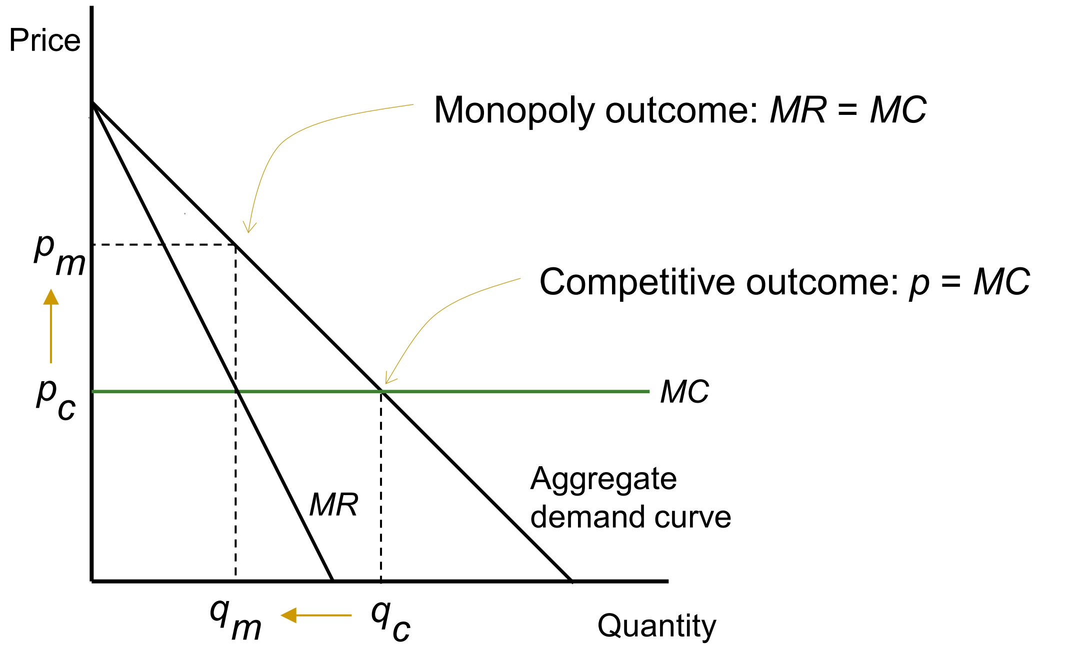
Review: Public policy on monopolies

- Modern view on why monopolies are bad:
  1. Increase price and decrease output
  2. Shift wealth from consumers to producers
  3. Create economic inefficiency (“deadweight loss”)

  - May (or may not) have other socially adverse effects
    - Decrease product or service quality
    - Decrease the rate of technological innovation or product improvement
    - Decrease product choice

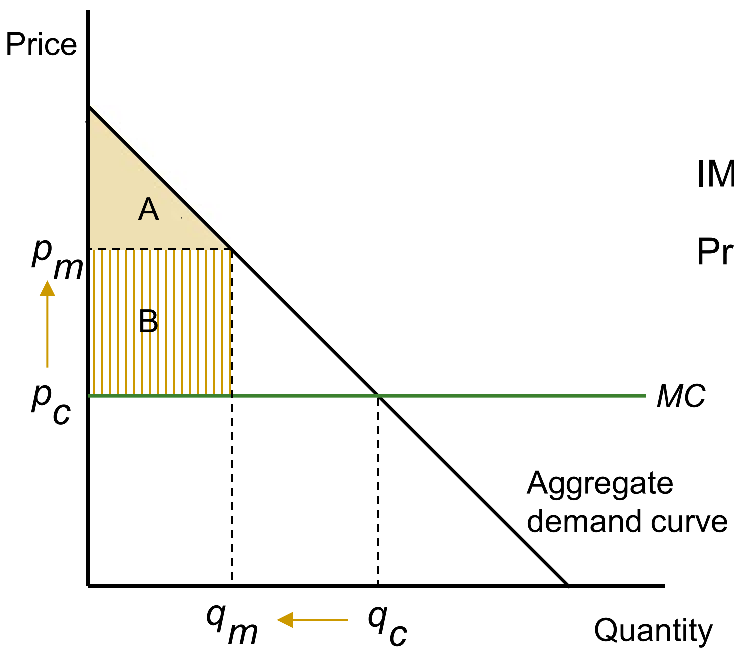
Review: Public policy on monopolies

- Output decreases: $q_c > q_m$
- Prices increase: $p_c < p_m$



Review: Public policy on monopolies

- Shifts wealth from inframarginal consumers to producers*
  - Total wealth created ("surplus"): A + B
  - Sometimes called a "rent redistribution"

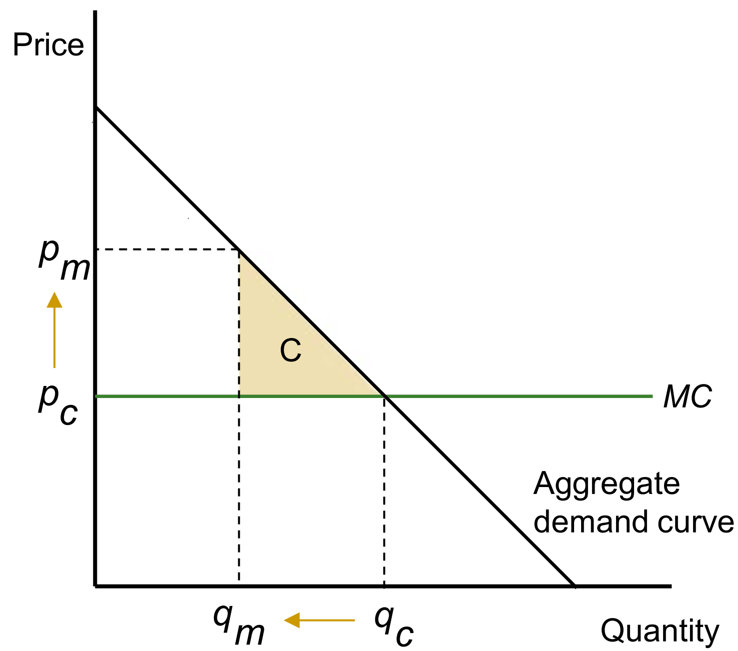


	Competitive	Monopoly
IM consumers	A + B	A
Producers	0	B

* Inframarginal customers here means customers that would purchase at both the competitive price and the monopoly price

Review: Public policy on monopolies

- "Deadweight loss" of surplus of marginal customers*
  - Surplus C just disappears from the economy
  - Creates "allocative inefficiency" because it does not exhaust all gains from trade

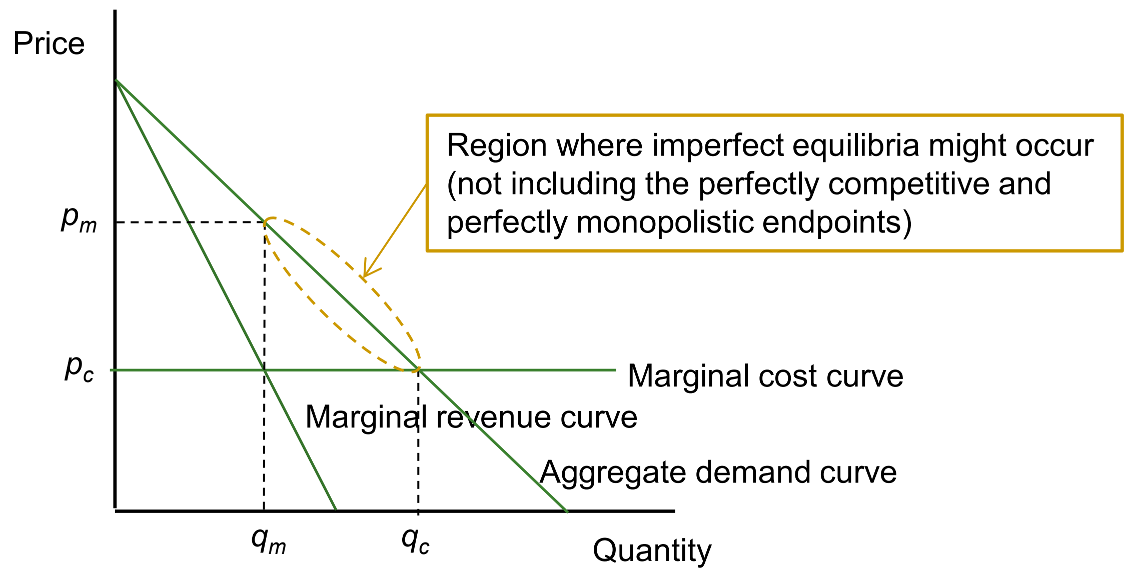


* Marginal customers here means customers that would purchase at both the competitive price and the monopoly price

Imperfectly Competitive Markets

Imperfectly Competitive Markets

- Range of imperfect equilibria
  - An imperfectly competitive equilibrium occurs when the equilibrium price and output on the demand curve falls strictly between the perfect monopoly equilibrium and the perfectly competitive equilibrium

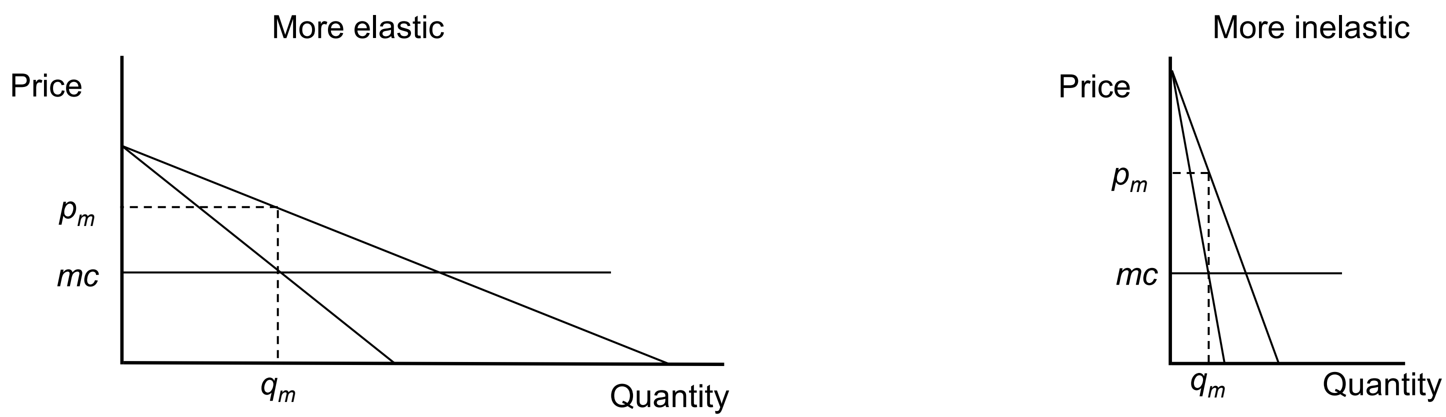


Market power

- Measuring market power
  - Economically, market power is the power of the firm to affect the market-clearing price through its choice of output level
  - The traditional economic measure of market power is the *price-cost margin* or *Lerner index L*, which is a measure of how much price has been marked up as a percentage of price:

$$L = \frac{p - mc}{p}$$

    - In a competitive market, $L = 0$ since because $p = mc$
    - In a perfectly monopolized market, L increases as the aggregate demand curve becomes steeper (more inelastic):



Market power

- The Lerner index for an imperfectly competitive market
  - The Lerner index is usually used as a measure of the market power of a single firm
  - The market Lerner index is defined as the sum of the Lerner indices of all firms in the market weighted by their market share:

$$L \equiv \sum_{i=1}^{n} L_i s_i ,$$

  - Where there are *n* firms in a homogeneous product market, with each firm *i* having a Lerner index L_i and a market share s_i, the aggregate Lerner index is:

$$L \equiv \sum_{i=1}^{n} L_i s_i = \sum_{i=1}^{n} s_i , \frac{p - c_i}{p}$$

Measures of market concentration

- The Herfindahl-Hirschman Index (HHI)
  - *Definition*: The Herfindahl-Hirschman Index (HHI) is defined as the sum of the squares of the market shares of all the firms in the market:

$$HHI \equiv s_1^2 + s_2^2 + \cdots + s_n^2 = \sum_{i=1}^{n} s_i^2$$

The HHI is the principal measure of market concentration used in antitrust law in all markets (not just Cournot markets)

    where the market has n firms and each firm i has a market share of s_i.
  - Example
    - Say the market has five firms with market shares of 50%, 20%, 15%, 10%, and 5%. The conventional way in antitrust law is to calculate the HHI using whole numbers as market shares:

$$\begin{aligned} HHI &= 50^2 + 20^2 + 15^2 + 10^2 + 5^2 \\ &= 2500 + 400 + 225 + 100 + 25 \\ &= 3250 \end{aligned}$$

In whole numbers, the HHI ranges from 0 with an infinite number of firms to 10,000 with one firm

    - In some economics applications, however, the HHI is calculated using fractional market shares:

$$\begin{aligned} HHI &= 0.50^2 + 0.20^2 + 0.15^2 + 0.10^2 + 0.05^2 \\ &= 0.25 + 0.04 + 0.0225 + 0.01 + 0.0025 \\ &= 0.3250 \end{aligned}$$

In fractional numbers, the HHI ranges from 0 with an infinite number of firms to 1 with one firm

Homogeneous product models

- Homogeneous product models
  - Characterized by products that are undifferentiated (that is, *fungible* or *homogeneous*) in the eyes of the customer
  - Common examples:
    - Ready-mix concrete
    - Winter wheat
    - West Texas Intermediate (WTI) crude oil
    - Wood pulp
  - Two properties of homogeneous products
    1. Customers purchase from the lowest cost supplier → This forces all suppliers in the market to charge the same price
    2. Since the goods are identical, their quantities can be added

$$Q(p) = \sum q_i(p)$$

  - Adding all individual consumer demands at price *p* gives aggregate demand (*Q*)
  - Adding all individual firm outputs at price *p* gives aggregate supply

Cournot oligopoly models

A control variable is the variable the firm can set (control) in its discretion

- The setup
  - The standard homogenous product model is the *Cournot model*
  - In a Cournot model, the firm's control variable is *quantity*
    - The (download-sloping) demand curve gives the relationship between the aggregate quantity produced Q and the market-clearing price p:

$$p = p(Q), \text{ where } Q = \sum_{i=1}^{n} q_i,$$

in a market with n firms

  - The profit equation for firm i is:

$$\pi_i = p(Q)q_i - T_i(q_i), \quad i = 1,2,\ldots,n$$

NB: Each firm i choses its level of output q_i, but the aggregate level of output determines the market prices

  - First order condition (FOC) for profit-maximizing firm:

$$m\pi_i(q_i) = mr_i(q_i) - mc_i(q_i) = 0$$

This generates n equations in n unknows and can be solved for each q_i

You should know the setup—You do not need to know how to solve the system of equations

Cournot oligopoly models

- Production levels in Cournot models
  - A simple example
    - Compare the competitive, Cournot, and monopoly outcomes in this example

Demand curve: $Q = 100 - 2p$

	Price	Quantity
Perfectly competitive	5 (= mc)	90
Cournot (n = 2)	20	60
Perfect monopoly	27.5	45

    - Note that the perfect monopoly output is one-half the perfectly competitive output (with linear demand and constant marginal costs)
  - When demand is linear and there are *n* identical firms in a Cournot model, then:

$$Q_{Cournot} = \frac{n}{n+1} Q_{Competitive}$$

NB: As the number of firms *n* gets large, the ratio $n/(n+1)$ approaches 1 and the Cournot equilibrium approaches the competitive equilibrium

$q_{competitive}$	90	90	90	90	90	90	90	90	90
n	9	8	7	6	5	4	3	2	1
$q_{cournot}$	81	80	78.8	77.1	75	72	67.5	60	45

Cournot oligopoly models

- Relationship of the Lerner index to the Herfindahl-Hirschman Index
  - *Proposition*: In a Cournot oligopoly model with n firms, the Lerner index may be calculated from the HHI and the market elasticity of demand:

$$L = \frac{HHI}{|\varepsilon|},$$

    where *L* is the market Lerner index and *ε* is the market price-elasticity of demand
  - This proposition is the reason antitrust law uses the HHI as the measure of market concentration
    - *WDC*: It is not a great reason, but is it generally accepted as better than the alternative measures (especially the four-firm concentration ratios used from the 1950s through the 1970s)
    - The HHI was adopted as the measure of market concentration in the 1982 DOJ Merger Guidelines and by the end of the 1980s has been accepted by the courts

The following slides prove the proposition. The proof is (very) optional, but if you are comfortable with a little calculus, you might find it interesting

Cournot oligopoly models

- Relationship of the Lerner index to the Herfindahl-Hirschman Index
  - *Proof* (optional):
    - Firm *i*'s Lerner index L_i is:

$$L_i = \frac{p(Q) - c_i}{p(Q)},$$

where $p(Q)$ is the single market equilibrium price (determined by aggregate production quantity Q) and c_i is firm i's marginal cost of production

    - The first order condition for firm i's profit-maximizing quantity is:

$$\frac{d\pi_i}{dq_i} = p(Q) + q_i \frac{dp(Q)}{dq_i} - c_i = 0$$

    - Now

$$\frac{dp(Q)}{dq_i} = \frac{dp(Q)}{dQ}\frac{dQ}{dq_i} = \frac{dp(Q)}{dQ}$$

Equals 1 under the Cournot assumption that all other firms do not change their behavior when firm i changes output

Cournot oligopoly models

- Relationship of the Lerner index to the Herfindahl-Hirschman Index
  - *Proof* (optional) (con't)
    - Substituting and rearranging the top equation:

$$p(Q) - c_i = q_i \frac{dp(Q)}{dQ}$$

    - Dividing both sides by $p(Q)$ and multiplying the right-hand side by Q/Q:

$$\frac{p(Q) - c_i}{p(Q)} = \frac{q_i}{Q} \frac{dp(Q)}{dQ} \frac{Q}{p(Q)} = \frac{s_i}{|\varepsilon|}$$

    - Multiply both sides by s_i:

$$\frac{p(Q) - c_i}{p(Q)} s_i = \frac{s_i^2}{|\varepsilon|}$$

Cournot oligopoly models

- Relationship of the Lerner index to the Herfindahl-Hirschman Index
  - *Proof* (optional) (con't)
    - Summing over all firms:

$$\sum_{i=1}^{n} \frac{p(Q)-c_i}{p(Q)} s_i = \sum_{i=1}^{n} \frac{s_i^2}{|\varepsilon|} = \frac{1}{n} \sum_{i=1}^{n} s_i^2$$

    - The left-hand side is the market Lerner index and the right-hand side is the HHI divided by the absolute value of the market price-elasticity:

$$L = \frac{HHI}{|\varepsilon|}$$

Q. E.D.

Cournot oligopoly models

- Mergers and price increases in Cournot oligopoly
  - From the previous slides:

$$L = \frac{HHI}{|\varepsilon|},$$

  - Then:

$$L^{\text{Postmerger}} - L^{\text{Premerger}} = \frac{HHI^{\text{Postmerger}}}{|\varepsilon|} - \frac{HHI^{\text{Premerger}}}{|\varepsilon|} = \frac{\Delta HHI}{|\varepsilon|}$$

This probably is the justification for the emphasis in the Merger Guidelines on changes in the HHI (the "delta") resulting from a merger

In other words, the difference in the share-weighted average percentage markup resulting from the merger is $\Delta HHI/|\varepsilon|$

Cournot oligopoly models

- Some final observations on the HHI and Cournot models
  - The HHI and ΔHHI are fundamental to modern merger antitrust law
  - The rationale for using these measures is grounded in their relationship in the Cournot model to percentage price-cost margins measured by the Lerner index

Cournot oligopoly models

- Some final observations on the HHI and Cournot models (con't)
  - BUT—
    - Price-cost margins typically cannot be calculated directly
      - Prices, while seemingly observable, can be empirically difficult to measure given the existence of discounts, variations in the terms of trade, and price and quality changes over time
      - Marginal costs are even more difficult to measure
        - *Time period*: There is the conceptual issue of the time period over which to assess marginal cost. As the time period becomes longer, some fixed costs such as real estate rents or workers' salaries become marginal costs. There is nothing in the theory that tells us what is the proper time period.
        - *Complex production processes*: In the real word, production functions are often joint and are used to produce multiple products. The is a conceptual problem of how to allocate costs associated with joint production to each individual product type.
        - *Dynamic market conditions*: Marginal costs can fluctuate rapidly in dynamic markets due to changing supply and demand conditions, input price volatility, or disruptions in the production process.
    - The Cournot oligopoly model is an abstraction that may not (and probably does not) accurately characterize any real-world market

Cournot oligopoly models

- Some final observations on the HHI and Cournot models (con't)
  - HHIs to some extent allow us to infer the magnitudes of percentage price-cost margins and how these margins may change with changes in market structure
  - BUT—
    - Antitrust law tests just look at the HHI and ΔHHI—antitrust law does not modulate its HHI tests for market elasticity of demand as the Cournot model suggests it should
      - So two mergers in a Cournot model may have the same HHI and ΔHHI but have dramatically different premerger postmerger percentage price-cost margins
        - A higher aggregate elasticity of demand yields lower percentage price-costs margins than a less elastic demand even with the same HHI and ΔHHI.
      - In any event, there are no accepted "thresholds" in antitrust law when percentage price-margins become "anticompetitive"

Bertrand oligopoly models

- The setup
  - In a Bertrand model, the firm's control variable is *price*
    - Compare with the Cournot model, where the firm's control variable is *quantity*
    - The (download-sloping) residual demand curve gives the relationship between the firms choice of price and the quantity consumers will demand from the firm at that price
  - The profit equation for firm *i* is:

$q_i(p_i)$ is the residual demand function for firm *i*

$$\pi_i(p_i) = p_i q_i(p_i) - T_i(q_i(p_i)), \quad i = 1, 2, \ldots, n$$

To see the first order conditions in operation, let's first look at profit-maximization for a monopolist whose control variable is price

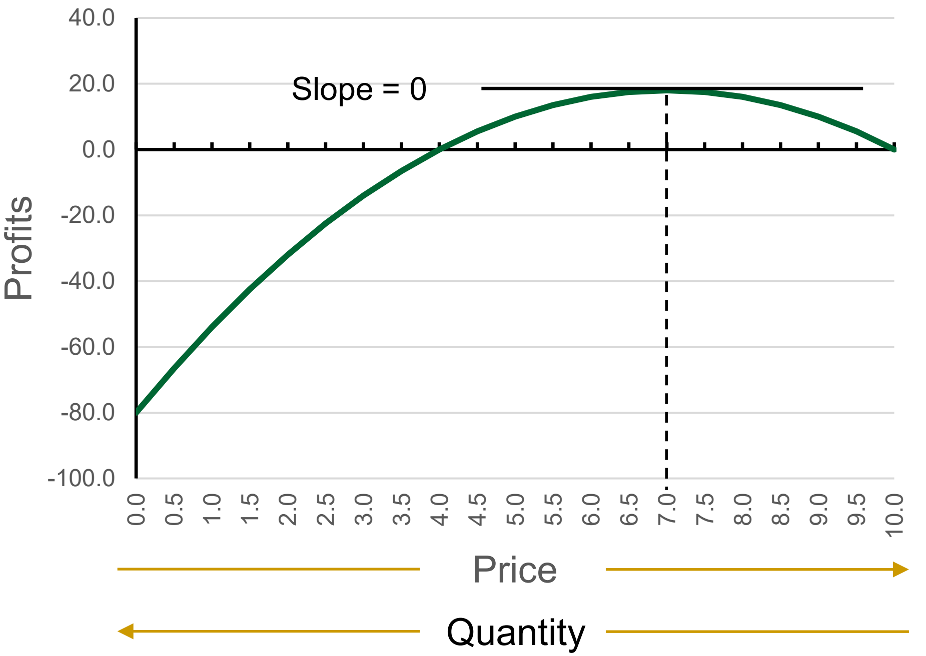
Bertrand oligopoly models

- Profits as a function of price: Example for a monopolist

Price p	Quantity q	Revenues r	Costs T	Profits Π
0.0	20	0.0	80	-80.0
0.5	19	9.5	76	-66.5
1.0	18	18.0	72	-54.0
1.5	17	25.5	68	-42.5
2.0	16	32.0	64	-32.0
2.5	15	37.5	60	-22.5
3.0	14	42.0	56	-14.0
3.5	13	45.5	52	-6.5
4.0	12	48.0	48	0.0
4.5	11	49.5	44	5.5
5.0	10	50.0	40	10.0
5.5	9	49.5	36	13.5
6.0	8	48.0	32	16.0
6.5	7	45.5	28	17.5
7.0	6	42.0	24	18.0
7.5	5	37.5	20	17.5
8.0	4	32.0	16	16.0

Demand: $q = 20 - 2p$
Fixed costs = 0
Marginal costs = 4 (for units)

Profits as a Function of Price



Bertrand oligopoly models

- Observations
  - The profit curve as a function of price is a parabola
    - Although different in shape than the profit curve as a function of quantity
  - The profit maximum is when the slope of the profit curve is zero
  - So:

Marginal profit (as a function of price)	=	Marginal revenue (as a function of price)	–	Marginal cost (as a function of price)
	=	0 at the firm's profit maximum		

NB: In Bertrand models, the marginal quantities are calculated for a one unit increase in price, not *a one unit increase in quantity as in Cournot models*

Bertrand oligopoly models

- Profit-maximization when a monopolist sets price: Example

Demand: $q = 20 - 2p$ Marginal costs ($mc(q)$) = 4
Fixed costs = 0

- Revenues:

$$r(p) = pq(p)$$
$$= p(20 - 2p)$$
$$= 20p - 2p^2$$

This describes the parabola on Slide 102

- Marginal revenues:

$$mr(p) = 20 - 4p$$

Remember, if $y = ax + bx^2$ is the function, then the marginal function is $a + 2bx$

- Cost: $C(q(p)) = mc(q) * q(p) = mc(q)(20 - 2p)$

$$= 4(20 - 2p)$$
$$= 80 - 8p$$

Constant marginal cost

Note: If $y = a + bx$ is the function, then the marginal function is b

- Marginal cost: $mc(p) = -8$

NB: This is marginal cost as a function of p (not q). Why is it a negative number?

- FOC: $mr(p^*) = mc(p^*)$

$$20 - 4p^* = -8$$

So $p^* = 7$ and $q^* = 6$

Bertrand oligopoly models

- Homogeneous products case with equal cost functions
  - Consider two firms producing homogeneous (identical) products at constant marginal cost *c* and use price p_i as their control variable
  - Consumers also purchase from the lower priced firm
    - If both firms charge the same price, they split equally consumer demand
  - Profit function for firm *i*:

$$\pi(p_i) \begin{cases} = p_i Q(p_i) - c(Q(p_i)) & \text{if } p_i < p_j \\ = \dfrac{p_i Q(p_i) - c(Q(p_i))}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

    - That is, firm *i* gets 100% of market demand $Q(p_i)$ at price p_i if p_i is the lower price of the two firms; the two firms split the market demand if their prices are equal; and firm *i* gets nothing if it has the higher price
    - *Equilibrium*: $p_1 = p_2 = mc$, so that both firms price at marginal cost (i.e., the competitive price) and split equally market demand and total market profits

Bertrand oligopoly models

- Homogeneous products case with asymmetric cost functions
  - Now consider two firms producing homogeneous (identical) products but with different cost functions costs, with firm 1 have lower marginal costs than firm 2 (i.e., $mc_1(q(p)) < mc_2(q(p))$)
  - The profit function is the same as before:

$$\pi(p_i)\begin{cases} = p_i Q(p_i) - c(Q(p_i)) & \text{if } p_i < p_j \\ = \dfrac{p_i Q(p_i) - c(Q(p_i))}{2} & \text{if } p_i = p_j \\ = 0 & \text{if } p_i > p_j \end{cases}$$

  - *Equilibrium*: Firm 1 prices just below firm 2 and captures 100% of market demand
  - *Idea*: Firm 1 and Firm 2 compete the price down to firm 2's marginal cost as in the symmetric cost case. Then firm 1 just underprices firm 2 and captures 100% of the market demand

Bertrand oligopoly models

- Differentiated products case
  - When products are differentiated, a lower price charged by one firm will not necessarily move all the market demand to that firm
    - Consider a market with only red cars and blue cars
    - Some consumers like blue cars so much that even if the price of red cars is lower than the price of blue cars, there will still be positive demand for blue cars
    - Moreover, if the price of blue cars increases, some (inframarginal) blue car customers will purchase blue cars at the higher price, while some (marginal) customers will switch to red cars
    - This means that the demand for red cars (and separately for blue cars) is a function both of the price of red cars and the price of blue cars
    - It also means that the price of blue cars may not equal the price of red cars in equilibrium

Bertrand oligopoly models

- Differentiated products case
  - Simple linear model
    - Firms 1 and 2 produce differentiated products and face the following residual demand curves:

$$q_1 = a - b_1 p_1 + b_2 p_2$$
$$q_2 = a - b_1 p_2 + b_2 p_1$$

NB: Each firm's demand decreases with increase in its own price and increases with increases in the price of the other firm

Assume that $b_1 > b_2$, so that each firm's residual demand is more sensitive to its own price than to the other firm's price

    - Assume each firm has a cost function with no fixed costs and the same constant marginal costs:

$$c_i(q_i) = cq_i$$

    - Firm 1's profit-maximization problem:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2)$$

NB: This formulation does not take into account firm 2's reaction to a change in Firm 1's price. It assumes that Firm 2's price is constant.

      - Firm 2 solves an analogous profit-maximization problem
    - Derive the FOCs for each firm and solve for the Bertrand equilibrium:

$$p_1^* = p_2^* = \frac{a + cb_1}{2b_1 - b_2}$$

You do not need to know this. What is important is how the model is set up.

Dominant firm with a competitive fringe

- The setup
  - Consider a homogeneous product market with—
    1. a *dominant firm*, with a control variable q and which sees its output decisions as affecting price and so sets output so that $mr = mc$, and
    2. a *competitive fringe* of firms that are small and act as price takers, that is, they do not see their individual choices of output levels as affecting price and therefore price as competitive firms (i.e., they set their production quantities q_i so that $p = mc(q_i)$)
  - *Decision for the dominant firm*: Pick the profit-maximizing level for its output given the production of the competitive fringe
    - The model requires some constraint on the ability of the competitive fringe to expand its output. Otherwise, the competitive fringe will take over the market.
    - The constraint usually is either limited production capacity or increasing marginal costs

Dominant firm with a competitive fringe

- The model
  - At market price p, let $Q(p)$ be the industry demand function and $q_f(p)$ be the output of the competitive fringe.
  - The dominant firm derives its residual demand function $q_d(p)$ starting with the aggregate demand function $Q(p)$ and subtracting the output supplied by the competitive fringe $q_f(p)$ at price p:

$$q_d(p) = Q(p) - q_f(p)$$

  - The dominant firm then maximizes its profit given its residual demand function by solving the following equation for the market price p^* that maximizes the firm's profits:

$$\max_p \pi_D = p \times \left[Q(p) - q_f(p) \right] - T\left(q(p) \right)$$

  - The dominant firm then produces quantity $q^* = q_D(p^*)$

You do not need to know how to solve the dominant firm maximization problem. What is important is the how the model is set up.

Dominant firm with a competitive fringe

- Dominant oligopolies
  - The model can be extended to the case where the dominant firm is replaced by a dominant oligopoly
  - The key is to specify the solution concept for the choice of output by the firms in the oligopoly (e.g., Cournot). You then create a residual demand curve for the oligopoly and apply the solution concept to that demand curve.
- Fringe firms
  - As we saw in Unit 2, the DOJ and the FTC typically ignore fringe firms. The dominant oligopoly model with a competitive fringe provides a theoretical justification.

Appendix

Mathematical notation

- pq: p times q (equivalently, $p \times q$, $p \cdot q$, and (p)(q))
- $p(q)$: p evaluated when quantity is q ("p as a function of q")
- $p(q)q$: p (evaluated at q) times q (i.e., pq)
- Δq: The change in q to the new state from the old state (i.e., $q_2 - q_1$)
- $\sum_{i=1}^{n} a_i$: The sum of the a_i's (i.e., $a_1 + a_2 + \ldots + a_n$)
- $\frac{\Delta y}{\Delta x}$: The change in y divided by the change in x
- $|a|$: The absolute value of a (i.e., a without a positive or negative sign) (e.g., $|3| = |-3| = 3$)
- $\equiv$ Like an equals sign but means a definition

Mathematical notation

Optional calculus terms

- $\frac{dy}{dx}$: The derivative of *y* with respect to *x* (where *y* is a function of *x*)
- $\frac{\partial y}{\partial x}$: The partial derivative of *y* with respect to *x* (where *y* is a function of *x*)
- Derivatives
  - If $y = a + bx + cx^2$
    then the derivative of *y* with respect to *x* is $\frac{dy}{dx} = b + 2cx$