MERGER ANTITRUST LAW

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Georgetown University Law Center
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Tuesdays and Thursdays, 3:30 pm – 5:30 pm
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CLASS 10 WRITTEN ASSIGNMENT—INSTRUCTOR'S ANSWER

Instructions

Submit by email by 3:30 pm on Tuesday, September 26
Send to wdc30@georgetown.edu
Subject line: Merger Antitrust Law: Assignment for Class 10

Assignment

Calls for short answers. If you want to use math or graphs in your answers, please feel free to write your answers using a pencil and paper (rather than a computer). Attach either a scan or a photograph to your email.

Questions:

1. What is elasticity of demand?
2. What is cross-elasticity of demand?
3. Explain what it means for demand to be inelastic at a given point on the demand curve.
4. Explain why a monopolist will never price in the inelastic portion of its demand curve.
5. What is a perfectly competitive market?
6. What is the profit-maximizing condition for firms in a perfectly competitive market?
7. What is a perfect monopoly?
8. What is the profit-maximizing condition for a firm that is a perfect monopolist?
9. What is an imperfectly competitive market?
10. What is the Lerner index? What does it measure?
11. What is a control variable?
12. What is the control variable in a Cournot oligopoly model?
13. What is the control variable in a Bertrand oligopoly model?
14. Using the first-order condition $mr = mc$ for a profit-maximizing firm, derive the results in the table on Slide 54 comparing the competitive, monopoly, and two-firm Cournot equilibrium results for prices and aggregate output.

If you have any questions, send me an email. See you in class.

INSTRUCTOR'S ANSWERS
(with far too many words)

1. What is elasticity of demand?

    Elasticity of demand (usually denoted by ε) is defined to be the percentage change in demand that results from a given change in price:

"≡" means a definition

$$\varepsilon \equiv \frac{\text{Percentage change in demand}}{\text{Percentage change in price}} = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q}$$

Rearranging terms

    Elasticity of demand is also known as *own-elasticity of demand* to emphasize it pertains only to a single product. It is also known as *price-elasticity of demand* to emphasize that it pertains to changes in demand due to a change in the product's price.

    Elasticity measures the sensitivity of demand with respect to price changes. When demand is relatively sensitive to demand—that is, when the percentage change in demand is greater than the percentage change in price—demand is called *elastic*. If, for example, the elasticity of demand is -4, that means that for a 5 percent increase in price, demand will contract by 20 percent.[1]

    When demand is relatively insensitive to demand—that is, when the percentage change in demand is less than the percentage change in price—demand is called *inelastic.* If, for example, the elasticity of demand is -0.5, that means that for a 5 percent increase in price, demand will contract by 2.5 percent.

    Technically, elasticity of demand is a negative number. We can see this from the above equation. The term $\Delta q/\Delta p$ on the far right-hand side of the equation is the slope of the demand curve, which is a negative number because the demand curve is downward sloping. Since p and q are both positive numbers, the far right-hand side of the equation is a negative number.

    Thinking about elasticities in terms of negative numbers is counterintuitive to many people. As a result, economists have adopted a convention of sometimes speaking of elasticity of demand in terms of absolute values (that is, the magnitude of the number without any sign). In absolute value terms, demand becomes more elastic (more sensitive to price changes) as the absolute value of elasticity increases. So the demand for a product with an elasticity in absolute value of 4 is more elastic than the demand for a product with an elasticity in absolute value of 0.5. Unfortunately, economists often do not specify whether they use true or absolute values and leave it to the context to reveal the sign.

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1 This is not technically quite correct, since the elasticity may not be constant along the demand curve as the price increases by 5 percent. For our purposes and for most practical uses, however, the statement works fine as an approximation.

2. What is cross-elasticity of demand?

Cross-elasticity of demand (usually denoted by ε_{ij}) is defined to be the percentage change in demand for product i that results from a change in price in product j:

$$\varepsilon_{ij} \equiv \frac{\text{Percentage change in demand for product } i}{\text{Percentage change in price for product } j} = \frac{\frac{\Delta q_i}{q_i}}{\frac{\Delta p_j}{p_j}} = \frac{\Delta q_i}{\Delta p_j}\frac{p_j}{q_i}$$

Like elasticity, cross-elasticity measures the sensitivity of demand to price changes. However, unlike elasticity, which measures the price sensitivity of demand for a product with respect to a change in that product's price, cross-elasticity measures the price sensitivity of demand with respect to a change in *another product's* price.

In cross-elasticity, the sign matters. A positive sign means that $\Delta q_i/\Delta p_j$ is positive so that the demand for the first product increases as the other product's price increases. This means that customers view the products as *substitutes*. A negative sign means that $\Delta q_i/\Delta p_j$ is negative, so as the price of the other price increases, the demand for the first product decreases. This means that customers view the products as *complements*. If $\Delta q_i/\Delta p_j = 0$, then the demands for the two products are unrelated.

Cross-elasticity is an essential concept in antitrust law. The greater the cross-elasticity ε_{ij} between two products, the greater product i constrains product j's price. Remember, the ability of a firm to increase its price depends in part on the extent to which demand decreases in the wake of a price increase. So if the cross-elasticity ε_{ij} is large, then for a given percentage price change in product j, a disproportionately high percentage of j's customers will abandon product j and switch to product i. This means that product i puts significant downward pricing pressure on product j. Conversely, another substitute product k with a lower cross-elasticity ε_{kj} with respect to product j's price will put less downward pricing pressure on product j because fewer sales will divert from product j to product k for the same percentage price increase in product j. This is why merger antitrust law is concerned with horizontal mergers involving firms with uniquely close substitutes.

3. Explain what it means for demand to be inelastic at a given point on the demand curve.

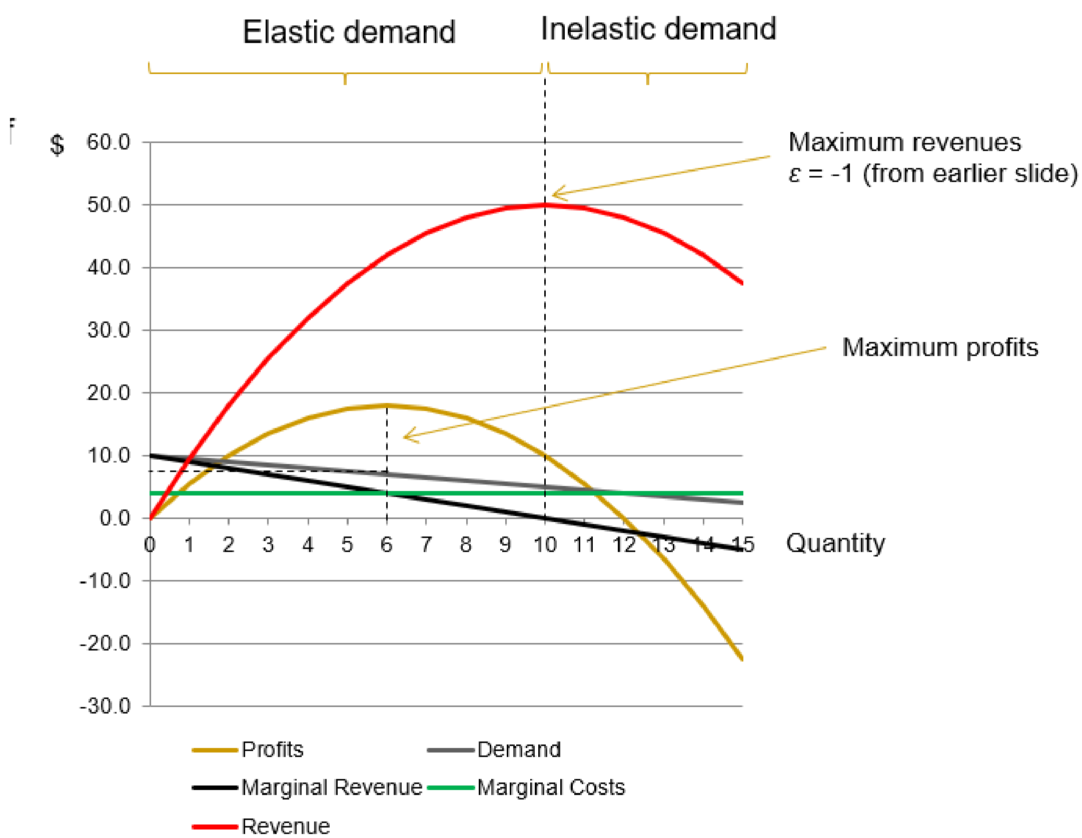
As noted in response to Question 1, demand is inelastic when the percentage change in the demand for the product accompanying a price change is less than the percentage price change. In other words, inelastic demand is not particularly sensitive to changes in the product's price. This also means there are no close substitutes, so the product's cross-elasticity of demand with respect to other products is also low.

4. Explain why a monopolist will never price in the inelastic portion of its demand curve.

Elasticity of demand goes from being elastic to being inelastic as quantity increases and price decreases. In a linear demand curve, for example, $\Delta q/\Delta p$ (the slope of the demand curve) is constant, but the fraction p/q gets smaller as p decreases and q increases going down the demand curve. Consequently, the absolute value of ε also becomes smaller and demand and therefore demand becomes more inelastic:

$$|\varepsilon| = \left|\frac{\Delta q}{\Delta p}\right| \frac{p}{q}$$

As the chart below shows, when a monopolist operates in the inelastic portion of its demand curve, the monopolist's marginal revenue is negative. We know that when marginal revenue is negative, the firm can increase in revenues by reducing production. Moreover, since the firm's total costs also decrease with a reduction in production, total profits increase with a reduction of production. These conditions continue to hold as long as the firm is operating in the inelastic portion of its demand curve. Hence, the firm will not operate in the inelastic portion of its demand curve.



(OPTIONAL—All the math with none of the intuitions): By definition, elasticity in the inelastic portion of the demand curve is between -1 and 0 (that is, $-1 < \varepsilon \leq 0$). Rearranging the definition of demand elasticity:

$$\varepsilon = \frac{dp}{dq}\frac{p}{q} \Rightarrow \frac{1}{\varepsilon} = \frac{dq}{dp}\frac{q}{p} \Rightarrow \frac{p}{\varepsilon} = q\frac{dq}{dp}$$

Revenue is equal to price times quantity. Look at marginal revenue:

$$mr = \frac{d(pq)}{dq} = p + q\frac{dq}{dq} = p + \frac{p}{\varepsilon} = p\left(1 + \frac{1}{\varepsilon}\right)$$

Since ε is small and negative, $1/\varepsilon$ is negative and less than -1. Hence, the term in parenthesis at the end is negative. Since price is positive, marginal revenue is negative when the firm operates in the inelastic portion of its demand curve.

Finally, we know that when marginal revenue is negative, revenues increase with a reduction in production. Moreover, since total costs also decrease with a reduction in production, total profits increase with a reduction of production. These conditions continue to hold as long as the firm is operating in the inelastic portion of its demand curve. Hence, the firm will not operate in the inelastic portion of its demand curve.

5. What is a perfectly competitive market?

   A perfectly competitive market is one in which all of the firms are price-takers, that is, each firm operates as if its choice of production level cannot affect the market price.

6. What is the profit-maximizing condition for firms in a perfectly competitive market?

   All profit-maximizing firms operate so that their marginal revenue equals their marginal cost. In a perfectly competitive market, each firm perceives that its choice of production level will not affect the market-clearing price. This means that the firm believes that it can sell q+1 units at the same price that it can sell q units, and so perceives that its marginal revenue is the market-clearing price p. Accordingly, the firm's profit-maximizing condition in a perfectly competitive market is that price equals marginal cost, so the firm expands output until its marginal cost equals the market-clearing price.[2]

7. What is a perfect monopoly?

   A perfect monopoly is one in which there is only one firm operating in the market.

8. What is the profit-maximizing condition for a firm that is a perfect monopolist?

   As with all firms, the monopolist's maximizes its profits when its marginal revenue equals its marginal cost.

[2] As the class notes point out, for the model to close and had a sensible solution, firm marginal costs must be increasing as production increases, otherwise firms would produce infinite amounts of production (or at least each firm would exhaust its capacity).

9. What is an imperfectly competitive market?

An imperfectly competitive market is a market that is neither perfectly competitive nor perfectly monopolized. Instead, there is more than one firm in the market, and some or all firms perceive that their choice of production level will have some influence on the market-clearing price.

10. What is the Lerner index? What does it measure?

The Lerner index L is defined to be price minus marginal cost, all divided by price:

$$L = \frac{p - mc}{p}.$$

The quantity $p - mc$ is the *dollar gross margin*. The Lerner index is a measure of market power, that is, the power to increase price over the perfectly competitive level (which is equal to marginal cost). L has a minimum at 0, where $p = mc$. L has a maximum at the price p that a profit-maximizing monopolist would charge.

11. What is a control variable?

A control variable is a variable that the firm can set when operating in the market.

12. What is the control variable in a Cournot oligopoly model?

The control variable in a Cournot oligopoly model is the firm's level of production q. The market-clearing price is determined by the aggregate demand curve given the sum of the production levels of each firm in the market.

13. What is the control variable in a Bertrand oligopoly model?

The control variable in a Bertrand oligopoly model is the firm's price p.

In a homogeneous product market (where all the products are identical and firms only compete on price), the firms in a Bertrand market will compete price down to the level of marginal cost. Thus, only two firms are necessary in a Bertrand model for the market to be perfectly competitive.

In a differentiated products market, the firms compete with one another, but each produces a product with its own individual attributes. Different customers in the market value these attributes differently, and so each firm faces its own separate residual demand curve. Each firm then sets its price to maximize its profits subject to the residual demand curve the firm faces.

14. Using the first-order condition $mr = mc$ for a profit-maximizing firm, derive the results in the table on Slide 60 of the class notes comparing the competitive, monopoly, and two-firm Cournot equilibrium results for prices and aggregate output.

The aggregate demand function on Slide 60 is $Q = 100 - 2p$. The marginal cost of production for each firm in the market is constant at 5.

The perfectly competitive market. Each firm in the market sets its marginal revenue equal to its marginal cost but perceives its marginal revenue to be equal to price (that is, the competitive firm does not perceive that its output decision changes the market price). Hence, in a competitive equilibrium, firms in the market, while acting individually, produce a level of output in the aggregate so that the market clears when the market price is 5. Substituting $p = 5$ into the aggregate demand function yields the equilibrium market output:

$$\begin{aligned} Q &= 100 - 2p \\ &= 100 - (2)(5) \\ &= 90 \end{aligned}$$

So aggregate quantity produced and demanded is Q = 90.[3]

The perfectly monopolized market. In a perfectly monopolized market, there is only one firm in the market. The firm's residual demand curve is then the same as the market's aggregate demand curve. The firm sets its production level so that its marginal revenue equals its marginal cost. The monopolist's marginal cost is equal to 5. To find the monopolist's marginal revenue function, we first need to convert the firm's residual demand function (i.e., the aggregate demand function) to the firm's inverse demand function:

$$q = 100 - 2p,$$

so

$$\begin{aligned} p &= \frac{100 - q}{2} \\ &= 50 - \frac{1}{2}q. \end{aligned}$$

We know if the inverse demand function is of the form $p = a + bq$, then the marginal revenue function is of the form $mr(q) = a + 2bq$. So

$$\begin{aligned} mr(q) &= 50 - 2\frac{1}{2}q \\ &= 50 - q \end{aligned}$$

[3] While the aggrege production will be 90 units, when—as in this cse—marginal costs for all firms are equal and constant, we cannot determine how much each firm individually will produce. All we know is that in the aggregate they will produce 90 units. If they collectively produce more than 90 units, then the market-clearing price will drop below 5 and the firms will not be able to recover their marginal costs. If they collectively produce less than 90 units, then the market-clearing price will increase above 5 and some firms will increase the production in order to make more profit.

Now we can solve the first order condition:

$$mr(q) = mc$$
$$50 - q = 5$$
$$q = 45$$

So q = 45 is also Q, or half of the output in a competitive equilibrium.[4] Returning to the inverse demand function to find the market-clearing price:

$$\begin{aligned} p &= 50 - \frac{1}{2}Q \\ &= 50 - \frac{1}{2}45 \\ &= 27.5 \end{aligned}$$

Cournot duopoly. The market contains two identical profit-maximizing firms that have production level as their control variable. From Slide 45, we know:

$$Q_{Cournot} = \frac{n}{n+1} Q_{Competitive}$$

So where n = 2 and $Q_{competitive}$ =90:

$$\begin{aligned} Q_{Cournot} &= \frac{2}{2+1} 90 \\ &= 60 \end{aligned}$$

If $Q_{cournot}$ = 60, then from the aggregate demand function:

$$Q = 100 - 2p$$
$$60 = 100 - 2p$$

So p = 20.

[OPTIONAL] We can also solve the Cournot problem without the formula by looking at each firm's first order conditions. First, let q_1 and q_2 be the quantities produced by Firm 1 and Firm 2, respectively. Then:

$$Q = q_1 + q_2.$$

From the aggregate demand curve, we know from the analysis of the perfectly monopolized market that the aggregate inverse demand function is:

4 The result that the output in a perfectly monopoly equilibrium is half of the output in a competitive equilibrium holds whenever the aggregate demand firm is linear and marginal costs are equal and constant for all firms in the market. Absence these conditions, the result does not generally hold.

$$p = 50 - \frac{1}{2}Q$$
$$= 50 - \frac{1}{2}(q_1 + q_2)$$
$$= 50 - \frac{1}{2}q_1 - \frac{1}{2}q_2,$$

which yields a revenue function for Firm 1 of:

$$r(q_1) = pq_1 = \left[50 - \frac{1}{2}q_1 - \frac{1}{2}q_2\right]q$$
$$= 50q - \frac{1}{2}q_1^2 - \frac{1}{2}q_2 q_1.$$

Take the partial derivative of revenue with respect to q_1 gives the marginal revenue function for Firm 1:

$$mr(q_1) = \frac{\partial r(q_1)}{\partial q_1} = 50 - q_1 - \frac{1}{2}q_2.$$

We can correspondingly derive a marginal revenue function for Firm 2:

$$mr(q_1) = \frac{\partial r(q_2)}{\partial q_2} = 50 - q_2 - \frac{1}{2}q_1.$$

Setting both of these marginal revenue functions equal to marginal cost ($mc = 5$) yields the first order conditions for each firm:

$$50 - q_1 - \frac{1}{2}q_2 = 5$$
$$50 - q_2 - \frac{1}{2}q_1 = 5$$

Rearranging the two equations so that q_1 and q_2 are on the left-hand side:

$$q_1 = 45 - \frac{1}{2}q_2$$
$$q_2 = 45 - \frac{1}{2}q_1.$$

These are called *reaction functions*, because they tell each firm what it should produce given the production level of the other firm. The equations are symmetrical, so it is not surprising that with a little manipulation we can see that $q_1 = q_2$ (which also should have been expected since the two firms are identical). Substituting q_1 for q_2 in the first equation and solving for q_1 yields:

$$\left[50-\frac{1}{2}q_1\right]-q_1=5$$

$$50-\frac{3}{2}q_1=5$$

$$q_1=30.$$

Since $q_2 = q_1$, $q_2 = 30$ and $Q = q_1 + q_2 = 60$. Substituting $Q = 60$ into the aggregate inverse demand function yields the market-clearing price:

$$\begin{aligned} p &= 50-\frac{1}{2}Q \\ &= 50-\left(\frac{1}{2}\right)(60) \\ &= 20. \end{aligned}$$