A Marginal Revenue Problem Step-by-Step (by Step)

A firm faces a residual demand curve of $q = 10 - \frac{1}{2}p$. What is the firm's marginal revenue at a production level q =4?

Answers (using lots of words)

ANSWER

Discrete case.[1] Marginal revenue is the net amount of revenue that the firm would earn if it increased its production by one unit. So the problem is asking for the difference in the revenue that the firm earns if it were to increase its production to 5 units.[2]

Let $r(q)$ and $mr(q)$ be the revenue and marginal revenue, respectively, of the firm at a production level q. Then:

$$mr(q) = r(q+1) - r(q). \tag{1}$$

Moreover, revenues are equal to prices times quantity:

$$r(q) = p(q)q. \tag{2}$$

We write $p(q)$ to remind us that that price is a function of quantity. The demand curve tells us the market-clearing price for any production level.

In the problem, the demand curve is given as $q = 10 - \frac{1}{2}p.$ This expresses q as a function of p, but we would like to express p as a function of q so that we can use Equation 2. To do this, we need to convert the demand curve in Equation 2 into the inverse demand curve. First, rearrange Equation 2 so that the fraction with p is on the left-hand side of the equation and q is on the right-hand side:

$$\frac{1}{2}p = 10 - q. \tag{3}$$

Now multiply both sides by 2 to clear the fraction:

$$p = 20 - 2q. \tag{4}$$

This gives us the inverse demand curve. Now substitute the inverse demand formula for p in Equation 2:

[1] A *discrete case* is when we count production levels in whole numbers. Contrast this is the *continuous case*, where production levels can be any number, which allows incremental increases in production levels to be arbitrarily small. Analysis of the discrete case uses algebra, while analysis of the continuous case uses calculus.

[2] NB: To keep things straight, remember that this is the marginal revenue of the firm at 30 units, *not* 31 units.

$$\begin{aligned} r(q) &= p(q)q \\ &= [20-2q]q \\ &= 20q-2q^2. \end{aligned} \tag{5}$$

We know have a formula for revenues that depends only on the firm's level of production. Now we can calculate *r*(31) and *r*(30) for use in Equation 1:

$$\begin{aligned} r(5) &= (20\times 5)-2\times 5^2 \\ &= 100-50 \\ &= 50 \\ r(4) &= (20\times 4)-2\times 4^2 \\ &= 80-32 \\ &= 48. \end{aligned} \tag{6}$$

Returning to Equation 1:

$$\begin{aligned} mr(4) &= r(5)-r(4) \\ &= 50-48 \\ &= 2. \end{aligned} \tag{7}$$

So in the discrete case, the firm's marginal revenue at a production level of q = 4 is 2, that is, the firm would increase its revenue by 2 by increasing its production from 4 units to 5 units.

This is called a *brute force calculation*: we just plug in the numbers to get an answer without gaining any real insight into what is happening. There is another way to obtain the answer that gives us more insight. Let q and p be the current production level and price for the firm. If the firm increases its production by one unit to p+1, because the firm faces a downward-sloping demand curve it must also decrease its price to clear the market. Although we could determine the necessary price decrease from the demand curve (actually, the inverse demand curve), for now let us call this change in price Δp, that the new price necessary to clear the market at $q = q+1$ is $p+\Delta p$.

Now we can create formulas for $r(q+1)$ and $r(q)$:

$$\begin{aligned} r(q+1) &= (p+\Delta p)(q+1) \\ &= pq+p+\Delta pq+\Delta p \\ r(q) &= pq. \end{aligned} \tag{8}$$

So from Equation 1 $mr(q)$ is:

$$\begin{aligned} mr(q) &= r(q+1)-r(q) \\ &= [pq+p+\Delta pq+\Delta p]-[pq] \\ &= p+\Delta pq+\Delta p \\ &= p+\Delta p[q+1]. \end{aligned} \tag{9}$$

Let's interpret Equation 9. The first term on the right-hand side (p) is the additional revenue the firm earns from producing one more unit *without* considering the need for a price change to clear the market with the one-unit increase in production. We can use inverse demand curve (Equation 4) to determine p:

$$\begin{aligned} p &= 20 - 2q \\ &= 20 - 2 \times 4 \\ &= 12. \end{aligned} \tag{10}$$

So the firm's revenue would increase by 12 by selling the 5^{th} unit *without* considering the need for a price change to clear the market with the one-unit increase in production.

Now look at the second term on the right-hand side of Equation 9: $\Delta p\left[q+1\right]$. This is the revenue adjustment for the price change. Since the new price would apply to all 5 units of production—the 4 previously produced units plus the one incremental unit—the revenue adjustment for the price change will be the price change (Δp) times the total number of units sold (q+1). Using the inverse demand curve, we can calculate Δp:

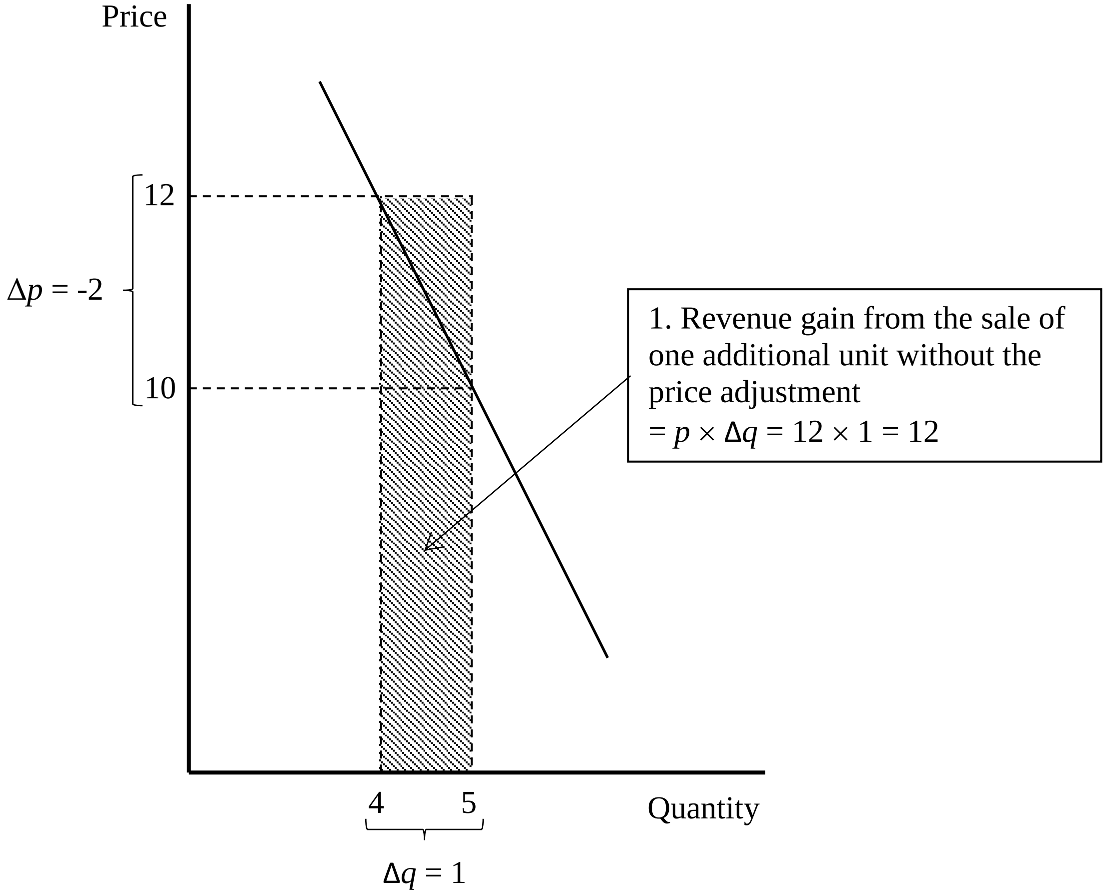
$$\begin{aligned} \Delta p &= p\left(31\right) - p\left(30\right) \\ &= \left[20 - 2 \times 5\right] - \left[20 - 2 \times 4\right] \\ &= 10 - 12 \\ &= -2. \end{aligned} \tag{11}$$

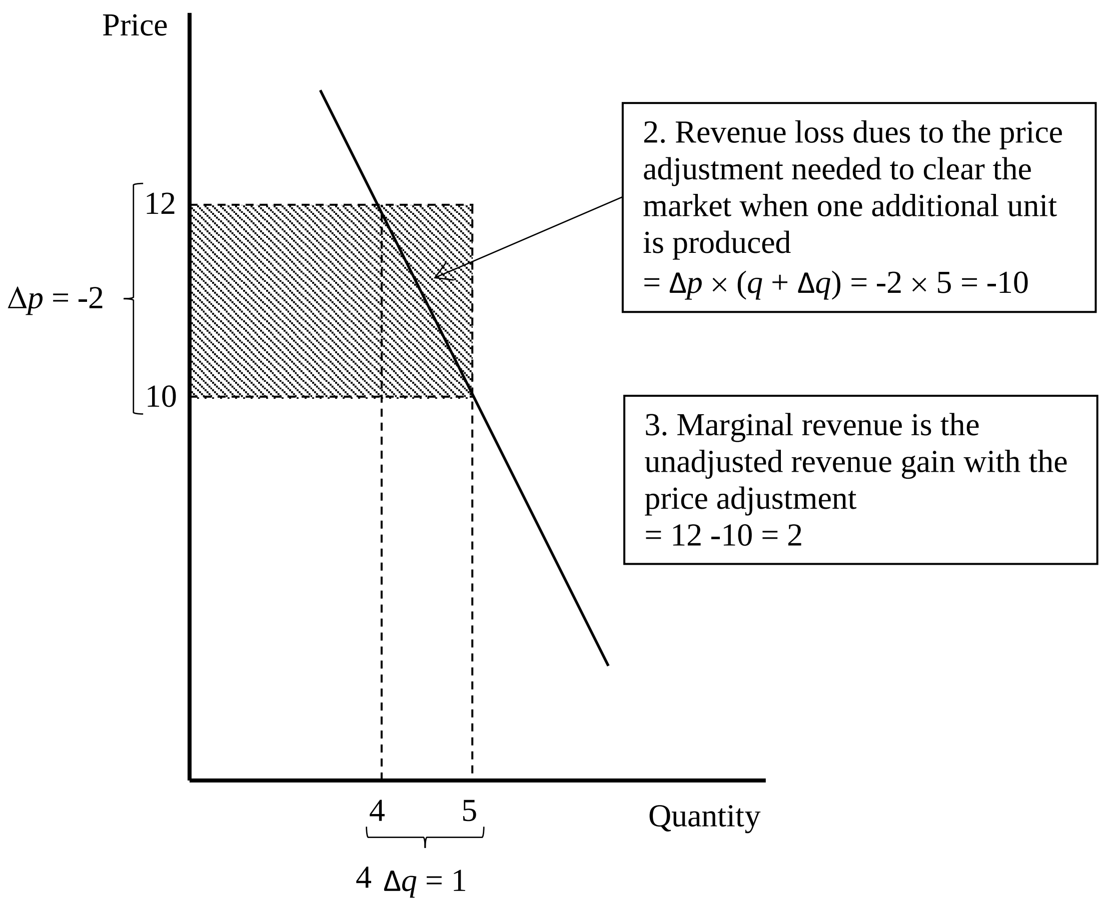
We can actually see this directly from the slope (-2) of the inverse demand curve. In any event, we can now calculate the revenue adjustment due to the price change:

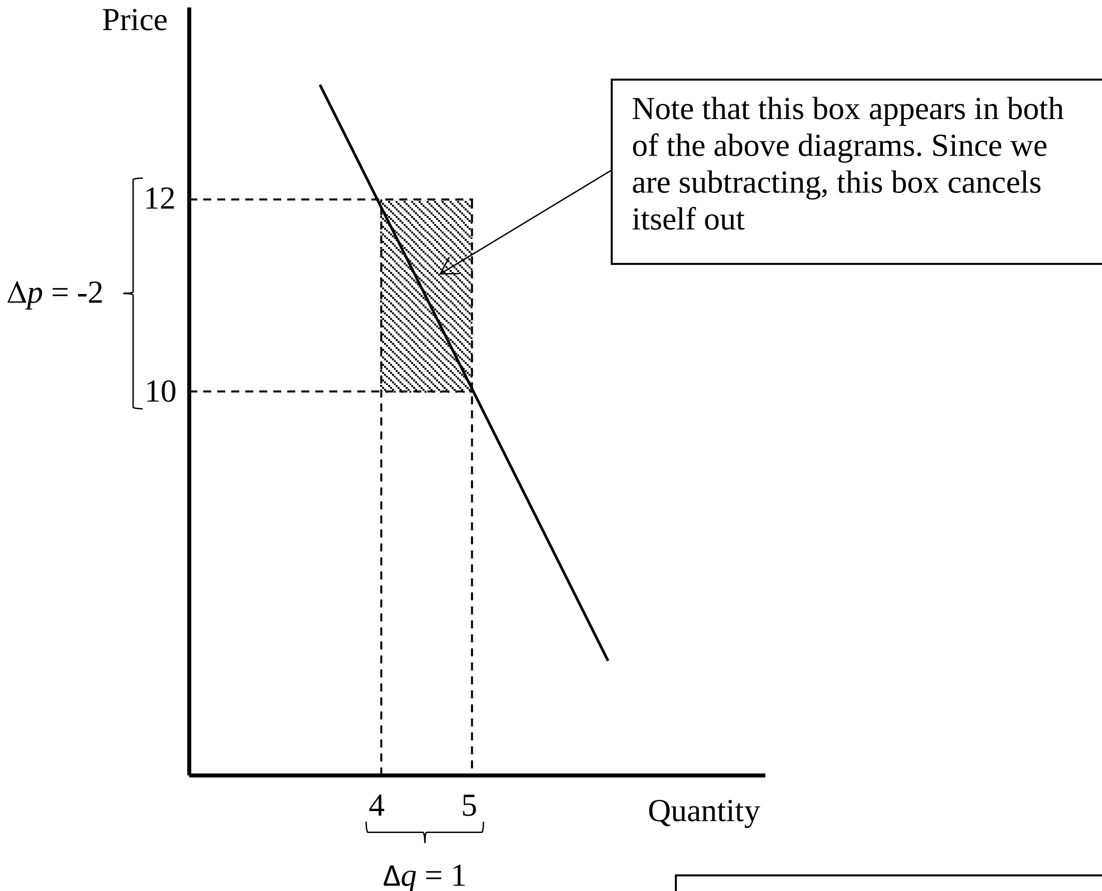
$$\begin{aligned} \Delta p\left[q+1\right] &= -2 \times 5 \\ &= -10. \end{aligned} \tag{12}$$

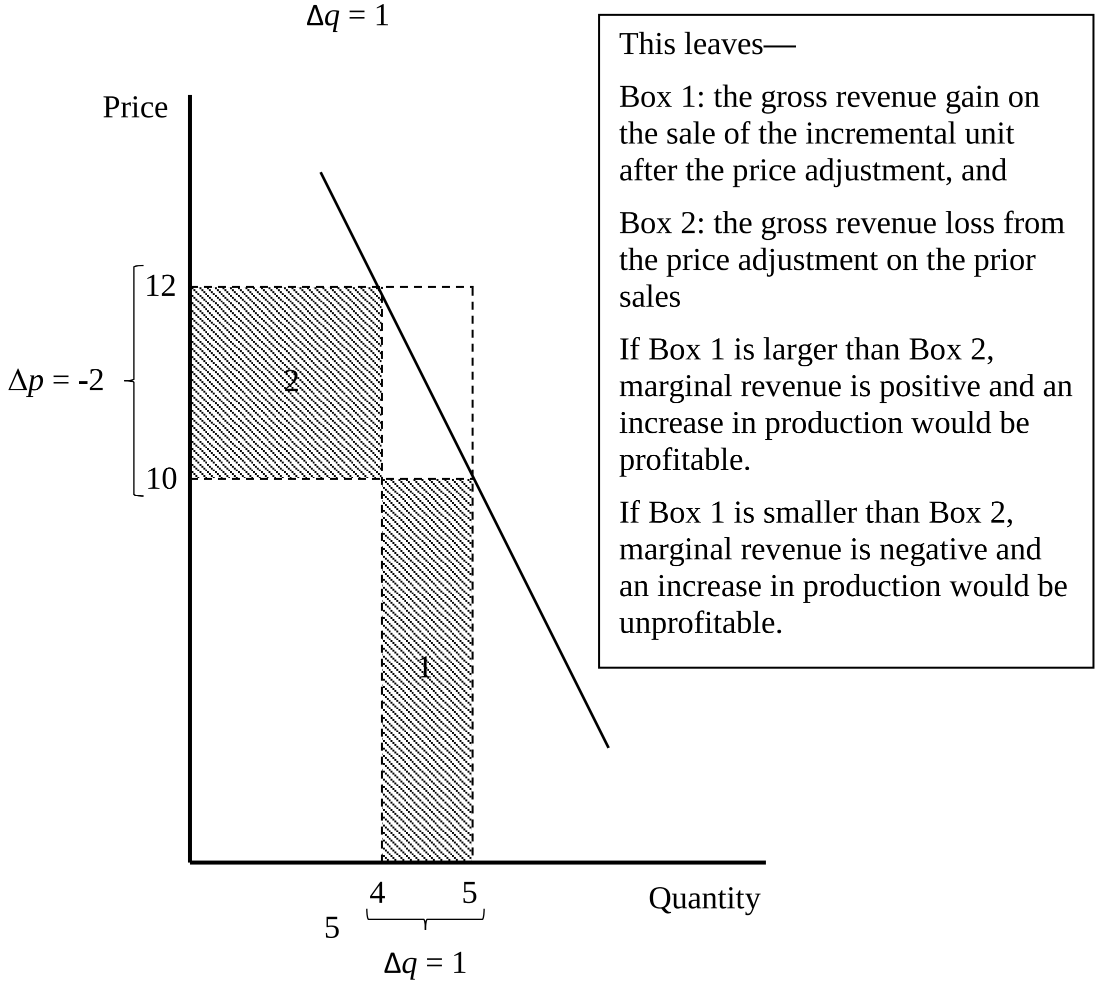
From Equation 9, marginal revenue at production level q = 4 is then the unadjusted revenue increase from the sale of an additional unit (12 from Equation 10) minus the adjustment for the price change on all units sold (-10 from Equation 12), which equals 2 (which is the same result as in Equation 7).

We can see this graphically on the next page.


Price
12
Δp = -2
10
1. Revenue gain from the sale of one additional unit without the price adjustment
= p × Δq = 12 × 1 = 12
4
5
Quantity
Δq = 1


Price
2. Revenue loss dues to the price adjustment needed to clear the market when one additional unit is produced
= Δp × (q + Δq) = -2 × 5 = -10
12
Δp = -2
10
3. Marginal revenue is the unadjusted revenue gain with the price adjustment
= 12 -10 = 2
4
5
Quantity
4 Δq = 1


Price
Note that this box appears in both of the above diagrams. Since we are subtracting, this box cancels itself out
12
Δp = -2
10
4
5
Quantity
Δq = 1


This leaves—
Box 1: the gross revenue gain on the sale of the incremental unit after the price adjustment, and
Box 2: the gross revenue loss from the price adjustment on the prior sales
If Box 1 is larger than Box 2, marginal revenue is positive and an increase in production would be profitable.
If Box 1 is smaller than Box 2, marginal revenue is negative and an increase in production would be unprofitable.
Price
12
Δp = -2
2
10
1
4
5
Quantity
5
Δq = 1