MERGER ANTITRUST LAW

LAWJ/G-1469-05
Georgetown University Law Center
Fall 2024

Tuesdays and Thursdays, 3:30 pm – 5:30 pm
Dale Collins
wdc30@georgetown.edu
www.appliedantitrust.com

CLASS 9 WRITTEN ASSIGNMENT—INSTRUCTOR'S ANSWERS

Instructions

Submit by email by 3:30 pm on Tuesday, September 24
Send to wdc30@georgetown.edu
Subject line: Merger Antitrust Law: Assignment for Class 9

Assignment

Calls for short answers. Since the assignment calls for some equations, feel free to write your answers using a pencil and paper (rather than a computer) if that is easier. Attach either a scan or a photograph to your email.

Assumptions: Consider a single firm, Avco.

1. The market price is p (whatever that may be)
2. The firm's residual demand function is $q = 100 - 8p$
3. The firm has fixed costs $f = 25$ and constant marginal costs $c = 5$

Questions:

1. Explain the concept of a demand curve. Why is it downward sloping?
2. Graph Avco's demand curve. Be sure to label the axes.
3. What is the equation for the firm's inverse demand curve?
4. Graph Avco's inverse demand curve. Be sure to label the axes.
5. If $q = 20$, what is Avco's market clearing price?
6. What is the equation for Avco's revenues as a function of Avco's production level? What are Avco's revenues when $q = 20$?
7. Explain the concept of marginal revenue and how it relates to gross revenue gains and losses associated with incremental sales. What is Avco's marginal revenue (in the discrete case) when $q = 20$?
8. What is Avco's marginal revenue (in the continuous case) when $q = 20$?

9. Explain the concepts of total cost, fixed cost, total variable cost, average variable cost, marginal cost, and constant marginal cost. What are the equations for these variables? What are Avco's various costs when $q = 20$?
10. Explain why the firm maximizes profit when marginal revenue equals marginal cost.
11. Given the above assumptions, what are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output?
12. Say Avco's fixed costs f decrease to 20. What are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output in this new situation?
13. Say Avco's fixed costs f remain at 25, but its (constant) marginal cost c decreases to 4. What are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output in this situation?
14. Explain what Avco should do and why if it finds that marginal revenue is greater than its marginal cost at current production (say because of a shift in demand).

If you have any questions, send me an email. See you in class.

Note: If you need an online calculator, look at Math Papa (https://www.mathpapa.com/algebra-calculator.html). It is actually an algebraic calculator, so it solves equations. I use it all the time.

INSTRUCTOR'S ANSWERS

1. Explain the concept of a demand curve. Why is it downward sloping?

Every customer has a maximum willingness to pay for the firm's product. Some customers will not value the product very much and will have a low maximum willingness to pay, while other customers will place greater value on the product and have a higher willingness to pay. Customers will only purchase a product if the price is at or below the customer's maximum willingness to pay. Accordingly, at low prices, more customers will purchase the product than at higher prices. For example, say there are four customers in the market with the following willingness to pay:

Customer	MWP
A	8
B	6
C	4
D	2

Then one customer would purchase the product at a price of 8, two at 6, 3 at 4, and 4 at two. This traces out a downward-sloping demand curve.

2. Graph Avco's demand curve. Be sure to label the axes.

Avco's demand curve is $q = 100 - 8p$.

Avco's demand curve is linear, that is, it has the form $q = a + bp$. We can create the demand curve if we know two points on the curve. The easiest points to find are when $p = 0$ and when $q = 0$:

When $p = 0$: $q = 100 - (8)(0) = 100$ (NB: This is *a*, the *y*-intercept in a linear equation)
When $q = 0$: $0 = 100 - 8p$. Solving, $p = 12.5$

Plot the two points on a graph. The demand curve is the line connecting them:

3. **What is the equation for the firm's inverse demand curve? If q = 20, what is the market clearing price?**

From the assumptions, the firm's residual demand function is $q = 100 - 8p$. The firm's inverse demand function is the price that clears the market (for the firm's product) at a production level q. To obtain the firm's inverse demand function, rearrange the demand function to express price p as a function of quantity q (that is, create an equation with only q on the left-hand side):

Demand function: $q = 100 - 8p$

Add $8p$ to both sides: $q + 8p = 100 - \cancel{8p + 8p} = 100$

Subtract q from both sides $\cancel{q} + 8p - \cancel{q} = 100 - q$

So $8p = 100 - q$

Divide both sides by 8: $p = \dfrac{100 - q}{8}$

Simplify: $p = 12.5 - \dfrac{1}{8}q$

The last expression is the firm's inverse demand function when the demand function is q=100–8p.

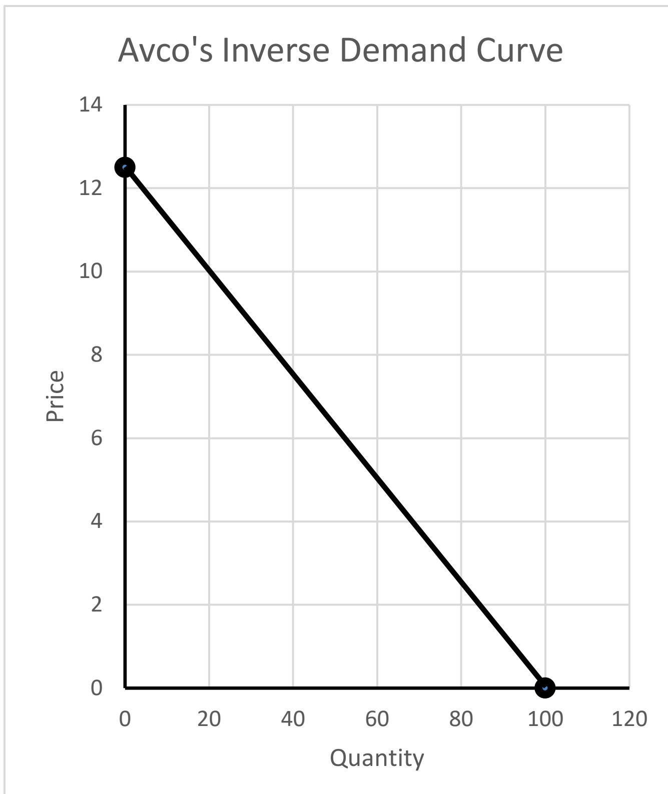
If q = 20, then the market-clearing price p is 12.5 – 1/8 × 20 =12.5 – 2.5 = 10.

4. Graph Avco's inverse demand curve. Be sure to label the axes.

If the demand curve is linear, then the inverse demand curve will also be linear. We could use the same technique as in Problem 2 to graph Avco's inverse demand curve. But from Problem 2, we already know two points on the inverse demand curve:

When $p = 0$: $q = 100$
When $q = 0$: $0 = 12.5$

All we need to do is to make sure that we plot the points on the correct axis (which are reversed from the demand curve):



5. If $q = 20$, what is Avco's market clearing price?

The market clearing price is the price at which the demand by consumers for Avco's product is equal to Avco's production level. This comes out of Avco's inverse demand curve:

$$p = 12.5 - \frac{1}{8}q$$

At $q = 20$:

$$p = 12.5 - \left(\frac{1}{8}\right)(20) = 10$$

So at $q = 20$, Avco's market clearing price is $p = 10$.

6. What is the equation for Avco's revenues as a function of Avco's production level? What are Avco's revenues when q = 20?

Revenues r are equal to the price p times the quantity sold q. So $r = pq$.

Using the inverse demand function to express p as a function of q:

$$r(q) = p(q)q = \left(12.5 - \frac{1}{8}q\right)q$$
$$= 12.5q - \frac{1}{8}q^2$$

When $q = 20$, we can use the revenue function to calculate r:

$$r(20) = (12.5)(20) - \frac{1}{8}20^2 = 200$$

But there is a much easier way. When $q = 20$, p equals 10 from the inverse demand function. So $r(20) = 10*20 = 200$.

7. Explain the concept of marginal revenue and how it relates to gross revenue gains and losses associated with incremental sales. What is Avco's marginal revenue (in the discrete case) when *q* = 20?

Heuristically, marginal revenue is the firm's additional net revenue with the sale of one additional unit of the product. The general formula (in the discrete case) is:

$$mr(q) = r(q+1) - r(q)$$

As at the end of Problem 6, use the inverse demand curve to calculate p at $q = 20$ and $q = 21$:

When $q = 20$: $p = 10$ $\quad$ $r(20) = (20)(10) = 200$
When $q = 21$: $p = 9.875$ $\quad$ $r(21) = (21)(9.875) = 207.375$

So:

$$\begin{aligned} mr(20) &= mr(21) - mr(20) \\ &= 207.375 - 200 \\ &= 7.375 \end{aligned}$$

8. What is Avco's marginal revenue (in the continuous case) when *q* = 20?

In the continuous case (that is, when an additional unit is infinitesimally small compared to Avco's total output), marginal revenue is the slope of the revenue curve. This would be the case, for example, if Avco's demand curve was expressed in millions of units.

If the inverse demand curve is of the form $p(q) = a + bq$, then the revenue curve will be:

$$\begin{aligned} r(q) &= p(q)q \\ &= (a + bq)q \\ &= aq + bq^2 \end{aligned}$$

where b is negative, since the inverse is downward sloping

Marginal revenue (in the continuous case) is then of the form:

$$mr(q) = a + 2bx$$

See Slide 33 (Unit 8, pt. 1).[1] Given Avco's inverse demand function $p = 12.5 + (1/8)q$, $a = 12.5$ and $b = -1/8$. So Avco's marginal revenue in the continuous case at $q = 20$ is:

$$\begin{aligned} mr(20) &= a - 2b(20) \\ &= 12.5 - 2\left(\frac{1}{8}\right)20 \\ &= 7.5 \end{aligned}$$

In general, Avco's marginal revenue $mr(q) = 12.5 - (1/4)q$.

[1] Mathematical aside (*optional*): Marginal revenue is the limit of incremental unit revenue as $\Delta q/q$ approaches zero. In calculus terms:

$$\text{Revenue } (r) = pq$$

$$\text{Marginal revenue } (mr) = \frac{dr}{dq} = p + q\frac{dp}{dq}$$

Note that dp/dq is the derivative of a downward-sloping (inverse) demand curve and so is a negative number. For a linear function of the form $r(q) = aq - bq^2$, the derivative of r with respect to q is a $-2bq$.

9. **Explain the concepts of total cost, fixed cost, total variable cost, average variable cost, marginal cost, and constant marginal cost. What are the equations for these variables? What are these various costs when q = 20?**

Total cost $t(q)$ is the sum of all costs of manufacturing a production level q. Conventionally, costs are broken down into two types: *fixed costs* (f), which do not vary with the production level (such as the CEO's salary or the maintenance of the headquarters building), and (total) *variable costs* ($v(q)$), which change with the level of production.

$$t(q) = f + v(q)$$

Fixed cost (f) are costs that the firm must incur to enter into production (with a given maximum capacity) but which do not vary with the level of production output q, as long as q is less than the maximum capacity. f is a fixed number and is not a function of q.

Total variable costs $v(q)$ are costs of production that vary with the production level and that are incurred producing a level q. Total variable cost is a function of q.

Average variable cost (avc) are the total variable costs for producing output q divided by q:

$$avc(q) = \frac{v(q)}{q}$$

Marginal cost (c) is the cost to produce one additional unit. Since marginal cost may depend on the current production level q, it is a function of q.

$$\begin{aligned} m(q) &= t(q+1) - t(q) \\ &= f + v(q+1) - \left(f + v(q)\right) \\ &= v(q+1) - v(q) \end{aligned}$$

In general, marginal costs are a function of q. Marginal cost is not a function of fixed costs f.[2]

A *constant marginal cost* is a marginal cost that does not change with the production level. If c is a constant marginal cost, then $v(q) = cq$ and $t(q) = f + cq$.

[2] In calculus terms (optional):

$$\begin{aligned} t(q) &= f + v(q) \\ c(q) &= \frac{dt}{dq} = \frac{dv(q)}{dq}. \end{aligned}$$

Applying these formulae at q =20:

Total cost: $t(20) = f + v(20) = 25 + 100 = 125$

Fixed cost: $f = 25$ (from problem)

Total variable cost: $v(20) = cq = 5 \times 20 = 100$

Average variable cost: $avc(2) = v(20)/q = 100/20 = 5$

Marginal cost: $c(20) = 5$ (constant; from problem)

10. Explain why the firm maximizes profit when marginal revenue equals marginal cost.

If marginal revenue is greater than marginal cost, then by producing one additional unit, the firm earns more revenues than it expends in producing the additional unit. A profit-maximizing firm would therefore increase its production by one unit and then ask again whether marginal revenue would be greater than marginal cost for the production of yet another additional unit. If so, the firm should produce the additional unit. This iterative process should continue until marginal revenue is less than marginal cost, at which point the firm would lose profits by producing an additional unit. This means that the firm should produce a level of output so that marginal revenue equals marginal cost:

$$\text{Marginal revenue } (q) = \text{Marginal cost } (q)$$
$$mr(q) = mc(q),$$

when the firm maximizes its profits.[3]

[3] *In calculus terms (optional)*: The first-order condition for a profit maximum set the derivative of profits with respect to quantity equal to zero, so marginal revenue is equal to marginal total costs (which is equal to marginal cost):

$$\pi(q) = r(q) - TC(q)$$
$$\frac{d\pi}{dq} = \frac{dr}{dq} - \frac{dTC}{dq} = 0$$
$$= MR - MC = 0.$$

11. What are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output?

At the firm's profit-maximizing level of output q^*, the firm's marginal revenue is equal to its marginal cost.

When the firm's residual demand curve is $q = 100 - 8p$, the firm's inverse demand curve is $p = 12.5 - 1/8\, q$ (from Question 3). From Question 8, the formula in the slides for marginal revenue in the case of a linear demand curve, marginal revenue $mr = 12.5 - 1/4q$ (in the continuous case—see Problem 8). The assumptions state that marginal cost is constant at $c = 5$.

Setting marginal revenue equal to marginal cost at the profit-maximizing level of output q^*:

$$mr\left(q^*\right) = mc\left(q^*\right)$$
$$12.5 - \frac{1}{4}q^* = 5,$$

where q^* is Avco's profit-maximizing production level.

Solving for q^* yields $q^* = 30$. Then:

$p^* = 12.5 - 1/8*30 = 8.75$	from the inverse demand curve
$r^* = p^*q^* = 30*8.75 = 262.5$	from the definition of revenues
$t^* = f + \text{v}(q^*)$	
$= f + cq^*$	since marginal costs are constant
$= 25 + 5*30 = 175$	since $f = 25$ and $c = 5$
$\pi = r(q^*) - t(q^*)$	
$= 262.5 - 175 = 87.5$	

12. Say Avco's fixed costs *f* decrease to 20. What are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output in this new situation?

None of the equations for the firm's profit-maximizing price, output, revenues, or total variable costs depend on f. So those are the same as in Problem 11:

$q^* = 30.$

$p^* = 8.75$

$r^* = 262.5$

$v^* = 150$

Fixed costs, however, decrease to 20. So total cost becomes:

$t^* = f + v^* = 20 + 150 = 170$

So profits increase by 5:

$\Pi^* = r(q^*) - t(q^*)$

$= 262.5 - 170 = 92.5$

13. Say Avco's fixed costs *f* remain at 25 but its (constant) marginal cost *c* decreases to 4. What are Avco's price, output, revenues, total costs, and profits at the profit-maximizing level of output in this situation?

At the profit-maximizing of output, marginal revenue is equal to marginal cost. Since marginal cost has changed, the profit-maximizing level of output will change as well.

Setting marginal revenue equal to the new marginal cost $c = 4$ at the profit-maximizing level of output q^*:

$$12.5 - \frac{1}{4}q = 4$$

Solving for q^* yields $q^* = 34$. Then:

$p^* = 12.5 - 1/8*34 = 8.25$	from the inverse demand curve
$r^* = p^*q^* = 8.75*34 = 280.5$	from the definition of revenues
$t^* = f + v(q^*)$	
$= f + cq^*$	since marginal costs are constant
$= 25 + 4*34 = 161$	since $F = 25$ and $c = 4$

$\pi^* = r(q^*) - t(q^*)$

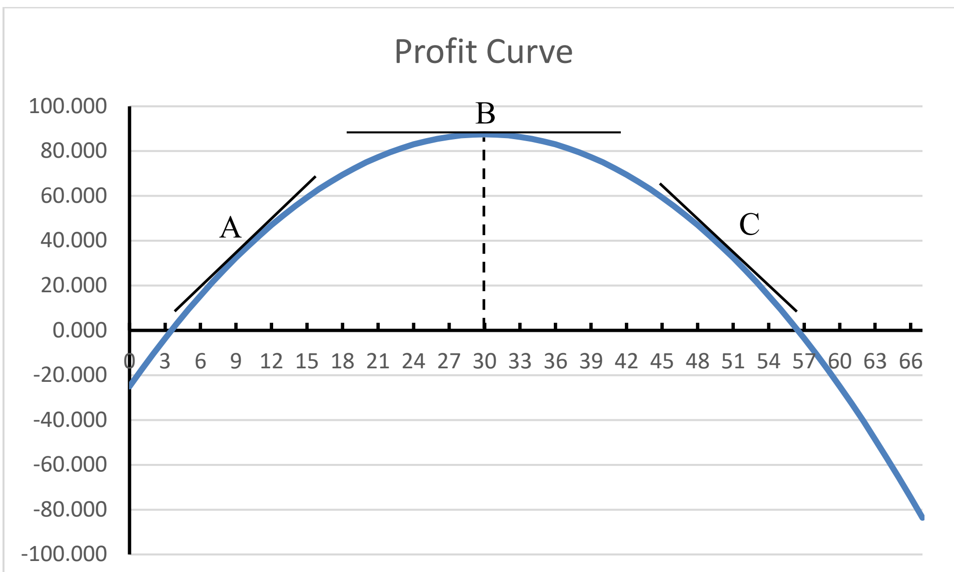
$= 280.5 - 161 = 119.5$

As we can see, although a change in fixed costs will not affect the profit-maximizing level of output or price, a change in marginal cost will, In particular, a decrease in marginal cost will increase output and lower the market-clearing price, while a increase in marginal cost will decrease output and increase the market-clearing price.

14. Explain what Avco should do and why if it finds that marginal revenue is greater than its marginal cost at current production (say because of a shift in demand).

For the reason explained in the answer to Question 10, whenever the firm finds that its marginal revenue is greater than its marginal cost, it should increase its output until marginal revenue is again equal to marginal cost.

More generally, the profit curve is a parabola and the slope of the curve at any point *q* is the marginal profit.



At point A on the profit curve, the slope is positive, meaning that marginal revenue is greater than marginal costs and so the firm would increase profits by increasing output. At point C on the profit curve, the slope is negative, meaning that marginal revenues are less than marginal costs and so the firm would increase profits by decreasing output. At point B on the profit curve, the slope is zero, meaning that marginal revenues are equal to marginal costs. At this point, the firm would lose money if it either increased or decreased its production level. According, point B (where $q = 30$) is the profit-maximizing point.